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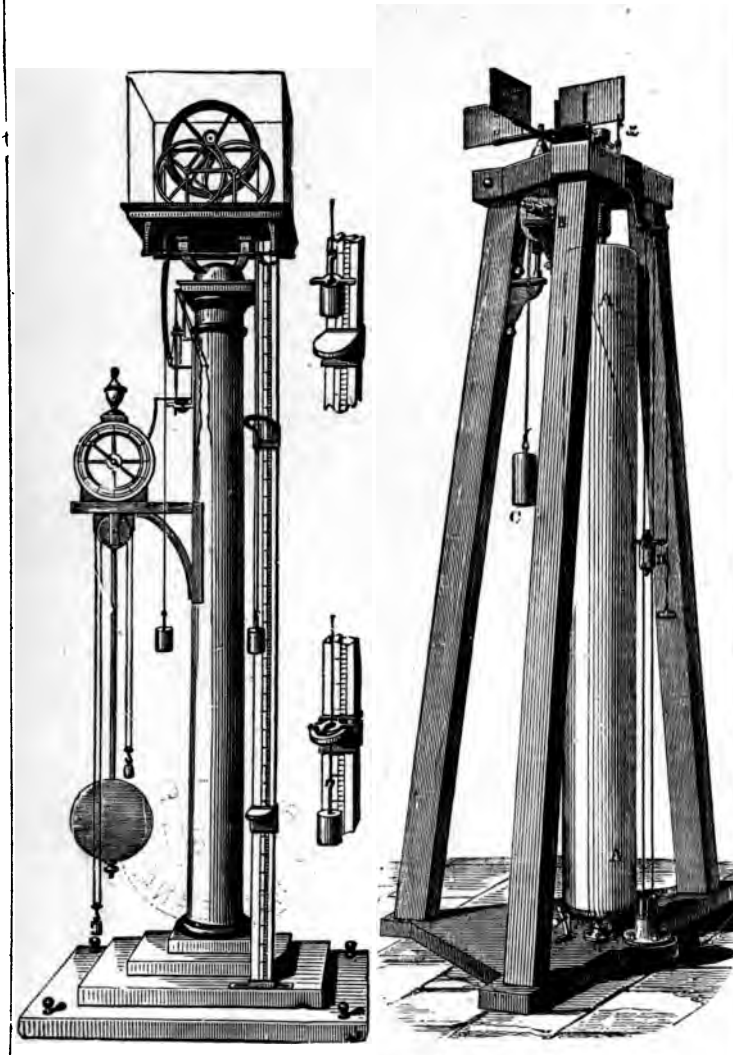

EXPERIMENTS ON THE LAWS OF MOTION.

HANDBOOK

OF

NATURAL PHILOSOPHY

BY

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MECHANICS.

NEW EDITION.

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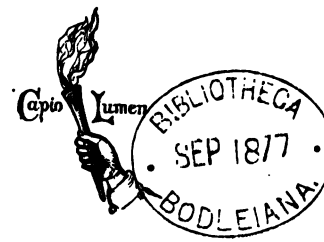

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ADVERTISEMENT TO THE NEW EDITION.

In the present edition, nearly the whole of the elementary exposition of the Principles of Mechanics of the late Dr. Lardner has been retained, but this portion has now been carefully revised and brought in accord with modern scientific terminology, wherever this was possible without detracting from the perspicuity of the original. On the other hand, portions of the descriptive chapters on machines, clockwork, etc., at the end of the work, which have been rendered nearly obsolete by the rapid progress in all modern mechanical appliances, have been cut out, and replaced by new matter, which the editor trusts will be found more serviceable to the general reader as well as to the student who enters upon a systematic study of Mechanics and Physics. With this view many additional illustrations, and a great number of worked problems on the most important statical and dynamical principles, have been interspersed in the text, in their proper places. The modern units of force

and work, and the principal definitions and notions connected with the doctrine of energy, its conservation and transmutation, have been explained and illustrated in a very elementary form, suitable to intelligent beginners.

March, 1877.

PREFACE.

THIS work is intended for all who desire to attain an accurate knowledge of Physical Science, without the profound methods of Mathematical Investigation. Hence the explanations are studiously popular, and everywhere accompanied by diversified elucidations and examples, derived from common objects, wherein the principles are applied to the purposes of practical life.

It has been the Author's especial aim to supply a manual of such physical knowledge as is required by the Medical and Law Student, the Engineer, the Artisan, the superior classes in Schools, and those who, before commencing a course of Mathematical Studies, may wish to take the widest and most commanding survey of the field of inquiry upon which they are about to enter.

Great pains have been taken to render the work complete in all respects, and co-extensive with the actual state of the Sciences, according to the latest discoveries.

Although the principles are here, in the main, developed and demonstrated in ordinary and popular language, mathematical symbols are occasionally used to express results more clearly and concisely. These, however, are never employed without previous explanation.

The present edition has been augmented by the introduction of a vast number of illustrations of the application of the various branches of Physics to the Industrial Arts, and to the practical business of life. Many hundred engravings have also been added to those, already numerous, of the former edition.

For the convenience of the reader the series has been divided into Five Treatises, which may be obtained separately.

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The Five Volumes taken together will form a complete course of Natural Philosophy, sufficient not only for the highest degree of School education, but for that numerous class of University Students who, without aspiring to the attainment of Academic honours, desire to acquire that general knowledge of these Sciences which is necessary to entitle them to graduate, and, in the present state of society, is expected in all well-educated persons.

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MECHANICS.

BOOK THE FIRST.

PROPERTIES OF MATTER.

CHAPTER I.

UNITS OF MAGNITUDE—STATES OF AGGREGATION.

1. **Object of physics.**—All human knowledge falls under two great departments, popularly called *Matter* and *Mind*; philosophers, further, employ the terms External World and Internal World. The organs of sense are the instruments by which the mind is enabled to determine the existence and the qualities of external things. The effects which these objects produce upon the mind through the organs are called sensations, and these sensations are the immediate elements of all human knowledge. Matter is the general name which has been given to that substance which, under forms infinitely various, affects the senses. Metaphysicians have differed in defining this principle. Some have even doubted of its existence. But these discussions are beyond the sphere of mechanical philosophy, the conclusions of which are in nowise affected by them. Our investigations here relate not to matter as an abstract existence, but to those qualities which we discover in it by the senses, and of the existence of which we are sure, however the question as to matter itself may be decided. When we speak of 'bodies' we mean those things, whatever they be, which excite in our mind certain sensations; and the powers to excite those sensations are called 'properties' or 'qualities.' To inquire into the *properties* of matter is the province of the physicist.

2. **Properties of matter are either inseparable or particular.**—Upon reviewing those properties of bodies which the senses most frequently present to us, we observe that very few of them are essential to, and inseparable from, matter. The greater number may be called particular or peculiar qualities, being found in some bodies but not in others. Thus, the property of attracting iron is peculiar to the loadstone, and not observable in other substances. One body excites the sensation of green, another of red, and a third is deprived of all colour. A few characteristic and essential qualities are, however, inseparable from matter in whatever state, or under whatever form it exists. Such properties alone can be considered as tests of materiality. Where their presence is neither manifest to sense, nor demonstrable by reason, there matter is not. The principal of these qualities are, *magnitude*, or extension in space, and *impenetrability*.

3. **Magnitude.**—Every body occupies space; that is, it has magnitude. This is a property observable by the senses in all bodies which are not so minute as to elude them, and which the understanding can trace to the smallest particle of matter. It is impossible, by any stretch of imagination, even to conceive a portion of matter so minute as to have no magnitude.

The quantity of space which a body occupies is sometimes called its magnitude. In colloquial phraseology the word *size* is used to express this notion; but the most correct term, and that which we shall generally adopt in speaking of the magnitude of bodies, is *volume*. Thus, we say, the volume of the earth is so many cubic miles; the volume of a room is so many cubic feet.

The external limits of the magnitude of a body are lines and surfaces, lines being the limits which separate the several surfaces of the same body. The linear limits of a body are also called edges. Thus the line which separates the top of a chest from one of its sides is called an edge.

Magnitude is either *linear*, *superficial*, or *solid*.

Linear magnitude is length or distance. Length or distance is expressed numerically by means of some unit; the unit adopted being, as a matter of convenience, small or great, according to the magnitude of the length or distance to be expressed. Thus, we say, the side of this leaf is so many *inches*, the length of the room is so many *feet*, the distance from London to York is so many *miles*.

Superficial magnitude, sometimes called *area*, has length and breadth. Its quantity is expressed by so many *square inches*, so

many *square feet*, or so many *square miles*. Thus, we say, the area of this page of paper is so many *square inches*, that of the floor of this room so many *square feet*, and the entire surface of the earth is so many *square miles*.

Solid magnitude has length, breadth, and depth, height or thickness.

Its quantity is expressed by so many *cubic inches*, so many *cubic feet*, or so many *cubic miles*. Thus, we say, the volume of this book is so many *cubic inches*, the volume of this room is so many *cubic feet*, and the volume of the terrestrial globe is so many *cubic miles*.

The *solid magnitude* or *volume* of a body is the space which that body occupies or fills.

The *form*, *figure*, or *shape* of a line, surface, or solid, depends upon the relative position of its parts or limits.

Two lines may be of equal length, while they differ in form, shape, or figure. Thus, the arc of a circle and a straight line may both measure a foot in length, while their figure or shape are different. Again, two lines may have the same form or figure, but very different lengths : thus, the two straight lines which *form* the edges of this page have the same form, both being *straight*, but they have different *lengths*. In like manner, two different arcs of the same circle will have like forms but different lengths, and two arcs of different circles may have equal lengths, and will both be curved and both circular, but they will, nevertheless, have different forms, inasmuch as the curvature of one will be greater than that of the other.

Solid bodies are bounded by *surfaces*. Thus, a globe is included within its spherical surface, a cube is included within six plane square surfaces.

Surfaces are bounded by *lines*. Thus, the surface of this page is bounded by four straight lines, forming its edges ; the surface of a circle is bounded by the line called its circumference.

4. **French units of magnitude adopted in scientific investigations.**—More recently the *metre* and its subdivisions are frequently used for measuring dimensions, and in scientific investigations this unit has now been almost generally introduced.

The *metre* is very nearly the $\frac{1}{40,000,000}$th part of a meridian, that is, of a great circle supposed to be drawn through the two poles of the earth, on the surface of the latter. The metre is divided into 10 *decimetres*, into 100 *centimetres*, and into 1,000

millimetres. For linear, superficial, and solid units, we have thus the following table :—

Measures of Length.

Metre		Decimetre		Centimetre		Millimetre
1	=	10	=	100	=	1,000
		1	=	10	=	100
				1	=	10

Measures of Area.

Square metre		Square decimetre		Square centimetre		Square millimetre
1	=	100	=	10,000	=	1,000,000
		1	=	100	=	10,000
				1	=	100

Measures of Volume.

Cubic metre		Cubic decimetre		Cubic centimetre		Cubic millimetre
1	=	1,000	=	1,000,000	=	1,000,000,000
		1	=	1,000	=	1,000,000
				1	=	1,000

It may be useful to state here that the length of 1 metre is equal to 39·37079 English inches.

The cubic decimetre, when used for the measurement of liquid bodies, is called *litre.* A litre is thus equal to 1,000 cubic centimetres.

5. **Magnitude is unlimited in its increase as in its diminution.**—There is no magnitude so great that we cannot conceive a greater, and none so small that we cannot conceive a smaller. The diameter of the earth measures about 8,000 miles ; but it is very small compared with the diameter of the sun, which measures nearly 900,000 miles ; and this, again, is itself very small compared with the distance between the earth and the sun, which measures little less than 100,000,000 miles ; and even this last space, great as it is, vanishes to nothing, compared with the distance between the sun and the fixed stars.

In like manner, there is no limit to the diminution of which magnitude is susceptible. There is no line so small that may not be bisected, or halved, or reduced, in any desired proportion. Let P 1 (*fig.* 1) be a line of any proposed length, as, for example, the tenth of an inch.

Draw P O at right angles to it, and a line 1, 10 parallel to P O. Take upon this line distances 1, 2, 3, 4, &c. successively equal to P O : the line O 2 will cut off half P 1 ; O 3 will cut off one-third of P 1 ; O 4 will cut off one-fourth of it ; O 5 one-fifth, and so on. Now, as there is no limit to the number of equal parts that may be taken successively on the parallel 1, 10, so there is no fraction so minute that a more minute one may not be cut from the line P 1.

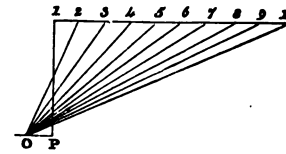

Fig. 1.

6. **Bodies consist of parts.**—All bodies consist of component parts, similar in their qualities to the whole, and into which, as will presently be shown, they are separable or divisible. Thus, a mass of metal may be reduced to powder, by filing, grinding, and a variety of other expedients ; each particle of this powder will have the same qualities as the entire mass of metal of which it constituted a part.

These component parts of bodies are observed to exist in three distinct *states of aggregation*, which are distinguished in mechanical science by the terms *solid*, *liquid*, and *gaseous*.

7. **Solid state.**—A solid body is one of which the component parts cohere with such force that it maintains its figure, unless submitted to some action more or less violent, by which it will be fractured, bruised, or otherwise changed in form. Thus, a solid body laid upon a plane surface will rest upon it in any position, without dropping asunder by the action of the weight of its component parts.

Stones, *metals*, *wood*, are examples of solids.

8. **Liquid state.**—A liquid body is one of which the component parts do not cohere with sufficient force to prevent their separation by the mere influence of their weight. Thus, a mass of liquid placed upon a plane will separate by the separate weights of its particles, and will spread itself in a film, more or less thin, over the surface of the plane.

If placed in a vessel within which it is confined by a bottom and sides, it will flow into all the inferior parts of the vessel, and its surface will become level ; for if any part of such surface were more elevated than another, the particles forming such elevation would fall down by their own weight to the level of the others.

9. **Difference between liquid and pulverised state.**—Solids in a state of pulverisation must not be confounded with

liquids, in the properties of which they do not participate. Sand or dust consists of a great number of *small solids*, each of which has the qualities of a solid as definitely as the largest mass of rock. If such particles be examined with a microscope, they will be found to have different forms, to be bounded by surfaces and lines, and to maintain their figure in virtue of cohesion.

10. **Gaseous state.**—A body in the gaseous or aeriform state is one whose component particles not only are not held together by mutual cohesion, but appear to exert against each other a *repulsion*, in virtue of which they will separate, so that the whole mass has a power of expansion to which there is no known limit.

Atmospheric air is the most familiar example of this state of body. If we suppose a quantity of air to be confined in a cylinder, under a piston which moves air-tight, such piston being drawn upwards, the air will not rest in the bottom of the cylinder, as the same volume of liquid would do, leaving vacant the space above it, but it will expand in virtue of the repulsive force already mentioned, which prevails among its particles, and it will fill the entire space under the piston. To this expansion there is no practical limit; such a piston may be indefinitely raised, producing an indefinitely increased space in the cylinder under it, and the air will still continue to expand, so as to fill this space to whatever extent it be increased.

Atmospheric air, however, is by no means the only example of bodies in the gaseous form. Many varieties of such bodies are found existing in the material world, and still greater varieties result from experimental operations.

11. **Substances may exist in either state.**—There are numerous bodies which may exist in any of these three states of solid, liquid, or gas, according to certain physical conditions, which will be explained hereafter; and analogy renders it probable that all bodies whatever may assume, under different conditions, these several states. Water affords a familiar example of this: as ice it exists in the solid state, as water in the liquid state, and as steam in the gaseous state.

CHAPTER II.

IMPENETRABILITY—DIVISIBILITY.

12. **Impenetrability** is the quality in virtue of which a body occupies a certain space, to the exclusion of all other bodies.

This idea is so inseparable from matter, that some writers affirm that it is nothing but matter itself ; that is, that when we say that a body is impenetrable, we merely say that it is a body.

However this be, the existence of this quality of impenetrability is so evident as to admit of no other proof than an appeal to the senses and the understanding. No one can conceive two globes of lead, each a foot in diameter, to occupy precisely the same place at the same time. Such a statement would imply an absurdity, manifest to every understanding.

Even bodies in the gaseous form, the most attenuated state in which matter can exist, possess this quality of impenetrability as positively as the most hard and dense substances. Thus if we invert a common drinking-glass, and plunge its mouth in water, the water will be excluded from the glass, in spite of the pressure produced by the weight of the external water ; because the air which filled the glass at the moment of immersion is still in it, and its presence is incompatible with that of any other body. It is true, however, that in this case the water will rise a little above the mouth of the glass ; but this effect arises not from the penetrability of the air, but from another quality, viz. its compressibility, of which we shall speak further on.

The numerous examples of apparent penetration which will present themselves to all observers are only cases of displacement.

If we walk through the atmosphere, our bodies may be said in one sense to penetrate it ; but they do so only in the same manner as they would penetrate a liquid in passing through it. They displace, as they move, as much air as is equal to their own bulk.

If a sewing needle be plunged in a glass of water, it might appear that penetration took place ; but it is evident that the needle displaces a quantity of water precisely equal to its own bulk ; and if we had the means of measuring with the necessary precision the position of the surface of the water in the glass, we should find that on plunging the needle in the liquid the surface rises through a space corresponding precisely to a quantity of water equal in volume to the bulk of the needle.

13. **Divisibility.**—As all bodies consist of parts which are similar in their qualities to the whole, a question arises whether there is any limit to their subdivision or comminution. Are there, in short, any ultimate particles at which the process of division must cease ? or, to speak more correctly, are there any ultimate particles so constituted that any further division would resolve them into parts differing in quality from the entire *mass ?*

To make our meaning understood, let us take the example of the most common of all substances, water. The discoveries of modern chemistry have disclosed the fact that water is a compound body formed by the combination of two gases, called *oxygen* and *hydrogen*. These gases, in their sensible properties and appearance, are similar to atmospheric air, and have never separately assumed the liquid form; but, by certain means (see the volume on 'ELECTRICITY' in this series, page 242), they may be made to combine and coalesce, and when so combined they form water.

Hence it is certain that the liquid, water, consists of ultimate particles, or *molecules*, as they are sometimes called; which molecules are again composed of particles of the two gases above mentioned combined together, which are called *atoms*.

Now, the question is, can water be practically so minutely divided that its particles admit of no further subdivision, save and except that which would resolve them into their constituent gases oxygen and hydrogen?

To this it is answered, that, although the process of subdivision may be carried to an extent which has no practical limit, yet that by no process of art or science have we ever even approached to those ultimate constituent particles which admit of no other division save decomposition.

Nor is this unlimited divisibility and comminution peculiar to water. It is common to all substances, whether solid, liquid, or gaseous. They may all be reduced to particles of the most unlimited minuteness, and yet each of these minute particles will possess the same qualities as the largest mass of the same substance.

Although experiment fails to present us with the ultimate molecules of any substance, and still less with the atoms into which it may be resolved, yet many facts in chemistry, such as the invariability in the relative weights of the elements which combine with each other, would lead us to believe that the divisibility of bodies has an actual limit, and that the ultimate limit is reached in the *atoms of the chemical elements*.

14. **Examples of divisibility.**—Most solid bodies are capable of unlimited comminution, by a variety of mechanical processes, such as cutting, filing, pounding, grinding, &c. If a mass of marble be reduced to a fine powder by the process of grinding, and this powder be then purified by careful washing, its particles, if examined by a powerful miscroscope, will be found to consist of blocks having rhomboidal forms, and angles as perfect and as accurate as the finest specimens of calcareous spars. These

rhomboids, minute as they are, may be again broken and pulverised, and the particles into which they are divided will still be rhomboids of the same form and possessing the same character. The particles of such powder being submitted to the most powerful microscopic instruments, and the process of pulverisation being pushed to the utmost practical extreme, it is still found that the same forms are reproduced.

The polish of which the surfaces of certain bodies, such as steel, the diamond, and other precious stones, are susceptible, is an evidence at once of the limited sensibility of our organs, and the unlimited divisibility of matter. This polish is produced, as is well known, by the friction of emery powder or diamond dust, and consequently each individual grain of such powder or dust must leave a little trench or trace upon the surface submitted to such friction. It is evident, therefore, that after this process has been completed, the surface which presents to the senses such brilliant polish, and apparently infinite smoothness, is in reality covered with protuberances and indentations, the height and depth of which cannot be less than the diameter of the particles of powder by which the polish has been produced.

In the detection of matter in a state of extreme comminution, the sense of sight is infinitely more delicate than that of touch. If we rub a piece of gold upon a touchstone, we plainly see the particles of matter which are left upon the surface of the stone. The touch, however, cannot detect them.

In the preceding examples the comminution, however great, cannot be easily submitted to actual measurement. Certain processes, however, in the arts enable us to obtain exact numerical estimates of a minute divisibility which without them might appear incredible. If a thin tube of glass, being held before the flame of a blow-pipe until the glass is softened and acquires a white heat, be drawn end from end, a thread of glass may be obtained so fine that its diameter will not exceed the two-thousandth part of an inch. This filament of glass will have all the fineness and almost all the flexibility of silk, and yet a bore proportional to that which passed through the original tube will still pass through its centre. The presence of this bore may be rendered manifest by passing a fluid through it.

It has been ingeniously conjectured, that if a filament of this degree of fineness could be obtained of a material which would retain sufficient inflexibility, it might be made to penetrate the flesh without producing either pain or injury, inasmuch as its magnitude would be much less than that of the pores of the integuments.

15. **Wollaston's micrometric wire.**—In the application of the telescope to astronomical purposes, the distance between objects which are present at one and the same time within the field of view of the instrument, is measured by fine threads which are extended parallel to each other across the field of view, and which may be moved towards and from each other until they are made to pass through the objects between which we desire to measure the distance. An experiment, then, which determines the distance between these threads measures the distance between the objects.

But these threads, being placed before the eye-glass of the telescope, and therefore necessarily magnified in the same manner as the objects themselves, would, unless such filaments were of an extreme degree of tenuity, appear in the field of view like great broad bands, and would conceal many of the objects which it might be necessary to observe. It is therefore necessary to resort to the use of filaments of extraordinary minuteness for this purpose. The threads of spiders' webs are used with more or less success; but the late Dr. Wollaston invented a beautiful expedient by which metallic threads of any degree of fineness might be obtained.

Let us suppose a piece of platinum wire, one-hundredth of an inch in diameter—a fineness easily obtainable by the process of wire-drawing—to be extended along the axis of a cylindrical mould, one-fifth of an inch in diameter, the wire being thus the twentieth part of the diameter of the mould. Let the mould be then filled with silver in a state of fusion. When this is cold we shall have a cylinder of silver, having in its axis a thread of platinum the twentieth part of its diameter. This compound cylinder is then submitted to the common process of wire-drawing, during which the platinum in its centre is drawn with the silver, the proportion of their diameters being still maintained. When the wire is drawn to the greatest degree of fineness practicable, a piece of it is plunged in nitric acid, by which the surrounding silver is dissolved, and the platinum wire remains uncovered.

By this process Dr. Wollaston obtained platinum wire so fine, that 30,000 pieces, placed side by side in contact, would not cover more than an inch. It would take 150 pieces of this wire bound together to form a thread as thick as a filament of raw silk. Although platinum is the heaviest of the known bodies, a mile of this wire would not weigh more than a grain. Seven ounces of this wire would extend from London to New York.

16. **Minuteness of organic filaments.**—The natural filaments of wool, silk, and fur afford striking examples of the minute divisibility of organised matter. The following numbers show how many filaments of each of the annexed substances placed in contact, side by side, would be necessary to cover an inch :—

Coarse wool	500
Fine Merino wool	1250
Silk	2500

The hairs of the finest furs, such as beaver and ermine, hold a place between the filaments of Merino and silk, and the wools in general have a fineness between that of Merino and coarse wool.

All these objects are sensible to the touch.

It will be remembered that they are compound textures, having a particular structure, each containing very different elements, which are prepared by the processes of nutrition and secretion.

17. **Thickness of soap-bubbles.**—The optical investigations of Newton disclosed some astonishing examples of the minute divisibility of matter. A soap-bubble as it floats in the light of the sun reflects to the eye an endless variety of the most gorgeous tints of colour. Newton showed, that to each of these tints corresponds a certain thickness of the substance forming the bubble ; in fact, he showed in general, that all transparent substances, when reduced to a certain degree of tenuity, would reflect these colours.

Near the highest point of the bubble, just before it bursts, is always observed a spot which reflects no colour and appears black. Newton showed that the thickness of the bubble at this black point was the 2,500,000th part of an inch! Now, as the bubble at this point possesses the properties of water as essentially as does the Atlantic Ocean, it follows, that the ultimate molecules forming water must have less dimensions than this thickness.

18. **Other examples.**—The same optical experiments were extended to the organic world, and it was shown, that the wings of insects which reflect beautiful tints resembling mother-of-pearl owe that quality to their extreme tenuity. Some of these are so thin that 50,000 placed one upon the other would not form a heap of more than a quarter of an inch in height!

In the process of gold-beating the metal is reduced to laminæ or leaves of a degree of tenuity which would appear fabulous if

we had not the stubborn evidence of common experience in the arts as its verification. A pile of leaf-gold the height of an inch would contain 282,000 distinct leaves of metal! The thickness, therefore, of each leaf is in this case the 282,000th part of an inch. Nevertheless, such a leaf completely conceals the object which it is used to gild; it moreover protects such object from the action of external agents as effectually as though it were plated with gold an inch thick.

In the manufacture of embroidery, fine threads of silver gilt are used. To produce these, a bar of silver, weighing 180 ozs., is gilt with an ounce of gold; this bar is then wire-drawn until it is reduced to a thread so fine that 3400 feet of it weigh less than an ounce. It is then flattened by being submitted to a severe pressure between rollers, in which process its length is increased to 4,000 feet. Each foot of the flattened wire weighs, therefore, the 4,000th part of an ounce. But, as in the processes of wire-drawing and rolling, the proportion of the two metals is maintained, the gold which covers the surface of the fine thread thus produced consists only of the 180th part of its whole weight. Therefore the gold which covers one foot is only the 720,000th part of an ounce, and consequently the gold which covers an inch will be the 8,640,000th part of an ounce. If this inch be again divided into one hundred equal parts, each part will be distinctly visible without the aid of a microscope, and yet the gold which covers such visible part will be only the 864,000,000th part of an ounce.

But we need not stop even here. This portion of the wire may be viewed through a microscope which magnifies 500 times; and by these means, therefore, its 500th part will become visible.

In this manner, therefore, an ounce of gold may be divided into 432,000,000,000 parts, and each part will still possess all the characters and qualities found in the largest mass of the metal. It will have the same solidity, texture, and colour, will resist the same chemical agents, and will enter into combination with the same substances. If this gilt wire be exposed to the action of nitric acid, the silver within the coating will be dissolved, but the hollow tube of gold which surrounds it would still cohere and remain suspended.

Moreover, the representation of such small magnitudes is not beyond the limits of the skill of the mechanician. It is possible to rule lines on a plate of glass so close together that the bands of fine lines thus obtained cannot be resolved even by the most powerful microscopes. The German optician, *Nobert*, has ruled

bands containing about 224,000 lines to the inch. He regularly makes plates with bands consisting of from about 11,000 to 112,000 lines to the inch. These bands are numbered from the first to the nineteenth, and are used for microscopic tests. Photographs of the nineteenth band, containing over 112,000 lines to the inch, have been taken, and in projecting the magnified image on a screen the separate lines are distinctly visible.

19. **Composition of blood.**—The organic world affords most interesting and striking examples of the minute divisibility of matter. None can be selected more remarkable than that presented by the blood of animals, which is not, as it appears to the naked eye, a uniform red liquid, but consists of a transparent colourless fluid called *liquor sanguinis*, or blood plasma, in which innumerable small red corpuscles of solid matter float.

In different species these red corpuscles differ both in form and size. In the human blood, and in that generally of animals who suckle their young, they are circular discs, their surfaces being slightly concave, like the spectacle-glasses used by short-sighted persons. In birds, reptiles, and fishes, they are generally oval, the oval being more or less elongated in different species. The surface of the discs in these species, instead of being concave, are convex, like the spectacle-glasses used by long-sighted persons. The thickness of these discs varies from one-third to one-quarter of their diameter. Their diameter in human blood is the 3,500th part of an inch : they are smallest in the blood of the *Napu* musk-deer, where they measure only the 12,000th of an inch. It would require 50,000 of these discs as they exist in the human blood to cover the head of a small pin, and 800,000 of the discs of the blood of the musk-deer to cover the same surface.

It follows from these dimensions that in a drop of human blood which would remain suspended from the point of a fine needle there must be about 3,000,000 discs, and in a like drop of the blood of the musk-deer there would be about 120,000,000 ; yet these corpuscles are rendered not only distinctly visible to the senses by the aid of the microscrope, but their forms and dimensions are rendered apparent. Small as they are, they are divisible, and can be resolved into their elements by chemical agents.

20. **Size of animalcules.**—But these diminutive globules are exceeded in minuteness by innumerable creatures whose existence the microscope has disclosed, and whose entire bodies are inferior in magnitude to the globules of blood.

Microscopic research has disclosed the existence of animals,

a million of which do not exceed the bulk of a grain of sand, and yet each of these is composed of members as admirably suited to their mode of life as those of the largest species. Their motions display all the phenomena of vitality, sense, and instinct. In the liquids which they inhabit they are observed to move with the most surprising speed and agility; nor are their motions and actions blind and fortuitous, but evidently governed by choice and directed to an end. They use food and drink, by which they are nourished, and must, therefore, be supplied with a digestive apparatus. They exhibit a muscular power far exceeding in strength and flexibility, relatively speaking, the larger species. They are susceptible of the same appetites as the superior animals, and, though differing in degree, the satisfaction of these desires is attended with the same results as in our own species.

Spallanzani observed that certain animalcules devour others so voraciously that they fatten and become indolent and sluggish by over-feeding. After a meal of this kind, if they be confined in distilled water so as to be deprived of all food, their condition becomes reduced, they regain their spirit and activity, and once more amuse themselves in pursuit of the more minute animals which are supplied to them. These they swallow without depriving them of life, as, by the aid of the microscope, the smaller, thus devoured, have been observed moving within the body of the greater.

The microscopic researches of Ehrenberg have disclosed most surprising examples of the minuteness of which organised matter is susceptible. He has shown that many species of infusoria exist which are so small that millions of them collected into one mass would not exceed the bulk of a grain of sand, and a thousand might swim side by side through the eye of a needle. The shells of these creatures are found to exist fossilised in the strata of the earth in quantities so great as almost to exceed the limits of credibility.

By microscopic measurement it has been ascertained that in the slate found at Bilin, in Bohemia, which consists almost entirely of these shells, a cubic inch contains forty-one thousand millions; and as a cubic inch weighs two hundred and twenty grains, it follows that one hundred and eighty-seven millions of these shells must go to a grain, each of which would consequently weigh the 187,000,000th part of a grain.

All these phenomena lead to the conclusion that these creatures must be supplied with an organisation corresponding in beauty with those of the larger species.

21. **Divisibility by solution.**—If a grain of salt be dissolved in 1,000 grains of distilled water, each grain of the water will contain the 1,000th part of the grain of salt; and if a grain of this water be mixed with 1,000 grains of distilled water, the 1,000th part of a grain of salt which it holds in solution will be uniformly diffused through the latter, so that each grain of the latter solution will contain the millionth part of a grain of salt. The presence of the salt in this second solution can be detected by certain chemical tests. It is evident that this process may be continued to a still greater extent.

A grain of sulphate of copper, dissolved in a gallon of water, will impart to the whole mass of the liquid a plainly perceptible tinge of blue; and a grain of carmine will give its peculiar red to the same quantity of water. It follows, therefore, that a minute drop of such water will contain such a proportion of a grain of either of these substances as the drop bears to the gallon.

22. **Divisibility of odours.**—The sense of smell, although it does not inform us of the mechanical qualities of minute masses of matter, determines, nevertheless, their presence: thus, it is known that a grain of musk will impregnate the atmosphere of a room with its odour for a quarter of a century, or more, without suffering any considerable loss in its weight. Every particle of the atmosphere which produces the sense of the odour must contain a certain quantity of the musk.

23. **Fineness of spider's web.**—A thread of a spider's web, measuring four miles, will weigh very little more than a single grain! Everyone is familiar with the fact that a spider spins a thread, or cord, by which his own weight hangs suspended. It has been ascertained that this thread is composed of about six thousand filaments.

24. **Divisibility shown by the taste.**—The sense of taste, like that of smell, may determine the presence of matter, without manifesting, by direct evidence, anything concerning its mechanical qualities.

A portion of strychnine, so minute as to be scarcely perceptible to the sight, if dissolved in a pint of water, will render every drop of the water bitter. Now, it is evident, that in this case, the strychnine being uniformly diffused through the water, the minute portion of it above mentioned is subdivided into as many parts as there are drops of water in a pint.

In like manner, if chloride of silver be dissolved in a solution of hyposulphite of soda, and the whole be evaporated, a compound salt is obtained, of which a single grain will impart a

distinctly sweet taste to a gallon of water. Now, a gallon of water weighs 70,000 grains, and, as the effect of the salt is perceptible in each grain of the water, it follows that one grain of this salt is thus divided into 70,000 equal parts.

A small lump of sugar, dissolved in a cup of tea measuring half a pint, will sweeten the whole perceptibly. In this half-pint of tea there are 31,000 drops. Each drop, therefore, must contain the 31,000th part of the sugar dissolved, and each such drop is perceptibly sweet. But if the point of a needle be inserted in one of these drops, and withdrawn from it, a film of moisture will remain upon it, and the drop will not be visibly diminished. Yet this film of moisture will still be sweet, and will, therefore, contain a fraction of the 31,000th part of the lump of sugar, too minute to admit of numerical estimation.

25. **Proofs of the existence of molecules.**—It has already been stated that there are various reasons why we should hesitate to conclude from these various facts that the divisibility of matter is really infinite, and that there are no original constituent atoms of determinate magnitude and figure, at which all subdivision must cease; and one of these reasons, a result of chemical investigations, has been definitely pointed out. But besides this, although we are unable by direct observation to perceive the existence of molecules, or material atoms of determinate figure, yet there are many observable phenomena which render their existence in the highest degree probable, if not positively certain.

26. **Crystallisation.**—The most remarkable of such phenomena are observed in the crystallisation of salts. When salt is dissolved in distilled water, as in the preceding example, the mixture presents the appearance of a transparent liquid like water itself, the salt altogether disappearing from sight and touch. The presence of the salt in the water, however, can be established by weighing the solution, which will be found to exceed the original weight of the water by the exact amount of the salt dissolved.

Now, if this solution be heated to a sufficient temperature, the water will gradually evaporate; but this process of evaporation not affecting the salt, the remaining water will still contain the same quantity of salt in solution, and it will consequently become, by degrees, a stronger and stronger saline solution, the water bearing, consequently, a less and less proportion to the salt. The water will at length be diminished, by evaporation, to that point that a sufficient quantity does not remain to hold in solution the entire quantity of salt contained in it. When

this has taken place, each particle of water which is evaporated leaving behind it the salt which it held in solution, and this salt not being capable of being dissolved by the water which remains, it will float in such water in its solid and natural state, undissolved, just as particles of dust, or other matter not soluble in the water, would do. But the saline particles which thus remain floating in the liquid undissolved will not collect in irregular solid pieces, but will exhibit themselves in regular figures, terminated by plane surfaces, always forming regular angles, these figures being invariably the same for the same species of salt, but different for different species. There are several circumstances attending the formation of these crystals which merit attention.

If one of these be detached from the others, and the gradual progress of its formation be submitted to observation, it will be found to grow larger, always preserving its original figure. Now, since its increase must be produced by the continual accession of saline molecules, disengaged by the water evaporated, it follows that these molecules, or atoms, must have such a shape, that, by attaching themselves successively to the crystal, they will maintain the regularity of its bounding planes, and preserve the angles which these planes form with each other unvaried. In fact, they must be so shaped that the structure of the crystal they form may be built up by their regular aggregation into the form which it assumes. If one of these crystals be taken from the liquid during the process of its formation, and be broken, so as to destroy the regularity of its form, and then restored to the liquid, it will be observed soon to recover its regular form, the atoms of salt, successively dismissed by the evaporating water, filling up the irregular cavities produced by the fracture.

Two consequences obviously follow from this phenomenon: *First*, that the atoms of the salt dismissed by the water evaporated have such a form as enables them, by combination, to give to the crystals the shape which they exhibit; and, *Secondly*, that the atoms which are successively attached to the crystals in the process of formation attach themselves in a particular position, to explain which it is necessary to suppose that corresponding sides of the crystals have attractions for each other, so that the atoms of salt not only attach themselves to the sides of the crystals, but place themselves there in a particular position. In a word, we must suppose that the walls of the crystal are built with these atoms in the same manner, and with the same regularity, as the walls of a building are formed with bricks.

All these, and many similar details of the process of crystalli-

sation, are therefore, very evident indications of a determined figure in the ultimate atoms of the substances which are crystallised.

27. **Planes of cleavage.**—Closely related to crystalline form is the tendency to split along definite planes possessed by many minerals, which in each cleavable mineral species bears a constant relation to a certain face, or to certain faces, of the form in which the mineral crystallises. It is important to distinguish between fracture and cleavage, as the former is irregular, and not connected with the crystalline form of the mineral.

We must infer that these planes of cleavage are parallel to the sides of the constituent atoms of the crystals, and their directions form therefore many conditions for the determination of the shape of these atoms.

28. **Conclusions.**—It follows, therefore, from these effects, and the reasoning established upon them, that the substances which are susceptible of crystallisation consist of ultimate atoms of different figure. Now, all solid bodies whatever are included in this class, for they have severally been found in, or are reducible to, a crystallised form. Liquids crystallise in freezing: several of the gases have been already reduced to the liquid and solid forms, and analogy justifies the conclusion that all are capable of being reduced to this form.

Hence it appears reasonable to presume that all bodies whatever are composed of ultimate atoms, having determinate shape and magnitude; that the different qualities with which we find different bodies endued, depend upon the shape and magnitude of these atoms; that these atoms cannot be disturbed or changed so long as the body to which they belong is not decomposed into other elements, as we find the qualities which depend on them unchangeably the same under all the influences to which they have been submitted.

We must conclude also that these atoms are so minute in their magnitudes that they cannot be observed by any means which human art has yet contrived, but nevertheless that such magnitudes have definite limits.

29. **Matter indestructible.**—The extreme division to which bodies are subjected in many natural and artificial processes, and especially when exposed to the application of heat or fire, has naturally suggested to minds not habituated to the rigid process of scientific reasoning the idea that bodies are destructible. The ancients, instead of the modern practice of inhumation, disposed of the bodies of their dead by burning them, upon the supposition that their component parts were by such operation de-

stroyed.

The more exact reasoning of modern philosophy, however, teaches us that a power to destroy matter would be as inconceivable in a finite agent as a power to create it. It is certain that the quantity of matter which exists upon and in the earth has never been diminished by the annihilation of a single atom. That matter is in fact indestructible by any known agency has been abundantly proved by experiments. It may be asked, then, what becomes of the matter composing a body which, being subjected to the action of fire, gradually and completely disappears. The answer is, that in this, as well as in all other cases of the apparent destruction of matter, nothing takes place except its subdivision and the change of its form and position.

When a body is subjected to the action of heat, its elements are decomposed, and its constituent particles separated ; many of them combine with other particles of matter, and form new substances possessing other qualities. Thus, when coal or other fuel is burned, the carbon enters into combination with one of the constituents of the atmosphere called oxygen, and forms a gaseous substance called carbonic acid, which rises into and mixes with the atmosphere. Another element, hydrogen, combines with the same constituent of the atmosphere and forms vapour of water, which also disperses in the atmosphere.

Sulphur, which is also occasionally present in fuel, combines with the same constituent of the air, forming a gas called sulphurous acid, which also escapes into the atmosphere. Thus the entire matter of the fuel, with the exception of a small portion of incombustible matter which falls into the ash-pit, is dispersed in the air, but no destruction or annihilation takes place.

That no portion of the matter of the fuel is destroyed or annihilated can be established by the incontrovertible experimental proofs of the chemist, for by the expedients of his science all the products of the combustion which have been just mentioned can be preserved, weighed, and decomposed. The oxygen which has entered into combination with each element of the fuel can be reproduced, as well as the constituents of the fuel itself, the latter of which being weighed, as well as the incombustible ash, the weight of the whole is found to be precisely equal to the weight of the fuel which was burned and apparently destroyed.

Liquids when subjected to heat are converted into vapour, and this vapour disperses in the atmosphere, so that the liquid seems to be boiled away ; but if the vapour be preserved, as it may be in a separate vessel, and exposed to cold, it will return

to the liquid form, and its weight and measure will be found to be precisely the same as that of the liquid evaporated.

30. **Destructive distillation.**—There is a process in chemistry which is called destructive distillation. The term is obectionable, because it implies a destruction where no destruction takes place. If a piece of wood, being previously weighed, be placed in a closed retort and submitted to what is called destructive distillation, it will be found that water, a certain acid, and several gases, will issue from it, all of which may be preserved, and mere charcoal will remain in the retort at the end of the process. If the water, acid, and gases which thus escape be weighed with the charcoal, the weight of the whole will be found to be precisely equal to that of the wood which was subjected to destructive distillation.

31. **General conclusion.**—Thus various forms of matter may be fused, evaporated, or submitted to combustion; animals and vegetables may die, organised bodies may be dissolved and decomposed, but in all cases their elementary and constituent parts maintain their existence. The remains of our own bodies after death are deposited in the grave, and enter into innumerable combinations with the materials of the soil, with the vegetation which covers it, and the air which circulates above it.

Consequently, these parts enter into an infinite series of other combinations, forming parts of other organised bodies, animal and vegetable, and these, after having discharged their functions, are thrown off again, mixing with the soil, the air, or organised matter, and once more running through the round of physical and chemical changes.

The constituent atoms of matter are thus constantly performing a circle of duties in the economy of nature with infinitely more certainty and regularity than is observed in the most disciplined army or in the best regulated manufactory.

CHAPTER III.

POROSITY, COMPRESSIBILITY, ELASTICITY, AND TORSION.

32. **Component molecules not in contact.**—The volume or magnitude of a body is, as has been already explained, the *space which is* included within its external surfaces. The mass

of a body, or the quantity of matter of which it consists, is the collection of atoms or molecules which compose it. If these atoms were in actual contact, the volume would be completely occupied by the mass; but numerous results of observation prove that this is never the case. There is no body so solid or compact that it cannot, by various processes to be explained hereafter, be forced into less dimensions. Now, if its atoms were in absolute contact, and had no unoccupied spaces between them, this compression could not take place. It follows, therefore, that between the atoms or molecules which form the mass of a body there are vacant spaces in the magnitude or volume, which therefore consists partly of the spaces occupied by the atoms, and partly of the spaces which intervene between these atoms.

33. **Pores.**—These intervening spaces which separate the constituent atoms of a mass of matter are called pores; and the quality of bodies in virtue of which their constituent atoms are thus separated by vacant spaces is called *porosity*. But we must clearly distinguish between two kinds of pores; *physical pores*, where the interstices between the molecules are so small that there is no apparent want of continuity, and the molecules remain within the sphere of each other's attracting or repelling forces; and *sensible pores*, which are actual cavities between the molecules, usually filled with a substance different from that of the body. In many organic bodies these pores are a consequence of their structure, and the mode in which the cells and filaments of which they consist are connected.

The existence of sensible pores may be demonstrated by a variety of experiments. These pores are often filled with air. This is found to be the case, for example, with certain sorts of wood, with chalk, sugar, and many other substances. If a piece of such wood, chalk, or sugar be pressed to the bottom of a vessel filled with water, the air which fills the pores will be observed to escape in bubbles and to rise to the surface, the water pervading the pores and taking their place.

If a cylinder of wood be inserted in the bottom of a cup which is attached to the mouth of a glass receiver placed upon the plate of an air-pump, the cup thus placed being filled with mercury, on withdrawing the air from the interior of the reservoir the pressure of the external atmosphere will force the mercury through the pores of the wood, and it will be observed to fall in a shower of silver within the receiver.

Wood in general is lighter, bulk for bulk, than water, and will therefore float in it; but this comparative lightness is in

some cases not a property of the wood, but of the air which fills its pores. To prove this, let a piece of such wood be held beneath the surface of water contained in a vessel placed under the receiver of an air-pump. On exhausting the receiver, the air contained in the piece of wood will force its way out by reason of its elasticity, and will rise in bubbles to the surface of the water. If, when the air has been thus expelled from the pores, the pressure of the atmosphere be made to act again upon the surface of the water by opening the cock which admits air to the receiver, the water will be forced into the pores of the wood and will fill the spaces deserted by the air, and the wood will then sink to the bottom.

34. **The Florentine experiment.**—There is no substance so dense as to be divested of pores. The celebrated Florentine experiment, performed at the Academia Del Cimento, 1661, and often repeated since that time with the same result, showed that gold itself has pores sufficiently large to admit the particles of water to pass through them. A globe of gold, being completely filled with water, was closed by a screw, and submitted to a severe pressure. As a globe is the figure which within the same surface contains the greatest possible volume, any change produced in its figure by external pressure must necessarily diminish its volume. When the globe, therefore, thus filled with water, is submitted to a pressure which changes it to a form slightly elliptical, or turnip-shaped, it would necessarily contain less liquid, and either of two effects must ensue, viz., the globe must burst, or a portion of the liquid must force its passage through the pores of the gold. The latter effect ensued ; and as the globe changed its form, the water was seen collecting in a dew on the external surface of the metal. This proved that the particles of water found their way through the pores of the gold without tearing, rupturing, or otherwise doing violence to its general structure.

35. **Other examples.**—The process of filtration, so extensively used in the arts and sciences, depends on the quality of porosity. The substance through which a liquid is filtrated has pores large enough to allow the particles of the liquid to pass, but too small to permit the passage of the foreign matter suspended in the liquid, and of which it is intended to purify the liquid by the process of filtration. The most ordinary filters are soft stone, paper, and charcoal.

Animal and vegetable petrifactions furnish striking examples of porosity, since the stony substance which petrifies *them must* have been infiltrated through their mass, so as to *penetrate all their fibres.*

Mineral substances are all more or less porous. Opaque stones are in general more porous than transparent ones. Chalk and marble are formed of the same constituents, with different degrees of porosity. If water be poured on chalk, it is instantly observed passing into its innermost pores; if it be poured on marble, it rests on the surface without penetrating. Stones, however, which resist the admission of water to their pores under ordinary circumstances, will be penetrated by the liquid provided an intense pressure be used for a sufficient length of time. Thus, stones taken from the bottom of the sea, especially if the depth be considerable, are found penetrated by water to their very centre. Among siliceous minerals, such as agate and flint, there is one called Hydrophane, whose porosity is attended by a singular phenomenon. A piece of this substance, in its common state, is nearly opaque; but if it be plunged in water it is found, on withdrawing it from the liquid, to be translucent and opalescent. In this case the water penetrates the stone exactly as oil penetrates paper; bubbles of air are disengaged from its pores, which are filled with absorbed water; this increases the optical uniformity of the whole, and thus allows a certain amount of light to be transmitted.

Large mineral masses existing naturally in the strata of the earth present examples of porosity still more striking. Water percolates through the sides and surfaces of caverns and grottos, and, being impregnated with calcareous and other earths, forms stalactites, or pendulous protuberances, presenting curious appearances, with which everyone is familiar.

36. **Compressibility.**—The quality in virtue of which a body allows its volume to be diminished without diminishing its quantity of matter is called *compressibility*, when the effect is produced by the application of external mechanical force; and *contractibility* when produced by change of temperature, or any other agency not mechanical. When the volume of a body is diminished, either by compression or contraction, its constituent atoms are brought into closer contiguity, its pores are consequently diminished, and its density proportionally increased.

One of the most conclusive proofs that all bodies are porous is derived from the fact that all known bodies, whatever be their nature, are capable of having their dimensions reduced without diminishing their mass.

In general, the more porous a body is the more easy it is to reduce its dimensions by external force. Many organised bodies which are easily penetrated by fluids can be easily diminished in volume by the application of pressure; and in this

case, if they have been previously filled with fluids, these fluids are expelled by the pressure exactly as water is squeezed from a piece of sponge. Innumerable processes in the arts supply examples of this.

Wood of even the hardest kind, in its natural state, is so porous as to absorb both air and water in considerable quantities. When such wood is used in the arts, in cases where extreme hardness is required, it is previously submitted to extreme pressure, by which the fluids absorbed are expelled from the pores, the volume diminished, and the density increased. The wooden wedges used in fastening the rails of railways in their chairs are prepared in this manner.

Even the most solid stone, when loaded with a considerable weight, is found to be compressed. The foundations of buildings, and the columns which sustain incumbent weights in architecture, supply numerous proofs of this.

Malleable metals are compressed by percussion or hammering : they become thus more compact and dense. In the process of coining, medals and pieces of money are struck by a severe pressure, by which they are made to receive the impression and characters upon them more accurately than softened wax would from the pressure of the hand. Under the blow of the press they not only change their form, accommodating themselves to the characters and figures sunk upon the die, but they are at the same time compressed and rendered more dense, so that the coin or medal has a volume sensibly less than the blank piece had before it was struck.

37. **Compressibility of liquids.**—Liquids in general are less easily compressed than solids ; so much so, that practically they are regarded as incompressible.

They are, however, strictly speaking, capable of a slight compression under the operation of considerable mechanical force.

In the year 1761, the compressibility of water and other liquids was established. It was found that water, submitted to a mechanical pressure, amounting to fifteen pounds on a square inch, would be diminished in its volume by forty-five parts in a million ; that is to say, a million of cubic inches would be reduced to about forty-five cubic inches less.

In more recent experiments, a quantity of water was enclosed in a piece of cannon and submitted to a mechanical pressure amounting to fifteen thousand pounds per square inch. Under this pressure it was diminished by one-twentieth of its volume, and the cannon enclosing it was burst.

The most recent experiments on the compressibility of liquids are treated of more in detail in the treatise on 'HYDROSTATICS,' in this series, page 3, etc.

Although liquids are very little compressible, they are in a very high degree susceptible of contraction.

If a quantity of water coloured with ink or other colouring matter be included in a glass bulb connected with a tube of small bore, it will be found that, when the bulb is exposed to cold, the level of the coloured water in the tube will descend. This is an effect of the contraction which the liquid undergoes in consequence of its diminution of temperature. This contraction by cold is an universal effect on matter, which will be explained more fully in the volume on 'HEAT,' in this series.

38. **Compressibility of gases.**—Of all forms of matter the gases are the most susceptible of compression. This quality has already been briefly noticed. There appears to be no practical limit to the compression of which this form of matter is susceptible, its volume being diminished in the exact proportion of the compressing force applied to it.

This property of gases is more fully described on page 125, etc., of the treatise on 'HYDROSTATICS.'

39. **Elasticity.**—This is the quality in virtue of which a body, after having been compressed, recovers its former dimensions, on being relieved from the force which compresses it. Bodies which retain their compressed state after the force ceases to act, and do not resume their original dimensions, are said to be inelastic.

The class of bodies which affords the most striking examples of elasticity are the gases and aeriform bodies. If a quantity of air be included in a syringe under a piston, and be compressed by a force applied to the piston, on the removal of that force the air, by virtue of its elasticity, will force the piston upwards until it resumes the position from which it had been driven by the compressing force.

All liquids, when compressed, immediately recover their original dimensions when relieved from the compressing force, and therefore may be said to be perfectly elastic. The play of compressibility and elasticity, however, in the case of liquids, is so extremely limited, that for all practical purposes this form of body is treated as both incompressible and inelastic. All the theorems and results in those parts of physical science called *Hydrostatics*, *Hydrodynamics*, *Hydraulics*, etc., are based upon the principle that liquids are incompressible and inelastic; for although it be true, as has been stated, that within certain very

minute limits they are both compressible and elastic, yet these limits are so small as to produce no appreciable effects under ordinary circumstances.

Gaseous bodies are not only compressible and elastic without any practical limit, but also endued with unlimited dilatability. Thus, if a quantity of gas be included in any given volume, and this volume be augmented in any required proportion, the gas will spontaneously, and without the application of any external agency, dilate itself so as to fill the augmented volume, and this expansion will go on, no matter to what extent the volume be augmented.

The quality of elasticity is manifested in solid bodies, but in a less decided manner than in gases. In solid bodies elasticity may be developed not only by compression, but also by pulling, bending, and twisting. These effects will be discussed further on.

Caoutchouc, or india-rubber, is perhaps of all solid bodies that which has most elasticity. This quality, combined with the methods recently discovered of varying the form of this substance, has extended considerably the application of it to the useful purposes of life.

The following experiments will illustrate the quality of elasticity as found in solid bodies :—

If a flat and hard surface be smeared with a thin coating of oil, and an ivory ball be allowed to drop upon it, the ball will rebound by reason of its elasticity. On examining that part of the surface of the ball which struck the flat surface from which it rebounded, it will be found that a somewhat extensive circular space will have been stained with the oil. If the ball be brought gently into contact with the flat surface, a minute space only would be stained with the oil. Why, then, it may be asked, did a larger space receive a stain when the ball was allowed to drop with a certain force upon the surface? The answer to this is, that the force of the impact *flattens* the surface of the ball to a certain extent ; that, in virtue of its elasticity, the ball recovers its spherical figure ; and that the force with which it recovers this figure causes the rebound. The extent of the surface stained by the oil is a little, but not much, greater than the extent of the circle flattened by the impact.

If the ball be let fall from several different heights, it will be found that the circular space stained by the oil will be greater the more elevated the point from which the ball is allowed to depart. This effect is only what might have been anticipated ; the greater the height from which the ball falls, the greater will be the force of the impact, and consequently the greater will be

the extent over which its surface will be flattened, and the greater, consequently, will be the elastic force which produces the rebound.

If such an experiment be made with a ball composed of a substance softer than ivory, and equally elastic, the flattening may be rendered directly perceptible to the senses. This may be made evident by the large caoutchouc balls inflated with air, used in the plays of children. When they strike the ground, they are flattened at the surface over a circle of very considerable magnitude, and which flattening may be exhibited by pressing them on the ground by the force of the hand. This is only an exaggeration of what would actually take place in the case of a ball of ivory or glass.

Elasticity in bodies is sometimes manifested by their disposition to recover their form when disturbed by a force which does not affect their volume. For example, a plate of steel when bent would have the same dimensions which it had before the pressure, yet its elasticity will be rendered apparent by its immediately recovering its original form after the force which bends it had ceased to act. The play of springs of every form affords examples of this. When a straight bar of steel is bent into a curve, both compression and expansion of its molecules take place. The molecules which compose that side which becomes convex are forcibly drawn asunder, and those which form the surface which becomes concave are forcibly compressed. This is evident, inasmuch as the convex side becomes longer, and the concave shorter, by the change of form. The tendency of the molecules, by virtue of their elasticity, to recover their original position, causes those on the convex surface to contract, and those on the concave surface to expand, the combined effects of such contraction and expansion being the restoration of the bar to its original form.

40. **Limits of the elastic force.**—As elasticity results from a derangement of the component molecules of bodies, it will be easily understood that there must be limits beyond which such derangements cannot be produced without a permanent change in the form of the body; and there are consequently limits to the play of elasticity. These limits will be obviously different in different bodies.

In the case of the most elastic class of bodies, such for example as caoutchouc, these limits are very extensive. In ivory they are more extensive than in glass, for ivory will recover its figure after a compression which would cause the fracture of glass. The limits are narrow in the case of such metals as

lead; for although considerable compression will not cause the fracture of lead as it would that of glass, yet the derangement which such compression produces amongst the molecules of that metal is greater than their feeble elasticity can resist, and the metal accordingly takes permanently any form given to it.

41. **Elasticity of torsion.**—Elasticity is sometimes manifested by torsion. Thus, let a filament of raw silk be stretched by an attached weight and the weight be made to revolve several times in the same direction, so as to twist the silk, and then be left to itself: the fibre of silk in virtue of its elasticity will untwine itself, causing the weight to revolve in a direction contrary to that in which it was twisted originally; and this process of untwining will continue until the filament recovers its original position; but the twisting may have been continued to such an extreme as to exceed the limits of the elasticity of the silk; and in that case a permanent derangement of the molecules of the silk will take place, and it will not recover its original form.

The same effects would ensue if the weight had been suspended to a fine wire of copper, silver, or any other metal, but the limit at which the twisting would produce a permanent derangement of form, or, in other words, the limit of play of elasticity, would be different.

42. **Dilatability.**—When the extension or augmentation of the volume of a body is produced by any physical agency, such, for example, as heat, not coming under the denomination of mechanical force, it is called *dilatation*. All bodies whatever, when submitted to the action of heat, are susceptible of having their dimensions enlarged; and to this augmentation of magnitude or dilatation by increase of temperature there is no practical limit while a body maintains its state of aggregation.

Innumerable examples of the operation of this principle in the arts and sciences may be produced.

In the thermometer the dilatation of a liquid is used as the measure of the heat which produces it. This instrument consists of a glass bulb attached to a tube of small bore. The bulb and part of the tube are filled with a liquid. As the temperature to which the instrument is exposed is increased or diminished, the liquid affected by it expands or contracts in a much greater degree than does the glass in which the liquid is contained. The consequence of this is, that in order to find room for its increased volume, a portion of the liquid in the bulb is forced into the tube. The column in the ube consequently becomes longer, and its increase of length, measured by

a scale attached to the tube, becomes a measure of the increased temperature.

The dilatation and contraction of metal consequent upon change of temperature has been applied some time ago in Paris to restore the walls of a tottering building to their proper position. In the *Conservatoire des Arts et Métiers*, the walls of a part of the building were forced out of the perpendicular by the weight of the roof, so that each wall was leaning outwards. M. Molard conceived the notion of applying the irresistible force with which metals contract in cooling to draw the walls together. Bars of iron were placed in parallel directions across the building, and at right angles to the direction of the walls. Being passed through the walls, nuts were screwed on their ends outside the building. Every alternate bar was then heated by lamps, and the nuts screwed close to the walls. The bars were then cooled; and the lengths being diminished by contraction, the nuts on their extremities were drawn together, and with them the walls were drawn through an equal space. The same process was repeated with the intermediate bars, and so on alternately, until the walls were brought into a perpendicular position.

43. **General effects of dilatation and contraction.**—Since there is a continual change of temperature in all bodies on the surface of the globe, it follows that there is also a continual change of magnitude. The substances which surround us are constantly swelling and contracting under the vicissitudes of heat and cold. They grow smaller in winter, and dilate in summer. They increase their bulk on a warm day, and diminish it on a cold one. These curious phenomena are not noticed only because our ordinary means of observation are not sufficiently accurate to appreciate them. Nevertheless, in some familiar instances, the effect is very obvious. In warm weather the flesh swells, the vessels appear filled, the hand is plump, and the skin distended. In cold weather, when the body has been exposed to the open air, the flesh appears to contract, the vessels shrink, and the skin shrivels.

CHAPTER IV.

SPECIFIC PROPERTIES.

44. THE qualities of matter which have been illustrated and explained in the preceding chapters, are those which are common, in a greater or less degree, to all bodies, in whatever form or under whatever circumstances they may exist. There remains to be noticed another group of properties which may be denominated for distinction, specific properties, being found in some species of matter and not in others, or at least varying in degree so extremely in different sorts of bodies as to give them specific characters.

45. **Elasticity, brittleness, and hardness.**—Although the property of elasticity in its general sense may be considered as one which, in various degrees, is common to all bodies, yet it is manifested in so peculiar a manner in bodies of different forms, that it may be not incorrectly considered as giving them a specific character. It is intimately connected with another mechanical quality which may be called *hardness*. This quality consists in a certain degree of coherence, by which the constituent molecules of a body keep their relative position so as to resist any force which tends to change the figure of the body.

Hardness is distinct from *density*, as we frequently find the most dense bodies possess this quality in a much less degree than lighter substances. Glass, for example, is harder than gold, or even than platinum, which is still harder and denser than gold. A piece of glass will scratch the surface of gold or platinum, an effect which shows that the particles of gold or platinum yield and are displaced more easily than those of glass.

Again, in comparing different species of metals one with another, their hardness is evidently independent of their densities. Gold and platinum, the most dense of metals, are softer than iron or zinc, which are much lighter. Among the hardest of the metals are iron, zinc, copper, manganese, nickel, titanium, and palladium. The softest of the common metals is lead, but the so-called alkaline metals, such as potassium and sodium, are so soft as to yield under the finger like putty.

Glass cannot be much altered in form without breaking. Such bodies are called *brittle*. But if glass be bent, and is not

broken, it immediately recovers its previous form, when left to itself. Brittleness and elasticity are therefore by no means opposite properties, but a body may possess both at the same time. The elasticity of glass is well shown by a narrow long strip of window-glass, but much better by a spiral made of a glass cylinder. Such a spiral, *fig.* 2, may be pulled at both ends with thumb and forefinger and the single coils will separate as far as $\frac{1}{10}$th of an inch or more from one another without breaking; if the strain ceases, they close again completely together.

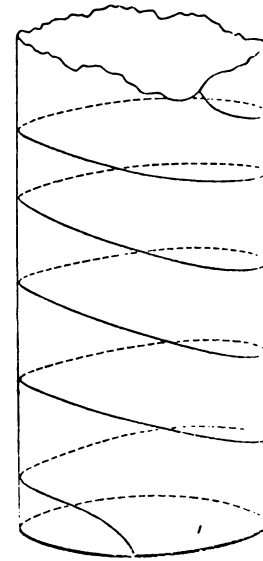

Fig. 2.

46. **Modifications of elasticity and hardness in the same body.**— Some metals are capable of having their structure modified without undergoing apparently the least chemical change, and the consequence is that their hardness or elasticity suffers a change of degree. Thus steel, when heated, and then suddenly cooled by being plunged in cold water, becomes harder than glass; but if it be cooled more gradually, then it becomes soft and flexible.

Elasticity manifests itself in various ways according to the form and character of the body to which it belongs. The elasticity of a flat and thin bar of steel is manifested by the force with which it will recover its figure when bent by lateral pressure; the elasticity of an ivory ball is manifested by the force with which it will recover its figure when flattened by impact against some hard surface. In the case of a steel spring, the body yields to a slight pressure, readily changing its forms; in the case of an ivory ball, the body does not yield to mere pressure, and requires the force of impact to produce change of form.

When the force with which a body recovers its form is equal to the force by which its form has been changed, the elasticity is said to be perfect. Thus, if a bent spring recover its position when relieved from the force which bent it with an energy equal to such bending force, then the spring is said to be perfectly

elastic; but when the restoring force is less than the bending force, the elasticity is imperfect. In the same manner, if an ivory ball flattened by a blow recover its form with a force equal to that of the blow which flattens it, the elasticity is perfect, but otherwise imperfect.

It is more exact to say that no body whatever is, in an absolute sense, either perfectly elastic or perfectly inelastic. All bodies possess some degree of elasticity, however small; but some bodies, such as the gases, for example, manifest elasticity in so high a degree, and others, such as the liquids, in a degree comparatively so small, that not only, in popular language, is the one considered elastic and the other inelastic, but it has been found convenient to assume hypothetically these two qualities of perfect elasticity and perfect inelasticity as the bases of those divisions of physical science in which the laws which regulate the phenomena of liquids and gases are discussed.

It has been already shown, however, that liquids themselves admit of some compression, and it may be added that they recover their volume with a force sensibly equal to the compressing force; and to this extent, and in this sense, they are therefore perfectly elastic.

The quality of elasticity is intimately connected with that of hardness; so much so, that it has sometimes been said that one quality is proportional to the other. This is, however, erroneous. Many of the gums, and eminently that called caoutchouc, are highly elastic, and yet these substances are among the softest of the solids. The elasticity of caoutchouc is nearly perfect, and yet this substance, especially when it is warm, has great softness. On the other hand, glass, flint, marble, ivory, afford examples of solids in which hardness is combined with great elasticity.

Putty, wet paste, moist clay, and similar bodies afford examples of substances nearly deprived of elasticity. The figure of any of these may be changed by pressure or by impact, and no tendency to recover the figure so changed is perceptible.

Sound, as will be explained hereafter, is produced by vibration imparted to the air by some solid body which is itself in a state of sympathetic vibration. It is obvious, therefore, that the metals best suited for bells, and other forms of matter intended to produce sound, must be those which are most elastic.

The hardness and elasticity, and several other physical properties, of metals are affected in a striking manner by their combination. It often happens that two metals, neither of *which is* eminently hard or elastic, produce by their combina-

tion in certain proportions, one which possesses these qualities in a high degree. Thus, bells formed of pure copper, or of pure tin, will have little sonorous quality; but if these two metals be united in a certain proportion, their combination will give a beautiful musical sound. The compound of the two metals which has this quality is accordingly known as *bell metal.*

47. **Flexibility and brittleness.**—When a body easily yields, and changes its form in obedience to a force exerted at right angles to its length, as, for example, when a bar being supported at the middle is pressed upon the ends, it is said to be *flexible*; but if, upon the action of such a force, instead of yielding and changing its form, it breaks, it is said to be *brittle.*

Flexibility and brittleness are specific qualities which bodies possess in an infinite variety of degrees.

In general, brittleness is connected with hardness, but is not, as has been shown in art. 45, and as might at first appear, at all inconsistent with certain forms of elasticity. Glass, for example, which is highly elastic, is also one of the most brittle of known substances. Brittleness, like hardness and elasticity, is a quality which the same body may acquire, or be deprived of, according to certain conditions to which it may be subjected. Thus, the metals iron, steel, brass, and copper, if they be heated and suddenly cooled, by being plunged in cold water, will become brittle; but if, when heated, they are buried in a hot sand-bath, and allowed to cool very gradually, then they will lose their brittleness, and acquire the contrary quality of flexibility.

48. **Malleability.**—This is a quality by which the metals in general are eminently distinguished, but which they possess in extremely different degrees. It is a property in virtue of which a substance admits of being reduced to thin plates or leaves under the blow of a hammer, or the intense pressure of rollers. No process is of more extensive use in the arts. In large iron works, great lumps of metal at a white heat, but still solid, are taken from the furnace, stuck upon the end of a long bar of iron, and placed under a sledge hammer of enormous weight, which rapidly strikes them, and reduces them to an elongated form approaching to that of an iron bar. The metal, being still red hot, is then passed between rollers, which are formed to the shape of the transverse section of the rails used on our railways. When pressed between and drawn through these rollers, the rail has acquired its proper form, but is still red and soft; and when received from the rollers is so flexible, that it bends by its own weight like a rod of wax. It is then laid on a flat surface, where

it cools and hardens, and assumes the condition of the rails on which we travel.

The malleability of bodies depends on the combination in them of the qualities of tenacity and softness. Without softness, they could not yield either to the impact of the hammer or the pressure of the roller; without tenacity, they would be fractured by the severe process of their fabrication.

The most malleable of the metals are gold, silver, iron, and copper.

The malleability of a metal varies in degree according to its temperature. There are certain temperatures in which this quality exists in the highest degree. Iron is most malleable when it first attains the white heat which follows the red; zinc becomes malleable at a much lower temperature, possessing this quality in the greatest degree between 300° and 400° on Fahrenheit's scale. Some metals possess the quality of malleability in so slight a degree as to be in this respect specifically different from metals in general: among them may be mentioned antimony, arsenic, bismuth, and cobalt, all of which are brittle. The metals may be rendered brittle as they are rendered hard, by being heated and then suddenly cooled, in which case they lose their malleability. This quality, however, may always be restored by again heating them and cooling them gradually, as before described.

49. **Annealing.**—This process of gradual cooling, which is of great importance in the arts, is called *annealing*.

Metals are also rendered brittle, and deprived of their malleability, by constant hammering. Thus, a bar of iron may be hammered until it entirely loses its flexibility. In this case, as before, the malleability may be restored by heating and annealing.

50. **Welding.**—Metals which are highly malleable admit of being united, piece to piece, by the process called *welding*. In this process, the two pieces of metal are raised to that heat at which they are most malleable, and the ends being laid one upon the other, are rapidly beaten by a welding-hammer. The particles are thus driven into such intimate contact, that they cohere, and form one uniform mass. Different metals may in some cases be thus welded together.

51. **Ductility.**—The property in virtue of which a metal admits of being drawn into wire, is called *ductility*. This quality is also eminently specific, being possessed by some sorts of metal in a very high degree, while others are entirely destitute of it.

Ductility is a quality which must not be confounded with

malleability; for the same metals are not always ductile and malleable, or, at least, do not possess these qualities in the same degree. Iron possesses ductility in a much greater degree than it possesses malleability, for it admits of being drawn into extremely fine wire, though it cannot be beaten into extremely thin plates. Tin and lead, on the other hand, are highly malleable, being capable of being reduced to extremely attenuated leaves; but they are not ductile, since they cannot be drawn into small wire. Gold and platinum possess both ductility and malleability in a high degree. Gold has been drawn into wire so fine that 180 yards' length of it did not weigh more than one grain, and an ounce weight would consequently extend over fifty miles.

52. **Tenacity.**—The property in virtue of which a body resists the separation of its parts, by extension in the direction of its length, is called *tenacity*. This manifestation of strength must be carefully distinguished from that of which the absence or feebleness is expressed by brittleness. The one form of strength may exist in the highest degree in a body in which the other is in the lowest degree. A thin rod of glass, if laid at its middle point on any support, will be broken by the slightest force pressing on its ends; but the same rod, if suspended by one end in a vertical position, will sustain an immense weight attached to the lower end without being broken. It has at once great brittleness and great tenacity; while its longitudinal strength is considerable, its lateral strength is almost nothing.

Different bodies vary extremely in their tenacity. Experiments have been made on an extensive scale for determining the tenacity of those bodies most used in the arts. The tenacity of metals has been tested by suspending a weight from the end of a wire.

In the following table, the greatest weights are given which were found to be supported by wires of the different metals, having a diameter of $\frac{655}{10000}$ths of an inch.

	Weights supported.		Weights supported.
Iron .	549·250 lbs.	Gold .	150·173 lbs.
Copper .	302·278 ,,	Zinc .	109·540 ,,
Platinum .	274·320 ,,	Tin .	34·630 ,,
Silver .	187·137 ,,	Lead .	27·621 ,,

The process of annealing, which improves the malleability and ductility of metals, is found in some cases, as, for example, in iron, copper, and the combinations of zinc and copper, to diminish their tenacity. In organised substances, those which

possess a fibrous texture have greater tenacity than those of cellular tissue. Hence, we find that cotton has much less tenacity than thread, rope, or silk.

L'Abbé Labillardière found that threads of the following substances, having the same diameter, were capable of supporting weights in the proportion of the annexed numbers:—

Silk	3400
New Zealand Flax	2380
Hemp	1633
Flax (common)	1175
Ditto (Pita) (*Agave Americana*) . . .	700

53. **Chemical properties.**—There is an endless variety of specific properties of bodies, the exposition and investigation of which belong properly to chemistry. It will be sufficient here to notice briefly the distinctions between these qualities and those which form the proper subjects of physics.

If two substances, being mixed together, retain respectively their separate qualities, the combination is said to be a mechanical mixture. Thus, if blue and yellow powders be mingled together, the mixture will appear green; but on examining it with a microscope, it will appear to consist of large specks of matter of the colours blue and yellow, these being the particles of the separate powders retaining their distinctive qualities, which are mechanically mingled. The combination produces upon the eye a green colour, the effect of the separate particles being too minute to be separated by unassisted vision.

There are two gases, called oxygen and hydrogen, which have the common mechanical properties of atmospheric air. If one ounce weight of hydrogen be mingled with eight ounces of oxygen, the gases will interfuse and mingle; but the entire mass will retain the same mechanical qualities as before, and the separate articles will remain side by side in the mixture, exactly as did the particles of blue and yellow powder in the preceding example. But if an electric spark be passed through this mixture, a striking change will take place. The mixture will in an instant be reduced to water, or rather to vapour, which, being cooled, will be soon converted into water. In fact, the oxygen has in this case chemically united with the hydrogen, and the mass has lost the mechanical qualities which it possessed, and has acquired those of the liquid water. This change constitutes a *chemical* phenomenon.

There is a metal called *sodium*, and a gas called *chlorine*, each of which, separately, is destructive of life, if taken into the

stomach or lungs. If these two substances be brought together, they immediately combine and burst into flame. If the substance resulting from this be preserved and cooled, it will be found to be common kitchen salt, one of the most wholesome condiments, and highly antiputrescent. Thus, two ingredients possessing the most noxious properties, when combined chemically, lose those properties, and produce a substance wholly different from either of them in form and *physical* as well as *chemical* qualities.

The investigation of this and all similar phenomena belongs to the province of chemistry, which deals with all those phenomena which are attended by a change of one or more of the materials employed into new substances. On the other hand, all material phenomena which are manifested without a loss of identity in the substances involved, belong to the province of Physics.

BOOK THE SECOND.

FORCE AND MOTION.

CHAPTER V.

INERTIA.

54. **All matter inert.**—The quality of matter which stands foremost in importance in all mechanical inquiries, forming the basis of the whole theory of force and motion, is *inactivity* or inertia; and important as this quality is, there is perhaps nothing that has given rise to so many erroneous conceptions.

These errors have chiefly arisen from the adoption of an ambiguous phraseology on the part of many writers on physics.

55. **Inertia.**—At any given moment of time a body, mechanically considered, must be in one or other of the two states, rest or motion. Inertia or inactivity is the total absence of all power in the body to change its state. If the body be at rest, it cannot put itself in motion; if the body be in motion, it can neither change that motion nor reduce itself to rest. Any such change must be produced from some external cause independent of the body.

The phrase *vis inertiæ*, or force of resistance, used in many treatises on physics, has been a fertile source of error. Such a phrase implies a disposition in matter to resist being put in motion when at rest. Now no such disposition is found to exist; and if it did exist, it would be as utterly incompatible with the quality of inactivity as is the power to produce spontaneous motion.

Innumerable effects which fall daily under our observation prove to us the inability of mere matter when at rest to put itself in motion, or when in motion to augment its speed; but,

on the other hand, we have not the same direct and manifest evidence of its inability to destroy or diminish any motion which it may have received ; and it happens, therefore, that while few will deny to matter the former effect of inertia, many will at first doubt or fail to comprehend the latter.

Philosophers themselves, so late as the epoch signalised by the writings of Bacon, held it as a maxim that matter is more inclined to rest than to motion ; and this being so, we cannot be surprised to find those who have not been familiar with physical science still slow to believe that a body once put in motion would continue for ever to move in the same direction and with the same speed, unless stopped by some external cause.

But a careful examination of the circumstances which affect the movement of the bodies around us with which we are most familiar will soon convince us that, in every case in which we observe the motion of those bodies gradually diminished, or entirely destroyed, such effects arise, not, as has been erroneously supposed, from any natural disposition of the bodies themselves to be retarded or brought to rest, but from the operation of causes of which there is no difficulty in rendering an account.

In some works on physics an experiment is mentioned as an example of the effect of inertia, and often explained on principles somewhat erroneous. A card being placed on the top of the finger, and a coin placed on the card, a sudden blow being given with the back of the nail to the edge of the card, it will be projected from its place between the coin and the finger, the coin remaining unmoved on the finger.

This has been explained by stating that the inertia of the coin is comparatively so great, that the friction produced between it and the card is insufficient to move it from its place.

If by these words it be understood that the coin resists the force exerted upon it by means of the friction, it is erroneous, and would be incompatible with that quality of inertia to which the effect is ascribed.

The correct explanation of the experiment is as follows. A part of the momentum given to the card by the blow is communicated to the coin in consequence of the resistance to the motion of the card produced by the friction which takes place between it and the coin. But the coin contains comparatively so much matter, that this moving force, when distributed among its component particles, which it will necessarily be, will give to the whole coin a velocity in the direction of the motion of the card incomparably smaller than that of the card, and so

small that the resultant of the forces produced by the weight of the coin upon the finger and this force scarcely deviates from the direction of the weight of the coin : consequently, although the coin remains on the finger, it does not remain precisely in the same position over the finger which it had when it rested on the card ; its position will be changed in a slight degree in the direction of the motion of the card.

When a stone is rolled along the surface of the ground, the inequalities of its form, as well as those of the ground on which it moves, present impediments which gradually retard its movement, and soon bring it to rest. Render the stone round and smooth, and the ground level, and the motion will be considerably prolonged ; a much longer interval will elapse, and a much greater space will be traversed, before it will come to rest. But asperities more or less considerable will still remain on the surface of the stone, and on the surface of the ground. Substitute for it a ball of highly polished metal, moving on a highly polished steel plane truly level, and then the motion will continue for a very long time.

But, even in this case, asperities will remain on the surface of the moving body, as well as on the surface on which it moves, which will gradually destroy the motion, and ultimately bring it to rest.

But, independently of the obstructions to the motion of bodies arising from the friction of the surfaces which move in contact with each other, all motions which take place on or near the surface of the earth are necessarily made in the fluid medium of the atmosphere. This fluid, however attenuated, still offers considerable resistance to the motion of bodies through it. An extensive flat surface spread at right angles to the direction of the motion will thus meet a powerful resistance.

This resistance arises from the body moved being compelled to push out of its way a volume of air proportional to the extent of the surface which the body presents in the direction of the motion. If on a calm day an open umbrella be carried with its concave surface presented in the direction in which we are moving, a powerful resistance will be encountered, which will increase with every increase of speed.

56. **Proofs and Examples of Inertia.**—As these causes of resistance to the motion of bodies are everywhere present on and near the surface of the earth, we are unable, by direct experiment, to establish the proposition that a body when once put in motion would continue for ever to move in the same *direction, and with* the same speed, if undisturbed ; but astro-

nomical observations supply an immense mass of evidence to establish this principle. In the heavens we find a vast apparatus, every movement of every part of which establishes incontrovertibly the inertia of matter, inasmuch as the reasoning by which all these motions are explained, and by which all these phenomena are predicted, is based upon the fundamental principle of the complete inertia of matter. The celestial bodies, removed from all the casual obstructions and resistances on the surface of the globe which disturb our reasoning, roll on in their appointed paths with unerring regularity, preserving undiminished all that motion which they received at their creation from the hand which launched them into space. These phenomena alone, unsupported by other reasoning, would be sufficient to establish the quality of inertia; but, viewed in connection with the other circumstances already mentioned, and with the whole super-structure of mechanical science, leading to innumerable truths verified by daily and hourly experience, no doubt can remain that this important principle is a universal law of nature.

The following examples will illustrate the quality of inertia:—

If a horse or a vehicle of any kind moving with considerable speed be suddenly stopped by any cause which does not at the same time affect the rider or those who are transported by the vehicle, then the body of the rider, or those who are transported, still retaining the progressive motion of which the horse or vehicle is suddenly deprived, will be projected forwards; and unless some means of resistance be adopted, the rider will be thrown over the head of the horse, and the passengers thrown forwards from the vehicle.

In the same manner, if a horse or vehicle being at rest be suddenly started forwards with considerable speed, the rider, or the persons placed upon the vehicle, not being as suddenly affected by the same forward motion will be thrown backwards.

In both these cases the effects are the consequence of the quality of inertia. In the one case they manifest the tendency of the bodies to continue the motion they have already received, and in the other they manifest the disposition of the same bodies to continue at rest.

If a passenger in a carriage which moves with considerable speed leap to the ground, he will fall in the direction in which the carriage is moving; for in descending to the ground his entire body will still retain all the progressive motion which it had in common with the carriage. When his feet touch the ground, they and they alone will be suddenly deprived of this

progressive motion, which being retained by the remainder of his body, he will fall as if he were tripped by some object impeding his motion, in the direction of the carriage.

The sport of coursing presents many amusing and instructive examples of the force of inertia. From the movements of the hare, one might suppose that it is, indeed, an expert mechanical philosopher. The hound which pursues it being a comparatively heavy body, and moving at the same or a greater speed, cannot suddenly arrest its course, because, in virtue of its inertia, it has a tendency to proceed forward in the same straight line. The hare, a comparatively light body, and moreover being prepared for the evolution, first gradually retards its motion so as to diminish the effects of inertia, and at the moment when the hound, urged to its extreme speed, is in the act of seizing the game, the hare dexterously turns at an acute angle to its former course, leaving the hound propelled forwards in the direction in which it was previously moving.

Thus, if the line A B (*fig.* 3) represent the direction in which the hound was pursuing the hare, the hare, having arrived at the point C, suddenly turns in the direction C D ; while the hound, unprepared for the trick, and hurried forward by the inertia of its motion, is carried on in the direction A B to the point B, while the hare has passed along the line C D to the point D. The distance now between the hound and the hare is the line B D, the base of the obtuse angle formed by two lines, C B and C D, simultaneously moved over by the hound and the hare.

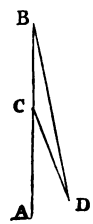

Fig. 3.

On a larger scale inertia finds many useful applications. Thus, in the corn warehouses in Liverpool, the corn

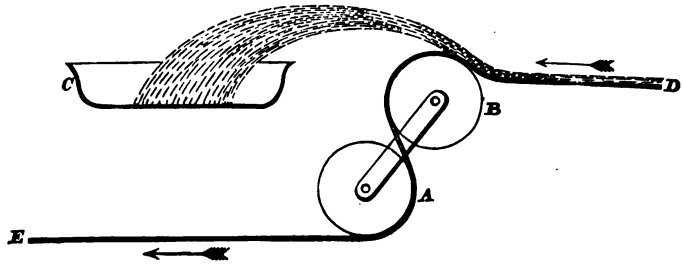

Fig. 4.

is carried upon a flat plain band, 18 inches broad, and made of

canvas or indiarubber. The point to be observed is the method of diverting the grain from one path into another during its passage. At the point where the change of path occurs, the carrying band D E is bent a little upwards, as shown at B in *fig.* 4. The result is that the stream of grain retains the velocity which is given to it by the band, and is carried forward in a jet over the top of B, just as if it were a stream of water. The spout C diverts the corn into a new channel, and may pass it on to another travelling band for transport in a new direction.

CHAPTER VI.

THE COMPOSITION AND RESOLUTION OF FORCES.

57. **Definition of force.**—Any agency or cause, which modifies or tends to modify the state of rest or motion of a body is called a *Force.*

To determine a force with precision, three things are necessary :

First, the point of the body to which it is applied, technically called its *point of application; Secondly*, its *intensity*, or *magnitude;* and *Thirdly*, its *direction.*

58. **Force expressed by weight.**—It is in general convenient and customary to express the intensity or magnitude of forces by equivalent weights. Weight is the kind of force with which we are most familiar. Everyone is acquainted with the effect produced by the pressure of a given weight; and whatever be the force whose intensity or magnitude it is required to express, a weight may be named which would produce the same effect. Thus, if a piece of iron, attracted by a magnet, be resisted by any surface, it will press against this surface with a certain force. A weight may in this case be named, which, being placed in the dish of a balance, would press upon the surface of the dish with the same force. The intensity of the attraction of the magnet on the iron would then be expressed by the amount of such an equivalent weight.

59. **Direction.**—When a force applied to any point of any body causes that point to move, the direction of its motion is the direction of the force. If the force do not produce motion,

but mere pressure, then the direction of the force is that in which the pressure is directed, and in which the point would move in obedience to the force, if it were free.

60. **Forces in same direction.**—If two or more forces act upon the same point, and in the same direction, their effect will be equivalent to a single force which is equal to their sum. This is so self-evident that it scarcely needs demonstration.

If a vehicle be drawn by three horses, one placed before the other, one horse pulling with a force of 80, another with a force of 60, and the third with a force of 40 lbs., then the combined action of the three horses upon the vehicle will be equal to the action of a single horse which should pull with a force of 180 lbs., which is equal to 80 lbs. + 60 lbs. + 40 lbs.

61. **Resultant force.**—A single force acting on a body which would thus produce the same motion or pressure as several forces acting together, is called technically the *resultant* of these forces. Thus, in this preceding example, the force of 180 lbs. acting on the vehicle in the same direction as the three independent forces of 80 lbs., 60 lbs., and 40 lbs., is the resultant of those three.

62. **Opposite forces.**—If two forces act upon a body in opposite directions, then the lesser of these forces will neutralise so much of the greater as is equal to its own quantity, and an effective force will remain in the direction of the greater, equal to their difference. This is also self-evident. If, for example, a vehicle be pulled backwards by a weight of 100 lbs. acting over a pulley, and that it be drawn forwards by a horse acting with a force of 150 lbs., then 100 lbs. of the horse's force will be neutralized by the weight which draws the vehicle backwards, and an effective force of 50 lbs. will remain in the direction of the horse's traction.

This principle is stated generally by saying that the resultant of two forces applied to the same point in opposite directions is equal to their difference and acts in the direction of the greater.

If any number of forces act upon the same point, some in one direction, and the others in the direction immediately opposed to it, then the resultant of such a combination of forces will be found by taking the difference between the sum of all the forces which act in the one direction, and the sum of all the forces which act in the other direction, the direction of such resultant being that of the forces whose sum is the greater.

63. **Forces in different directions.**—When two forces applied to the same point act in the directions of different and *diverging* straight lines, such as A X and A Y (*fig.* 5), then the

direction and magnitude of their resultant is not so evident as in the case just mentioned. It is indeed apparent that the combined effect of two such forces on the point A must be motion in some direction, such as A Z, intermediate between A X and A Y; but how this direction A Z divides the angle formed by the two components is not apparent.

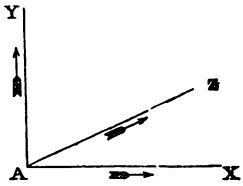

Fig. 5.

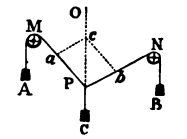

Fig. 6.

The following example, in which, as usual, weights are used to represent the forces in question, will, however, elucidate this.

Let two weights A and B (*fig.* 6) be attached to the extremities of a flexible cord which passes over two pulleys, M and N. Let another cord be knotted to this at any intermediate point, such as P; and let a third weight C be suspended from it. The weight C will then draw the cord which unites A and B into an angle M P N. The system after some oscillations will come to rest, and when it is at rest it will be evident that the point P is solicited by three forces; 1st, by the weight A acting in the direction of the line P M; 2ndly, by the weight B acting in the direction P N; and 3rdly, by the weight C acting in the direction of the line P C.

Now, it is evident that the weight C acting in the direction P C would equilibrate with an equal force acting in the opposite direction P *c*. Since, then, the weight C would precisely counterpoise an equal weight in the direction of P *c*, and that it is also in equilibrium with the weights A and B, which act in the directions P M and P N respectively, it follows that the resultant of the forces A and B acting in the directions P M and P N will be a single force equal to C acting in the direction P *c*.

It now remains to show in what manner this direction of the resultant of the two diverging forces M and N is connected with their magnitudes.

Let us suppose, for example, that the weight A is 6 oz., the weight B 4 oz., and the weight C 6½oz. If, then, we take upon the line P O a distance P *c* of 6½ inches, and if we draw two lines,

one *c a* parallel to P N, and the other *c b* parallel to P M, so as to form a parallelogram P *a c b*, we shall find, on measuring the side P *a*, that it is 6 inches, and on measuring the side P *b*, that it is 4 inches.

64. **Parallelogram of forces.**—Hence it appears, that while the diagonal P *c* consists of as many inches as there are ounces in the resultant of the two forces, the sides of the parallelogram which are in the direction of these two forces respectively consist of as many inches as there are ounces in these two forces. This result may be enunciated in general terms as follows :

If two forces acting upon the same point be represented in magnitude and direction by two lines drawn through that point, then the resultant of such forces will be represented in magnitude and direction by the diagonal of the parallelogram of which these lines are the adjacent sides.

The proposition here established by experiment is of the utmost importance in all mechanical investigations and is known as the *parallelogram of forces.* Its general truth may be independently demonstrated from the first axioms of mechanics in the following manner. The preliminary assumption necessary for the proof is that two *equal* forces, as P and P′ in *fig.* 7, which act on a particle A, have their resultant in the direction of the line R which bisects the angle P A P′. This is obvious, for there is no reason why the resultant should incline more to one force than the other. But as the diagonal of a parallelogram with equal sides bisects the angle between them, the proposition as far as the direction of the resultant is concerned will hold good in this case.

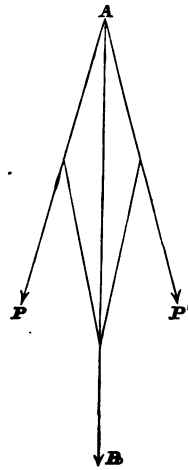

Fig. 7.

Let us now assume that the proposition holds good for two forces, p and q, in *fig.* 8, and also for two forces, p and r. It is then easy to show that it must also hold good for two forces, p and $q + r$.

Let p and q act at A in *fig.* 8 in the directions AD and AB, and be represented in magnitude by these lines respectively. We may consider the force r to be applied at B, a point in its direction, and let BC represent its magnitude. Let the parallelogram

ADFC be completed, also BE be made parallel to AD, and the diagonals AE, AF, and BF be drawn.

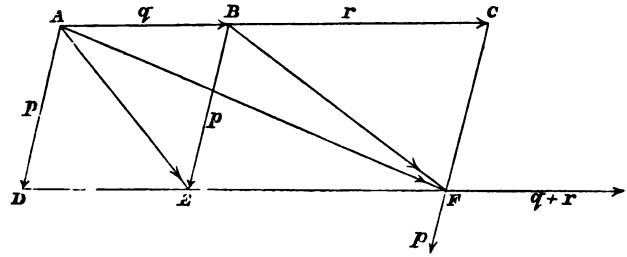

Fig. 8.

The force p acting in AD, and q in AB are by hypothesis equivalent to a force acting in AE, which we may suppose to act at the point E and to be resolved again parallel to the original directions; we have then a force p acting at E in the direction BE, and a force q acting at E in EF. We may thus suppose them to act at the points B and F in their directions. But p at B in BE, and r at B in BC are equivalent to a force in BF; therefore we may suppose all the forces to act at the point F, parallel to their original directions; and it follows that F must be a point in their resultant; or force p at A in AD, and forces q and r in AC have their resultant in the direction of the diagonal AF.

Now our assumption is true when q and r are each equal to p, therefore the proposition for the direction of the resultant is true for forces p and $2p$. Again, putting $q = 2p$, $r = p$, it is true

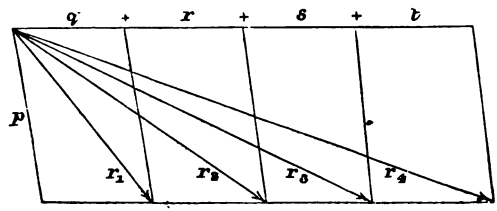

Fig. 9.

for forces p and $3p$, and so on for p and np; also putting np for p, and $q = p$, $r = p$, it is true for np and $2p$, and so on for all commensurable forces. We see also, in *fig.* 9, that the resultant lies nearer to the greater than to the weaker force; and if one

of the two forces be increased, the resultant lies still nearer to the increased force.

If the forces are incommensurable, the proposition still holds good. Let A B, A C in *fig.* 10, represent the incommensurable forces; complete the parallelogram, and draw A D; then A D is the direction of the resultant. If not, let A F be its direction. Take a quantity, A *m* which divides A B without remainder, and being applied to A C leaves a remainder less than D E. Complete parallelogram A B F G, and draw the diagonal A F. Now A B, A G, represent commensurable forces, and therefore their resultant is in the direction A F; but A E, the resultant of A B and a greater force, is nearer A B than the resultant of A B and the less force A G, which is impossible. Similarly it can be proved that no other direction than A D can be that of the resultant.

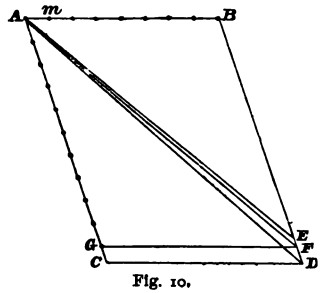

Fig. 10.

Next, to prove that the diagonal of the parallelogram represents the *magnitude* of the resultant also, when the sides respectively represent the component forces. Let P, Q, R, in *fig.* 11,

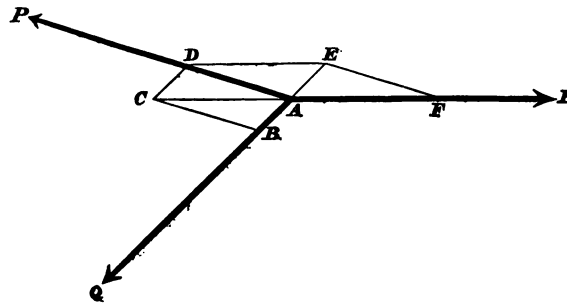

Fig. 11.

acting in the direction of the arrows, keep the point A at rest. Let the lines A D, A B, A F, represent respectively these forces. Complete the parallelograms A C, A E, and draw the diagonals. The resultant of any two of the forces must be equal in magnitude and opposite in direction to the third force, since there is equilibrium. Therefore B A E, C A F are straight lines. A E is

parallel to C D, and A C is parallel to D E, therefore A C D E is a parallelogram; hence A C = D E. But D E = A F by construction, and since A D, A B, represent P, Q, the force A C represents a force equal in magnitude to R and opposite in direction, and therefore is equal in direction and magnitude to the resultant of P and Q.

In virtue of this principle, whenever two forces in different directions act upon the same point of a body, a single force determined as above by the diagonal can be substituted for them without changing the mechanical state of the body; or, on the other hand, if a single force act upon any point of a body, two forces acting on the same point may be substituted for them, provided such forces can be represented, in quantity and direction, by the sides of a parallelogram whose diagonal represents in magnitude and direction the single force for which they are substituted. The former process is called the *composition* of forces, while the substitution of two or more forces for a single force is called the *resolution* of forces.

65. **Resultant of any number of forces.**—If any number of forces whatever act upon the same point of a body, and in any directions whatever, a single force can always be assigned which will be mechanically equal to them, and will therefore be their resultant. After what has been established, nothing is more easy than the solution of this question. Let the several single forces supposed to act upon the point in question be expressed by A, B, C, D, E, &c. 1st. Let the resultant of A and B be found by the principle of the parallelogram of forces explained above, and let this resultant be A′. 2nd. Let the resultant of A′ and C be found by the same principle, and let this resultant be B′. 3rd. Let the resultant of B′ and D be found, and let this resultant be C′; and so on. In this way we shall finally arrive at the determination of a single force, which will be equivalent to, and will therefore be the resultant of, the entire system.

66. **Forces applied at different points.**—In what precedes, we have supposed the forces whose combined effects are to be determined as applied to the same point of the body on which they act. It often happens, however, that the forces are applied to different points. We shall therefore now proceed to consider this case; and, first, we shall take the more simple condition under which the forces act in parallel directions.

67. **Parallel forces.**—As before, we shall consider the forces represented by weights.

Let P and P′ (*fig.* 12) be the points to which the two forces in question are applied, and let these two forces be represented in direction by parallel cords P M and P′ M′ passing over pulleys,

and let them be represented in quantity by two weights A and A′ suspended from these cords. Now the resultant of these two weights A and A′, or the single force which would be equal to them, may be determined by means precisely similar to those which we have adopted in the case of diverging forces —by ascertaining where a single force may be applied and what will be its amount, so as to balance the two forces A and A′.

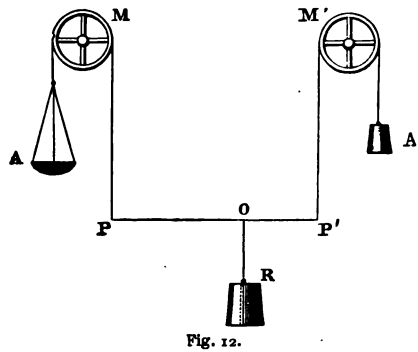

Fig. 12.

For this purpose, let us suppose a weight, R, to be suspended from a point, O, between P and P′.

Instead of suspending a determinate weight upon the string carried over the pulley M, let us suppose the dish of a balance to be suspended there, capable of receiving any heavy matter which may be placed in it. Things being thus arranged, let sand be poured into the dish A, until it is found that the three weights A, A′ and R are in equilibrium. Let us suppose, for example, that the weight A′ is 6 oz., and the weight R 10 oz. If the weight of the sand and of the scale which bears it at A be ascertained, it will be found to be 4 oz.; and it therefore follows that the sum of the two weights at A and A′ being 10 oz., is equal to the weight R. Hence it follows that the resultant of the two parallel forces A and A′ is in this case a force equal to their sum.

Now if the experiment be varied in any manner, the same result will still be obtained. Thus, if the weight A′ be 8 oz. and the weight R be 20 oz., then the weight of the sand in the dish A will be found to be 12 oz.; the sum of A and A′, 12 + 8, being still equal to R, which is 20. In a word, in whatever manner the weights A and R may be varied, so long as R is greater than A, the weight A′ will invariably be their difference; and we therefore conclude in general that *the resultant of two parallel forces acting in the same direction upon two different points of the same body, is a force parallel to their direction, and equal to their sum acting at some intermediate point.*

Now, it remains to determine what is the intermediate point between P and P′ at which this resultant will act.

After having established equilibrium by pouring the quantity

of sand into the dish A, if we measure the distances P O and P′ O, we shall find that they are invariably in the *inverse proportion* of the two weights A and A′; that is to say, if the weight A′ be 8 oz. and the weight A 12 oz., then the proportion of P O to P′ O will be 8 to 12, and this will be found to be invariably the case. If the position of the string supporting the weight R be varied, as it may be, it will always be found that the ratio of the two weights A and A′, which establish an equilibrium in the system, will be inversely as the distance of the points P and P′ from the point O, where the resultant is applied; while the distance P O represents in quantity the component A′, the distance P′ O will represent in quantity the component A.

This general principle, which is of great importance in mechanics, may be enunciated as follows :—

The resultant of two forces which act on different points of the same body in parallel lines, and in the same direction, is a single force equal to their sum acting parallel to them, and in the same direction, at an intermediate point which divides the line joining the two points of application of the components in the inverse proportion of the magnitudes of those components.

If the forces A′ and R be considered as components, the force A may be considered as the opposite of their resultant. It consequently follows that the resultant of A′ and R is a force equal in quantity to their difference, and applied at P, acting in a line parallel to them, and in the direction of the greater force R.

68. **Parallel forces in opposite directions.**—This general principle may be enunciated as follows :—

The resultant of the two forces which act on different points of the same body in parallel lines in opposite directions, will be a single force equal to their difference, and acting at a point beyond the greater of the two forces, and so situated that the point of application of the greater of the two forces will divide the distance between the lesser and the resultant in the inverse proportion of the magnitudes of the lesser and of the resultant.

Having the means of determining the resultant of two parallel forces, we can determine the resultant of any number of such forces by taking them respectively in pairs, as we have done in the cases of diverging forces. Thus, let any two forces of such a system be taken, and the resultant found. Then, considering such resultant as a component, let it be combined with a third component, and their resultant found ; and so on.

69. **Couple.**—There is a case of parallel forces which does not admit of a single resultant, and which is of considerable importance in mechanical inquiries. This case is that in which

two equal forces act upon two points of a body in parallel and opposite directions. The effect of such forces cannot be represented by any single force. In fact, such a combination of forces has no tendency to produce in a body any progressive motion, but has a tendency to cause it to revolve round a point intermediate between the direction of the two forces. Such a system of forces is called a *couple*.

70. **Mechanical effect of a couple.**—The mechanical effect of such a system depends consequently, on the intensity of the forces, the perpendicular distance between their lines of direction, and on the direction of the plane which passes through their lines of direction.

If P and P′ (*fig.* 13) be the points of application, and one of the forces acts in the direction of P M, while the other acts in the direction of P′ M′, then their effect will depend on their inten-

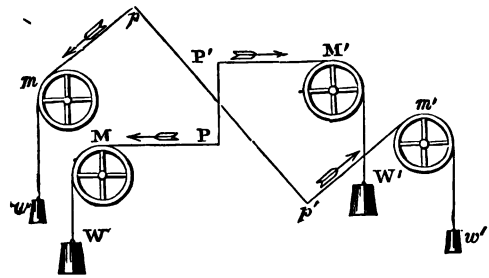

Fig. 13.

sity, on the length of the perpendicular distance P P′ between their directions, and on the direction of the plane in which the lines P M and P′ M′ lie.

Representing such forces by weights, as before, let us suppose strings attached to the points P and P′ carried over the pulleys M and M′, and supporting the two equal weights, W, W′.

The obvious tendency of these weights is to turn the line P P′ round in the direction in which the hands of a clock would move. Now this tendency cannot be counteracted by any single force, but it may be resisted by another *couple* applied to other two points of the body. Let us suppose, for example, that *p p′* be two other points of the same body, either situate in the same plane as the lines P M and P′ M′, or in any parallel plane, and that strings be applied to them extended by weights, in the same manner as in the former case, the strings lying in such parallel plane.

Let $p\ m$ and $p'\ m'$, carried over pulleys, support weights w and w', but let them be so applied to the line $p\ p'$ that they shall have a tendency to turn the body round *contrary* to the motion of the hands of a clock, and therefore contrary to the effect of the former couple. Now let the weights w and w' be so adjusted by trial, that this second couple shall exactly balance the first couple, and keep the body at rest, which may be done by using for the purpose the dish of a balance and sand, as in the former experiment. When the equilibrium is thus established, it will always be found that the weights W and w will bear to each other *the inverse proportion of the distance between the parallel cords*, that is to say, the weight W will be greater than the weight w in the exact proportion of the distance $p\ p'$ to the distance P P′; or to express this in the usual manner by algebraical symbols, we should have

$$W : w :: p\ p' : P\ P';$$

from which it follows that

$$W \times P\ P' = w \times p\ p'.$$

71. **Equilibrium of couples.**—This conclusion involves the entire mechanical *theory of couples*, and may be enunciated as follows :—

Two equal and parallel forces acting in contrary directions on a body, have a tendency to make that body revolve round an axis perpendicular to a plane passing through the direction of such two parallel and opposite forces; and such tendency is proportional to the product obtained by multiplying the intensity of the forces by the distance between their directions; and, consequently, all couples in which such products are equal and have their planes parallel are mechanically equivalent, provided that their tendency is to turn the body round in the same direction; but, if two such couples have a tendency to turn the body in contrary directions, then two such couples have equal and contrary mechanical effects, and would, if simultaneously applied to the same body, keep it in equilibrium.

72. **Conditions under which two forces admit a single resultant.**—Two forces not being parallel in their directions which are applied to different points of the same body, present two different cases, in one of which only they admit of a resultant.

1st. If the two forces applied at P and P′ (*fig.* 14), not being parallel, are nevertheless in the same plane, their directions P M and P′ M′, if prolonged, will necessarily meet at some point such as O. In this case we may imagine the two forces to be applied at O, and their resultant will be represented in magnitude

and direction by the diagonal o c of a parallelogram whose sides represent, in quantity and direction, the two forces, according to the principle already explained.

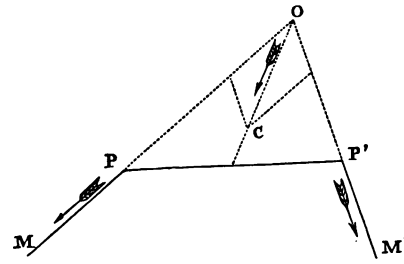

Fig. 14.

2nd. But if the forces applied at the points P and P′ (*fig.* 15), not being parallel, are at the same time in different planes, as P W and P′M′, then their directions, though indefinitely prolonged, will never intersect, and they will not have any single resultant; in other words, their mechanical effect cannot be represented by that of any single force.

73. **Two forces in different planes.**—It can be demonstrated, however, that the mechanical effect of such a system of two forces as here described, whose directions lie in different planes, and which, though not parallel, can never intersect, will be mechanically equal to the combined action of *a couple* such as already described, and a *single force*; in other words, such a system will have a double effect on the body to which it is applied: 1st, a tendency to produce revolution; and 2ndly, a tendency to produce a progressive motion; and if it were not held in equilibrium by some equal contrary forces, the body would at the same time move forward in some definite direction, and revolve round some determinate axis.

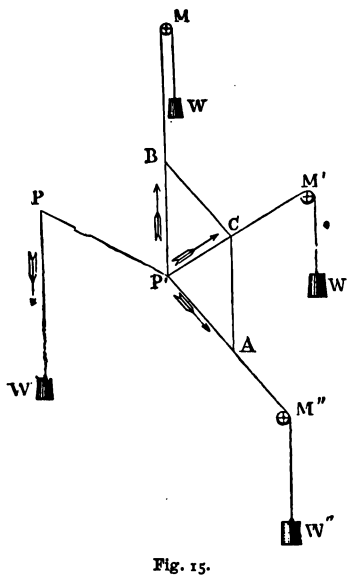

Fig. 15.

To render this intelligible, let us imagine the forces to be *represented by weights acting* on strings, and passing over pulleys.

In *fig.* 15, let P and P′ be the two points of application of the two forces; let the force acting at P be vertical, and be represented by the weight W, suspended by a string at the point P.

Let the other force applied at P′ be horizontal, and in a direction P′ M′ perpendicular to P′ P, and let it be represented by the weight W′ suspended by a cord which passes over the pulley M′. Attached to the point P′, let another string be carried vertically upwards to M, and then passed over a pulley, and let a weight be suspended to it equal to the weight W. Now take upon the line P′ M′ a distance P′ C, consisting of as many inches as there are ounces in the weight W′, or, which is the same, in the force which stretches the cord P′ C. Take also upon the vertical line P′ M as many inches P′ B as there are ounces in the weight W, and draw the line B C. From C draw C A parallel to P′ B, and from the point P′ draw P′ A, parallel to B C. Carry a string from P′ along the line P′ A, and let it pass over a pulley M″, and suspend from it the weight W″, consisting of as many ounces as there are inches in the line P′ A.

Now, according to this statement, the weight W will consist of as many ounces as there are inches in P′ B, and the weight W″ will consist of as many ounces as there are inches in the line P′ A. It follows, therefore, from the principle of the composition of forces already established, that the combined effects of these two forces W and W″ acting in the line P′ B, and P′ A upon the point P′, will be the same as the single action of the weight W′ acting in the direction P′ C upon the same point P′, and that it may be consequently substituted for the latter without changing the effects upon the body. Let us then detach the weight W′ and relieve the point P′ from its action, leaving the weights W and W″ acting in its place. The point P′ and the body to which it belongs will then be affected in the same manner by the three weights W, W, and W″ as it was by the two original weights W and W′. It follows, therefore, that the effect of the two original weights W and W′ is mechanically equivalent to the effect of the weight W″ and the two equal weights W.

This is equivalent to stating that the two forces acting at the points P and P′ in the directions P W and P′ C, are equivalent in their effect to three forces, viz., a single force, represented in intensity by the line P′ A, and acting along that line, and a *couple* acting in a vertical plane passing through the line P P′; the distance between the two forces being equal to P P′, and their intensities being equal to W.

The total effect, therefore, would be equivalent to a single

force acting in the direction P′ A, and a *couple* producing rotation round an axis perpendicular to a vertical plane through P P′.

74. **Examples.**—The principles upon which the jointed rods and links by which the floors of suspension bridges are supported depend, admit of easy explanation by the elementary principles of the composition of forces.

Supposing the floor of the bridge extending between the two quay walls to be represented by A B C D E F G (*fig.* 16), and to be sustained by the parallel and equidistant vertical rods M, N, P, Q, R, S, it is necessary that these rods severally should so support the flooring that no one part of it shall have a tendency to rise above or sink below the adjacent part; for it is evident that if any such inequality existed in the sustaining force, one of two consequences would ensue, either of which would be incompatible with the necessary conditions of such a

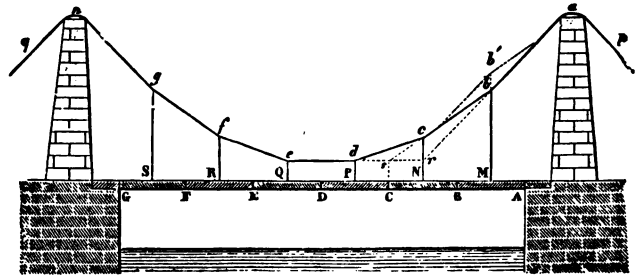

Fig. 16.

structure: if the flooring did not yield to these unequal forces, it would be subject to continual transverse strains, which would impair its strength; and if it did yield, its surface would not be level, but undulating. The condition, then, being admitted upon which the flooring is supported by the vertical rods in equilibrium in the horizontal position, we may imagine it divided at the middle points B, C, D, E, F of the successive intervals between the rods, as well as at the points A and G, where it meets the quay walls. In this way the several pieces A B, B C, C D, D E, E F, and F G are supported separately and severally by the rods M, N, P, Q, R, S. Although free to move up or down, being quite detached from each other, they will not do so, since such a derangement of their relative positions would be incompatible with the equilibrium of the forces supporting the flooring which has been supposed.

It remains now to determine the relative directions of the several links *a b*, *b c*, &c., which support the vertical rods. For this purpose we are to consider that the middle link *d e* is parallel to the flooring at a height above it altogether arbitrary, depending merely on the general height at which it may be found convenient to place the chain above the flooring. The half of the flooring extending from A to D, consisting of the three detached pieces A B, B C, and C D, may be considered as sustained exclusively by the links *a b* and *d e*; for if these last links were cut through, it is evident that the pieces suspended from the rods M, N, P, would fall. This being the case, the entire weight of the three pieces, acting vertically downwards at their centre of gravity N, will necessarily be in equilibrium with the tensions of the links *a b* and *d e*. It follows from this, by the principles of the composition of force, that if the line *e d* be continued to meet the vertical rod N at *r*, the direction of the link *a b* will be that of the line drawn from *r* to *a*; this, therefore, determines the direction of the first link *a b*. The direction of the second link *b c* is determined in precisely the same manner. We have only to consider the pieces B C and C D as being supported by the links *b c* and *d e*; and it follows, as before, that the weights of these pieces, B C and C D, acting vertically downwards from their centre of gravity C, will be in equilibrium with the tensions of the links *b c* and *d e*. If, therefore, the direction of *d e* be continued to meet the vertical through C at *s*, the line drawn from *s* to *b* will be the direction of the second link *b c*, that of the third link being necessarily the line *c d*.

The directions of the corresponding links at the other side of the centre will be the same.

The parts of the chain *p*, *q*, on the other side of the piers will be inclined to the vertical at the same angle with the links *a b* and *n g*; so that, the tensions being the same, the resultant of the two forces in each case will necessarily be vertical, and consequently the weight will be supported by the piers.

We have here considered only one chain. A bridge, however, must be at least supported by two, one at each side, and may be sustained even by four, two at each side. In such cases the force will be equally distributed among the chains, and the same principles will be applicable to determine the direction of their several links.

CHAPTER VII.

COMPOSITION AND RESOLUTION OF MOTION.

75. **Velocity and direction of motion.**—If we wish to define the state of a body which is in motion, we must state the direction in which it moves, and its *velocity*, that is the rate at which it moves or the speed which it has in such direction.

If the motion of a body be rectilinear, that is to say, if it move continually in the same straight line, then such straight line is its direction. But it is evident that a body may move in two opposite directions in the same straight line: thus, if the line of the motion be east and west, the body may move either from east to west, or from west to east, without departing from the line in question.

It is usual to distinguish the two opposite directions in the same straight line by the algebraical symbols + and −. Motion in one direction is thus defined as + or *positive*; and motion in the opposite direction as − or *negative*.

76. **Direction in a curve.**—If a body move in a curved path, such us A B (*fig.* 17), the direction of its motion is continually changed, but at any point of the curve, such as P, it is considered to have the direction of a tangent P T at that point.

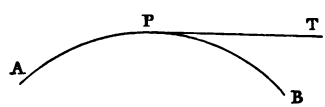

Fig. 17.

77. **Velocity defined.**—The velocity of a moving body is expressed by stating the relation between any space through which the body moves, and the time in which such motion is performed. Thus, we say that the speed with which a man walks is four miles an hour, the speed with which a stage-coach travels is ten miles an hour, and the speed of a railway train is thirty miles an hour.

It is evident that the same speed may be differently expressed, according to the different units of time or distance which may be adopted. We may express the velocity by stating the space moved over in a given time, or the time taken to move over a given space. The given time may be an hour, a minute, or a second; and the given space a mile, a foot, or an inch. If we say that a railway train moves at thirty miles an hour, or that it moves at eight hundred and eighty yards a minute, or

forty-four feet a second, we express exactly the same speed, the only difference being that different units of time and distance are adopted.

The selection of the units of time and distance for the expression of velocity is of course arbitrary. It is usual, however, to adopt such units that the velocity may be expressed by a number which is neither inconveniently great nor inconveniently small. If the motion of the body be extremely rapid, we express its velocity, by adopting a large unit of space or a small unit of time; and if, on the other hand, the motion be very slow, we express the velocity by a small unit of space or a large unit of time.

The following small table may facilitate the conversion of various units into each other :—

Comparison of different measures of Velocity.

	Miles per hour.		Feet per second.		Feet per minute.		Feet per hour.
	1	=	1·47	=	88	=	5,280
	0·632	=	1	=	60	=	3,600
	0·0114	=	0·017	=	1	=	60
	0·00019	=	0·0003	=	0·017	=	1
1 nautical mile per hour, or 'knot.'	1·1507	=	1·6877	=	101·262	=	6,075·74

As spaces or times which are extremely great or extremely small are more difficult to conceive than those which are of the orders of magnitude that most commonly fall under the observation of the senses, we shall convey a more clear idea of velocity, by selecting for its expression spaces and times of moderate rather than those of extreme length. The truth of this observation will be proved, if we consider how much more clear a notion we have of the velocity of a railway train, when we are told that it moves over fifteen yards per second, or between two beats of a common clock, than when we are told that it moves over thirty miles in an hour. We have a vivid and distinct idea of the length of fifteen yards, but a comparatively obscure one of the length of thirty miles; and, in like manner, we have a much more clear and definite idea of the duration of a second than we have of the duration of an hour.

78. **Table of Velocities.**—In the following table are collected examples of the velocities of various objects, which may be found useful for reference, and which will serve as standards of comparison for various classes of motion :—

Table showing the Velocities of certain moving Bodies.

Objects moving.	Miles per Hour.	Feet per Second.
Man walking	3	4½
Horse trotting	7	10
Swiftest racehorse	60	88
Railway train, English	32	47
,, ,, American	18	26½
,, ,, Belgian	25	36¾
,, ,, French	27	39
,, ,, German	24	35¼
Fast express train	42 to 50	62 to 75
Swift English steamboats navigating the Channel	14	20½
Swift steamers on the Hudson	18	26½
Fast sailing vessels	12	17½
Current of slow rivers	3	4½
,, rapid rivers	7	10¼
Moderate wind	7	10¼
Storm	36	52¾
Hurricane	80	117¼
Air rushing into vacuum	850	1247
Common musket-ball	850	1247
Rifle-ball	1000	1467
Cannon-ball	1090	1600
Bullet discharged from air-gun, air being compressed into a hundredth part of its volume	477	700
Sound when atmosphere is at 32° Fahr.	743	1090
,, ,, 60° Fahr.	765	1122
Earth moving round sun	68,182	100,000
A point at earth's surface at equator by diurnal motion	1040	1525

79. **Uniform velocity.**—The speed of a body in motion may be *uniform* or *varied*.

The speed is uniform when all equal spaces, great or small, are moved over in equal times.

It is possible to conceive a body in motion, moving over a mile per minute regularly, and yet the speed not to be uniform; for though each successive mile may be performed in a minute, the subdivisions of such mile may be performed in unequal times; thus, the first half of each successive mile might occupy forty seconds, and the other half twenty seconds. To constitute a uniform motion, therefore, it is necessary that all equal por-

tions of space, no matter how small they are, shall be passed over in equal times.

80. **Composition of Motion.**—As forces which produce pressure would, if the bodies on which they act were free to move, produce motion in the direction of such pressure, whose velocity would be proportional to the pressure, all the principles which have been established in the last chapter respecting the components and resultants of forces will be equally applicable to motion. Hence, without further demonstration, we may consider as established the following principles, called the composition and resolution of motion.

If a body A (*fig.* 18) receive at the same time two impulses, in virtue of one of which it would move in the direction A Y over the space A B, in one second; and in virtue of the other, it would move in the direction A X, over the space A C in one second; then the two impulses, acting upon it simultaneously, will cause it to move over the diagonal A D of the parallelogram, whose sides are A B and A C, in one second.

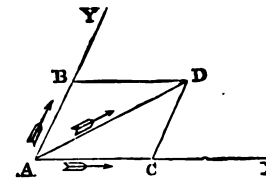

Fig. 18.

Conversely, also, a single motion A D may be considered as equal to the combination of two motions, A B and A C, along the sides of any parallelogram of which A D is a diagonal.

Now, as an infinite variety of parallelograms may have the same diagonal, it follows that any single force may be considered as equal to an infinite variety of combined forces or motions. In *fig.* 19, the line A D is the diagonal of the parallelograms A B D C, A B′ D C′, A B″ D C″, &c.

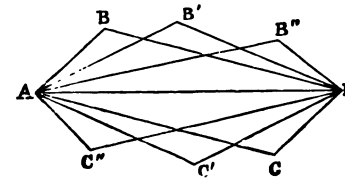

Fig. 19.

81. **Two motions in one direction.**—The effect of a force which acts upon a body in motion, must be either to change its velocity, or to change its direction, or to produce both these effects together.

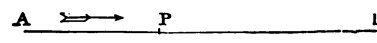

Fig. 20.

If a body being in uniform motion to a certain straight line, such as A B (*fig.* 20), receive at P the impulse of a force in the

direction in which it is moving, the effect of such impulse will be to augment its velocity ; and such increase of velocity will be exactly equal to the velocity which the same force would impart to the body, if the body had been at rest. Thus, if the body had been moving towards B at the rate of ten feet per second, and it received an impulse at P, in the direction of B, which would have imparted to it, being at rest, a velocity of five feet per second, then the velocity of the body after the impact will be fifteen feet per second. This is evident, because the previous motion which the body is supposed to have had in the direction of B cannot in any way impair the effect of a force tending to make it move in the same direction.

But if the impulse which it has received at P had been given in the direction of P A, contrary to that of its motion, then such impulse would deprive the body of just so much velocity in the direction P B, as it would have imparted to it in the direction P A had it been at rest. In this case, therefore, the velocity after the impact will be diminished from ten feet per second to five feet per second.

These consequences are obviously analogous to the corresponding conclusions, which were explained in the last chapter, respecting the combined effects of forces acting upon a body in the same or any parallel directions.

82. **Two motions in different directions.**—If a body, being in motion from A to B (*fig.* 21), receive at the point P an impact which, had it been at rest, would have caused it to move in the direction P C, then the body commencing from the point P will have a motion compounded of the motion which it had before receiving the impact at P, and the motion which that impact would have given to it had it been at rest. The motion, therefore, which it would have on leaving P in this case, will be determined by the parallelogram of forces, according to the principles already established.

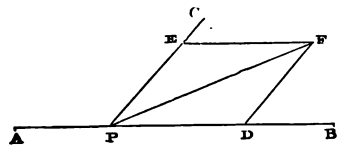

Fig. 21.

Thus, if we suppose that, by virtue of the motion which the body had along the line A B, it would have moved from P to D in a second, and that the motion which it would, being at rest at P, have received from the impact would have carried it from P to E in one second, then the motion which the body will actually have in leaving P after receiving the impact, will be

along the diagonal P F of the parallelogram whose sides are P D and P E.

If a body moving with a certain determinate velocity in the direction A B (*fig.* 22) encounter at P a fixed object that has a flat surface C D, its motion will not be destroyed, but will be modified by the resistance of this surface.

The effect of the surface, supposing it, as well as the body, to be perfectly hard and inelastic, will be to destroy so much of the motion of the body as is in a direction perpendicular to it.

To determine this, let us draw the line P M perpendicular to the surface C D, and, taking P O as the space which the moving body moves over in a second, from O draw O N parallel to P C, and O R parallel to P N, so that P N O R shall be a parallelogram, of which P O is the diagonal. The force, therefore, with which the body will strike the surface at P will be equal to the two forces, one in the direction of R P, and the other in the direction of N P, the sides of this parallelogram, inasmuch as these two forces are, by what has been already explained, equal to the single force represented by the diagonal O P. But the reaction of the surface C D will destroy the perpendicular force N P, and therefore the force R P alone wil remain in the direction R P D. The body, therefore, after it strikes the surface, will move from P towards D with a velocity, in virtue of which it will describe spaces equal to R P in one second.

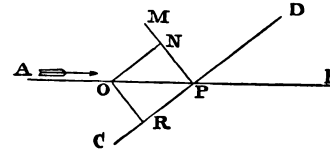

Fig. 22.

83. **Examples.**—The following examples will illustrate the principles of the composition and resolution of forces, and motions, which have been explained in the present and the last chapter.

If a swimmer direct his course across a river in which there is a current, his body will be at the same time subject to two motions: first, that which he receives from his action in swimming, which, if there were no current, would carry him directly across the river in a direction perpendicular to its course in a certain time, as, for example, fifteen minutes; and the other in virtue of the current, by which, if he were to float on the water without swimming, he would be carried down the stream with a velocity equal to that of the stream, at a rate, for example, of five thousand feet in fifteen minutes.

Now, in actually passing over the river, the swimmer is

affected at the same time by both these motions; and consequently his body will be carried in fifteen minutes along the diagonal of the parallelogram of which these motions are sides.

If S S′ and T T′ (*fig.* 23) be the banks of the river, the direction of the stream being expressed by the arrow, and P be the point of departure of the swimmer, let P N be the distance down

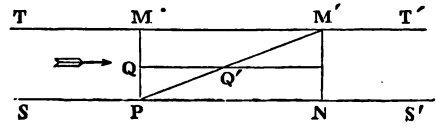

Fig. 23.

which the stream would carry the swimmer in the time which he would take, if there were no current, to cross the river from P to the opposite point M of the other bank. Then, as he crosses, he will be impelled by the two motions. By swimming, he will continually approach the bank T T′; at which he will arrive as soon as he would do if there were no current, because the current which crosses him laterally will not interfere with his course in swimming, which is perpendicular to it. Therefore, at the time that he would arrive at the middle point, Q, of the stream, if there were no current, he will still have arrived at the middle point, Q′, of the stream; but, in virtue of the current, that point will lie, not opposite the point from which he started but lower down on the stream at Q′; and, in fine, he will arrive at the opposite bank at the point M′ just so far below the point M as is equal to the space through which the current moves in the time which the swimmer takes to cross the river. The actual course followed by the swimmer, therefore, by the combined effects of his action in swimming and of the effect of the stream, will be the diagonal P M′ of the parallelogram one side of which, P N, represents the space through which the current moves in the time the swimmer takes to cross the river, and the other, P M, the space through which the swimmer would move in the same time if there were no current.

By a due attention to the principles of composition of motion the swimmer may be enabled, notwithstanding the current, to cross the river in a direction perpendicular to its banks.

As before, let P (*fig.* 24) be the point of the bank from which he starts, the current of the river being in the direction of the arrow.

Let P N be the distance through which the current would

run in the time which the swimmer would take to cross the river. From N draw the line N M to the point of the bank directly opposite to P, and from P draw the line P O parallel to M N. Now, by the principles of the composition of motion which have been already explained, a force in the direction of P M will be equal to two forces simultaneously acting, one in the direction of P N, and the other in the direction of P O; consequently, if the swimmer, leaving the point P, direct his course to the point O, and use such action in swimming as would enable him to pass in still water from P to O in the same time which the current requires to run from P to N, then his actual motion, in consequence of the combined effect of his action in swimming and the force of the current, will be in the direction P M, directly across the stream; as this direction will be the diagonal of the parallelogram, whose sides represent the two forces to which his body is exposed.

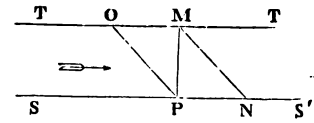

Fig. 24.

But, under these circumstances, although he will pass directly across the stream from P to M, instead of being carried diagonally down it, as in the last example, along the line P M′ (*fig.* 23), he will take a longer time and greater exertion to cross the river. In the former example, as his motion was perpendicular to the stream, the force of the current did not in any wise retard his course in swimming, but merely changed the point of the opposite bank at which he arrived; and although he passed over a greater space in the water, yet the increased distance over which he moved was due, not to his own exertion, but to the current. In the latter instance, however, a part of his exertion is expended in stemming the current, so as to prevent his descending the stream, and another part in crossing the river.

84. **Forces acting on a ship or boat.**—A vessel impelled at the same time by wind and tide in different directions presents another example of the composition of motion. If we suppose the wind to impel the vessel in the direction of the keel, and at the same time the tide to act at right angles to the keel, giving the vessel a lateral motion, the real course of the vessel will be intermediate between the direction of the keel and the direction of the tide at right angles to the keel. The exact direction of this course may be determined by the composition of motion, provided the force of the tide and the effect of the wind be known.

Let us suppose, for example, that the force of the tide is

four miles an hour, and that the effect of the wind is such that if there were no tide the vessel would be carried in the direction of its keel at the rate of seven miles an hour. The actual course of the vessel will then be the diagonal of a parallelogram, one side of which is in the direction of the vessel, measuring seven miles, and the other at right angles to the keel, and in the direction of the tide, measuring four miles.

The action of the oars in impelling a boat is an example of the composition of forces. Let A (*fig.* 25) be the head, and B the stern of the boat. The boatman presents his face towards B, and places the oars so that their blades press against the water in the directions C E, D F. The resistance of the water produces forces on the sides of the boat in the directions G L and H L, which by the composition of motion are equivalent to the diagonal force K L, in the direction of the keel.

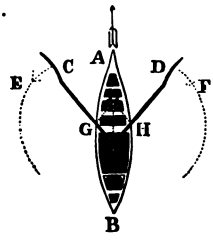

Fig. 25.

Similar observations will apply to almost every body impelled by instruments projecting from its sides, and acting against a fluid. The motion of fishes, the act of swimming, the flight of birds, are all instances of the same kind.

Numerous other examples may be derived from navigation, illustrative of the composition and resolution of forces and motion. The action of the wind on the sails of a vessel and the effects produced by the reaction of the water, is one of these.

Let A B (*fig.* 26) represent the position of the sail, and let the wind be supposed to blow in the direction C D oblique to the sail, its force being represented by the line C D. Let C D be considered as the diagonal of a parallelogram whose sides are E D and F D, the former perpendicular to the sail, and the latter in the direction of its surface. We may then consider the actual wind to be represented by two different winds, one of which blows in a direction perpendicular to the surface of the sail, while the other blows along the sail edgewise, and will therefore produce no effect.

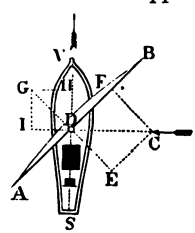

Fig. 26.

The effective part, therefore, of the wind C D will be represented by E D. Now, this force E D, thus acting perpendicularly to the sail, will still act obliquely to the keel. Let the force,

therefore, of the wind upon the sail be represented by D G, the diagonal of a parallelogram, one side of which, D H, is in the direction of the keel, and the other, D I, perpendicular to it. The component of the wind, therefore, expressed by D E, may be imagined to be replaced by two distinct winds, one, D H, in the direction of the keel, and the other, D I, at right angles to the keel.

The original force of the wind C D will thus be expressed by three distinct winds, one, D F, striking the sail edgewise, and therefore inefficient; another, D I, acting at right angles to the vessel, and therefore producing lee-way; and the third, D H, acting in the direction of the keel from stern to stem, and therefore propelling the vessel.

In this case, it appears that a wind blowing at right angles to the course of the vessel, and having, therefore, in itself no tendency to propel the vessel, is nevertheless, by the resolution of forces, shown to be efficient for propulsion.

It is easy to show that this principle may be pushed further, and that a wind which may blow in a direction nearly opposite to the course of the vessel may, by the proper application of the same principle of the resolution of force, be made to propel a vessel. In *fig.* 27 the wind is represented as blowing in the direction C D, forming an acute angle with the course of the vessel, and therefore to a great extent opposed to it. Let us suppose that the rigging admits of placing the sail so as to form a still more acute angle with the course of the vessel than does the wind. The sail will thus lie between the line C D, representing the direction of the wind, and the line D V, representing the direction of the keel. As before, C D, by two successive resolutions of forces, is shown to be equal to three different winds: 1st, a wind represented by F D, blowing edgewise along the sail, and therefore inefficient; 2nd, a wind, D I, blowing at right angles to the course of the vessel, and therefore producing lee-way; and 3rd, a wind, D H, in the direction of the keel, from stern to stem, and therefore efficient for propulsion.

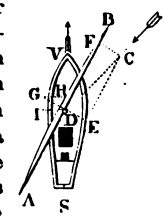

Fig. 27.

It is evident, however, that the more oblique the wind may be to the course of the vessel, the smaller will be the component of its force, represented in the diagram by D H, which is efficient for propulsion; and the greater will be the component F D, acting edgewise along the sail, and therefore inefficient. This will be apparent from the mere inspection of the two diagrams.

The limit of the practical application of this principle in sailing is determined by the play given by the rigging to the sails. There is in each species of rigging a practical limit beyond which it is impossible to place the sails obliquely to the keel. A wind which blows from the quarter near this limit cannot be rendered useful. It will be apparent, therefore, that different kinds of rigging supply different limits to the application of this principle; and we accordingly find that one form of vessel is capable of sailing nearer to the wind than another.

But even though the wind should blow in a direction immediately opposed to the course of the vessel, we are enabled, by another application of the principle of the resolution of force, to press such an adverse wind into the service, and to render it available in carrying the vessel onwards.

This object is attained by the expedient called *tacking*, which is nothing but a practical application of the resolution of force. Supposing the course which the vessel has to pursue to be due west, while the wind is blowing towards due east, then the course of the vessel due west is to be regarded as the diagonal of a parallelogram whose sides are directed alternately to some points north and south of west.

Let A W (*fig.* 28) be the course of the vessel departing from A; it sails from A to D, some points north of west; from D it sails to D′, some points south of west; from D′ it sails to D″,

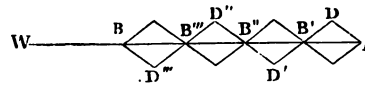

Fig. 28.

parallel to A D; and from D″ to D‴, parallel to D D′; and so on. Thus, at first, instead of sailing direct from A to B′, it reaches B′ by sailing over two sides A D and D B′ of a parallelogram whose diagonal is A B′. Again, instead of sailing directly from B′ to B″, it sails from B′ to D′, and from D′ to B″, over two sides of a parallelogram whose diagonal is B′ B″; and so on. The vessel is thus conducted over the sides of a series of parallelograms, the succession of whose diagonals forms its right course.

85. **Other examples on composition of motion.**—If a ball of lead be taken to the masthead of a vessel which is advancing under sail or steam, and be let drop downwards, it might at first be supposed that this ball would fall at a point vertically under that from which it was discharged, in which case it would necessarily strike the deck just so far behind the mast as would be

equal to the space through which the vessel had advanced in its course during the time of the fall.

But we find, on the contrary, that the ball strikes the deck at the foot of the mast precisely in the place where it would have struck it had the vessel been at rest.

This fact is explained by the principle of the composition of motion.

Let A B (*fig.* 29) be the course of the ship. Let B T be the position of its mast at the moment the ball is dropped; and let B′ T′ be the position of the mast at the moment the ball falls on the deck, the ship having advanced through the space B B′ during the interval of the fall. The ball by being dropped from the top of the mast has in common with the ship its progressive motion; and if it had not been dropped, it would have moved through the space T T′, precisely equal to B B′, in the time of the fall. This progressive motion during the fall is combined with the vertical motion, and if the vertical descent were made with a uniform velocity, the ball would, by reason of the combined effect of the two motions, move along the diagonal of the parallelogram, T B B′ T′. But the vertical descent of the ball not being uniform, but first moving more slowly, and then moving more rapidly, it will move not along the diagonal of the parallelogram, but along a curve represented by the dotted line in the figure, which curve will be described more in detail hereafter; but at the termination of the fall the ball will be found at the foot of the mast.

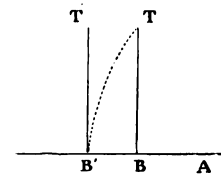

Fig. 29.

The same illustration is presented in a still more striking form by the movement of a carriage on a railway. Let us suppose that a carriage is moving along a line of railway, at the rate of 60 miles per hour, and a ball is dropped from it at the height of 16 feet as it moves. If this ball fell vertically it would strike the ground at a point 30 yards behind the point of the carriage from which it was dropped; for the time of falling 16 feet is one second, and in one second a carriage moving 60 miles per hour would move over 30 yards. The carriage would, therefore, be 30 yards in advance of the point at which the ball was let fall. But it is found that, as in the former case, the ball will meet the ground at a point vertically under the part of the carriage from which it was let fall, which renders it evident that during the fall the ball advances with the same progressive motion as the carriage.

Let T (*fig.* 30) represent the point from which the ball is disengaged, and B the point of the rails vertically under it, the height T B being 16 feet. Let B B′ be 30 yards. In one second after the ball has been disengaged, the point T will have been carried forwards to T′ vertically above B′. At the moment the ball was disengaged from T, it participated in the progressive motion of the carriage ; and consequently, having a motion in the direction of T T′, which, if it had not been disengaged, would have carried it to T′ in one second, during the fall this motion is combined with that of the vertical descent, and the combination of the two motions will cause the ball to move over the curve represented by the dotted line, and to arrive at B′, the point vertically under T′, at the end of a second.

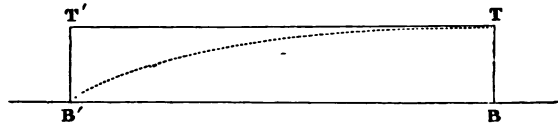

Fig. 30.

These effects may be illustrated in a still more obvious manner by supposing a vertical tube 16 feet high carried with the train, like the smoke funnel of the engine. If a ball were held over the centre of the top of the mouth of the funnel, and let fall, it would, if the train were at rest, strike the bottom of its centre point, falling directly along the centre or axis of the tube. If the tube be carried forward with any velocity, such as 60 miles an hour, the side of the tube will not be carried against the ball by such progressive motion, as might be imagined, but the ball during its descent will still keep exactly in the centre of the tube, as it would have done had the tube been at rest.

86. **Motion of billiard balls.**—The skill of the billiard-player depends on his dexterous application of the composition and resolution of motion. All the movements of the balls, in obedience to the cue, and whether reflected from each other or from the cushions, are determined by those principles. Let P (*fig.* 31) be a billiard ball driven in the direction P O by the cue. At O let it strike the cushion M N. The force of the impact may be represented by the dotted line O A, which is the diagonal of a parallelogram, one side of which is O B perpendicular to the cushion, and the other O C in the direction of the cushion. The *effect, therefore, is the* same as if, at the moment the ball struck

the cushion, it were influenced by two independent forces represented by O C and O B. The force O B being perpendicular to the cushion is destroyed by its reaction; but the ball, being elastic, receives a rebound in the contrary direction O B′. The force O C being in the direction of the cushion is not destroyed, and being combined with that of the rebound O B′ will cause the ball to move along the diagonal O D of the parallelogram, of which these two lines O C and O B′ are the sides. If O, instead of being a point upon the cushion, had been a point upon the surface of another ball, the effects would be the same; only in this case the line M N would represent, not the cushion, but a tangent plane to the ball at the point of impact.

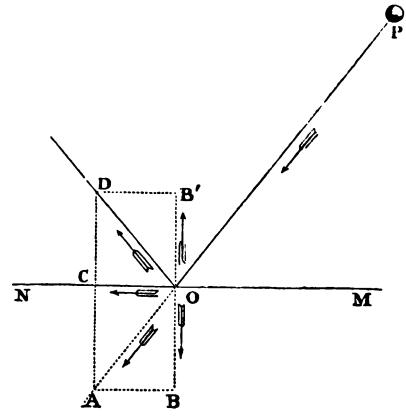

Fig. 31.

In *fig.* 32, the cushion of the table and the ball are shown, A B being the direction in which the ball strikes the cushion; B C′ that in which it is reflected, and B D the force of the blow on the cushion.

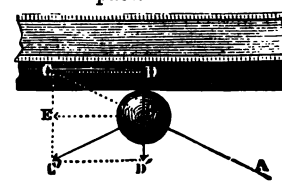

Fig. 32.

The skill of the billiard-player consists in a knowledge of the combination of such effects of the composition and resolution of motion; but, instead of deriving it from the physical principles, he usually obtains his knowledge by long-continued experience.

In *fig.* 33, a stroke is represented in which a *cannon* is made, after successive reflections from each of the four cushions, at the points marked O. The ball P is first directed in the line P O, upon the ball P′, so that, being reflected from it, it strikes the four cushions successively at the points marked O, and is finally reflected so as to strike the third ball P″. At the rebound from the ball P′ and at each of the reflections from the four cushions, the same composition and resolution of forces takes place as is

represented in *fig.* 31 ; and the diagrams showing such composition and resolution are given at the various points of collision in *fig.* 33.

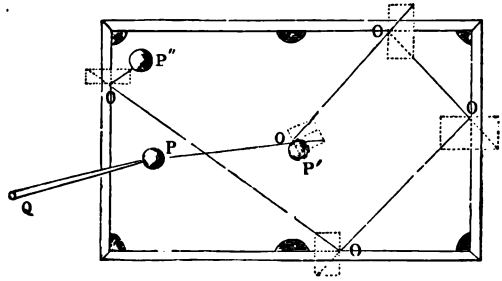

Fig. 33.

87. **Direct proof of the earth's rotation. Further illustrations.**—The principle of the composition and resolution of motion has been ingeniously applied for the purpose of obtaining direct demonstration of the diurnal motion of the earth.

If a high tower or steeple be erected on the surface of the earth, it is evident that, in consequence of the revolution of the globe upon its axis, the top of the tower will be moved in a greater diurnal circle than the base, being more distant from the common centre round which the entire globe is moved. The top of the tower, therefore, and anything placed upon it, has a greater velocity from west to east, which is the direction of the earth's rotation, than has the base.

Now if we imagine a heavy ball to be let fall from the top of the tower towards the base, this ball will be affected by two motions : 1st, that which it has in common with the top of the tower from west to east, in virtue of the earth's diurnal motion ; and 2nd, that vertical motion which it has in falling. The course it will follow will therefore depend on the combination of these two motions, and it will strike the ground at a point east of that which it occupied at the commencement of its fall, by a space equal to that through which the top of the tower is carried during the time of the fall. But during this same interval the base of the tower is also moving eastward, but, as has been explained, through a less space. Now as the ball is carried eastward through the space through which the top of the tower is carried, while the base of the tower is carried eastward through a less space, the ball, instead of falling at the *base* of the tower, which it would do if there was no diurnal

rotation of the earth, will fall just so much east of the base as is equal to the difference between the motion of the top and the motion of the bottom of the tower.

As the difference between its two motions must be an extremely minute quantity, it might be supposed that such an experiment, though beautiful in theory, would be impracticable, the quantity which would indicate the effect of rotation being smaller than could be accurately measured. The experiment, nevertheless, has been made; and the result has been, within the limits of error, such as would be produced by a diurnal revolution of the earth in twenty-four hours.

Many of the striking feats exhibited in gymnastic and equestrian exhibitions are easily explained by the principles of the composition and resolution of motion.

As an example, let us take the case in which, the exhibitor standing on the saddle, a table or other elevated object is held before him, above the height of the horse, but below that of the rider, so that the horse may pass under the table, which would obstruct the progress of the rider. In this case the rider leaps over the table, and returns to the same point of the saddle which he left when the horse had passed under the table. Now, this feat demands from the rider a muscular exertion extremely different from that which might be expected.

Let us suppose the course of the horse to be represented by A A′ (*fig.* 34), the table by T B, and the position of the rider upon the saddle when the horse approaches the table, and when the leap is about to be effected, by R D. At this point the rider leaps, not with a force which would project him over the table, but with one which would project him *vertically upwards* to the position represented by *r′ d′*. This motion, combined with the progressive motion which the rider has in common with the horse, and which is represented by R T, will cause the rider to move, not directly upwards, in immediate obedience to his exertion, but in the diagonal direction R *r*, so that at the moment the horse comes directly under the table the rider is directly over it at *r d*. The upward force of the leap being here expended, the body of the rider begins to fall, and, if not urged by a progressive motion,

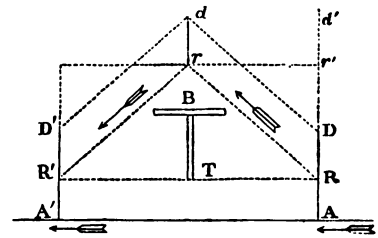

Fig. 34.

would fall on the table. But retaining the progressive motion, the descending motion during the fall is combined with the progressive motion, and the rider accordingly descends in the diagonal direction *r* R′, and arrives at the point R′ precisely at the moment that the saddle borne by the horse arrives at the same point, so that the rider returns to his position on the saddle at R′ which he left at R.

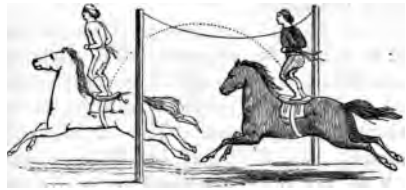

Fig. 35.

Strictly speaking, in this example, the motion of the rider does not take place in the right lines represented by the diagonals in the figure, but in lines slightly curved (*fig.* 35). This, however, makes no difference in the principle involved in the case.

88. **Forces acting on a kite.**—The flying of a kite presents an example of the principle of the composition of forces. Let K (*fig.* 36) be the point on which the force of the wind may be considered as concentrated, and let the direction and quantity of this force be represented by the horizontal line K W. This line K W may be taken as the diagonal of a parallelogram, one side of which, K L, is in the direction parallel to the surface of the kite, and the other, K M, perpendicular to it. The former component, K L, being in the direction of the surface of the kite, glides off without effect; the latter, K M, being perpendicular to the kite, is effective. If the string of the kite be in the direction K A, directly opposed to K M, then the tension of the string will balance the force K M, and the kite will remain suspended in the air, without rising higher; but if the direction of the string be K A′, making an angle with K M, then the tension of the string acting in the direction K A′,

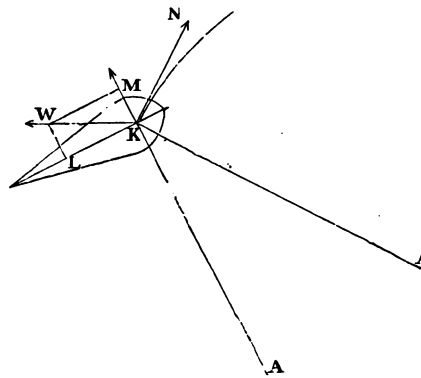

Fig. 36.

and the component of the wind acting in the direction of K M, will produce a resultant action in some intermediate direction such as K N, and this resultant will cause the kite to rise to the point N, describing a circle round the centre A′, and the kite will ascend in this circle until the string K A takes the direction of the component of the wind to which it is opposed. The weight of the kite may in these considerations be neglected, as it is very small when compared with the effect of the wind on the kite.

89. **Absolute and relative motion.**—Motion is usually distinguished into *absolute* and *relative*. In reality, however, we can only conceive of relative motion, because the fact that a body is in motion can only be ascertained by reference to some other body, which is not moving with it.

If a man walk upon the deck of a ship from stem to stern, he has a motion, relative to the deck, measured by the space upon it over which he walks in a given time; but while he thus walks from stem to stern, the ship and its contents, including himself, are carried in an opposite direction.

If it should so happen that his own progressive motion from stem to stern is exactly equal to the progressive motion of the ship, then he will be at rest with regard to the surface of the sea, and will remain vertically above the same point of the surface: for his own motion on the surface of the deck in one direction is, by this supposition, exactly equal to the motion of the ship in the other direction.

If he walk upon the deck at a slower rate than the progress of the ship, then he will have a motion relatively to the sea, in the direction of the ship's motion, equal to the difference between the rate of the ship and the rate of his own motion on the deck; but if he move from stem to stern on the deck at a more rapid rate than the ship advances, then he will have a motion relative to the sea contrary to that of the ship, and equal to the difference between the rate of the ship and the rate of his own motion on the deck.

But these motions are again compounded with that of the earth, in which the man who walks, the ship which sails, and the sea itself participate; and if the absolute motion of the man who walks upon deck is required to be determined, it would be necessary to combine, by the principles of the composition of motion, 1st, the motion of the man upon the deck; 2nd, the motion of the ship through the water; 3rd, the motion of the earth on its axis; 4th, the motion of the earth in its orbit; and finally, the motion of the solar system through space.

90. **Momentum.**—That a mass of matter moving in any

manner produces a certain effect upon any object which may lie in its way is a physical law which, of all others, is the most early, the most frequent, and the most universal result of observation and experience.

The moving mass of a hammer-head will produce an effect upon a nail sufficient to make it penetrate wood—an effect which no common pressure could produce. The effect of the hammer-head of the sledge-hammer will be to compress and vary the form of a mass of metal under it. By the effect of a descending mass of matter, the die impresses upon a piece of metal the image and characters of the coin with more fidelity and effect than the pressure of the hand could produce them, if similarly applied to wax.

A moving mass will cause a punch to penetrate a thick plate of iron, or shears to cut the same plate, with as much facility as a needle would penetrate, or common scissors cut this paper.

The moving masses of the spear, the javelin, and the arrow, and of the bullet and the cannon-ball, have been used as destructive projectiles in war, ancient and modern.

This quality equally appertains to matter in the liquid form.

The torrent, when a river overflows its banks, carries away buildings and levels towns. The moving stream is used to turn the mill and discharge various mechanical functions. They who have stood at the foot of the falls of Niagara have undoubtedly become conscious of the frightful energy of the mechanical power developed by the motion of the mass of waters forming that cataract.

The same quality belongs to matter even in the attenuated form of air. Air in motion carries the ship over the sea, and, acting upon the diverging arms, impels the mill. The tempest agitates the deep, and flings the largest vessel, with destructive force, upon the rock. The moving atmosphere in the hurricane devastates countries, overturning buildings and tearing up by the roots the largest trees.

It will, therefore, be understood how important it is to investigate the laws which govern a quality of matter which produces such effects.

It is not enough to know that matter in motion will exercise this effect on any object which it encounters ; we must be able to express, with arithmetical precision, the conditions on which the effect produced in each case depends ; we must be able to determine, when two different masses of matter are moving under known conditions, what is the proportion between the effect which they would respectively produce on any object *which they might strike.*

If a leaden ball, of a certain magnitude, moves with a certain velocity, and strikes an object, the effect produced will depend upon certain definite laws. To ascertain these laws we must first define *momentum*. If another leaden ball, of the same magnitude, moving by the side of the first ball, with the same velocity, strike the same object at the same time, it will evidently strike it with the same effect, so that the joint mechanical effect produced by the two balls will be precisely double that which would be produced by either of them.

But if we suppose the two balls moulded into one, so as to make a ball of double magnitude, the effect of the impact will still be the same, provided the velocity is the same. It results, therefore, in general, that if two bodies be moving with the same velocity, one having twice the quantity of matter of the other, the mechanical effect produced by the body containing double the mass will be twice that produced by the other.

By the same mode of reasoning, it may be shown generally that, in whatever proportion the mass of the body in motion be increased, the effect with which it would strike an object in its way will be increased in the same proportion. This effect, exerted by a mass of matter in motion, depends thus on the mass and the velocity of a body. Now the product of the mass into its velocity of motion, or the *quantity of motion* of a body, is called in mechanics by the term *momentum*.

When the velocity is the same, therefore, the momentum of bodies is directly proportional to their masses or quantities of matter.

It is found that any force which would impel a ball with a given velocity must be doubled, if the ball is to be impelled with double the velocity, and increased in a threefold proportion if the ball be required to be impelled with three times the velocity, and so on. It is evident, therefore, that the momentum of a body will be augmented in the exact proportion in which its velocity is increased, its mass or quantity of matter remaining the same. Thus the momentum with which a ball weighing an ounce and moving at ten feet per second will strike any object will be exactly ten times the momentum with which the same ball, moving at one foot per second, would strike such an object.

These fundamental principles are so obviously consistent with universal experience that they can scarcely be said to require proof. Yet, abundant proofs of the truth of these principles have been supplied by numberless investigations, in which the relation between moving masses, their magnitudes,

their velocities, and their mechanical effects have been subjected to accurate measurement.

91. **Examples.**—If we project a stone from the hand we give it but a slight impulse if our purpose be to impart to it a slow motion; but if we desire to project it with greater speed, we exercise a greater force of projection. Now, as the stone receives all the force communicated by the hand, it is evident that the increase of force in this case is exhibited by the increase of velocity of the motion of the projectile, the body projected being the same.

If we find a certain muscular exertion sufficient to project a ball of 10 lbs. weight with a certain velocity, we shall find it necessary to use double the force if we desire to project a ball of 20 lbs. weight with the same velocity.

A man's force exerted upon oars can propel a skiff weighing 100 lbs. through the water at the rate of ten feet per second; but the same force applied in the same manner to a vessel weighing a hundred tons would not move it faster than a sixteenth of an inch per second; for, since the same force would be then applied to a body more than two thousand times heavier, the speed of the motion would be more than two thousand times less.

To express momentum algebraically, let M express the mass of the body, and V its velocity; then $M \times V$ will express its momentum: and if M' express the mass of another body, and V' express its velocity, $M' \times V'$ will express its momentum.

Nothing can be more simple or easy than the use of such symbols, provided it be observed that the same units are used in each case for matter, and space, and time. Thus, for example, if in the first case the mass M be expressed by its weight in pounds, and the velocity V by the number of feet moved over per second, then it is necessary, in the second case, that the mass M' should also be expressed in pounds, and the velocity V' also in feet per second, if the momenta of these masses are to be made a subject of comparison.

For example, if the mass M be 10 lbs., and the velocity V be 5 feet per second, then the product $M \times V = 50$; the meaning of which is, that the momentum of the body M having the velocity V will be the same as the momentum of a body weighing 50 lbs., moving at the rate of one foot per second.

If the mass be 5, instead of 10, and the velocity 10, instead of 5, we should still obtain for the momentum

$$M \times V = 50,$$

showing that the momentum would in both cases be the same. But there is an essential difference between both cases with reference to the effects produced by both moving bodies, which will be explained hereafter.

In these two cases the momenta are equal, although the mass of one body be double that of the other; but it will be observed that the lighter mass obtains as much force by its superior velocity as the other has by its superior quantity.

92. **General condition of the equality of momenta.**—These examples being generalised, we obtain the following theorem, which is of considerable use in mechanics:—*When the momenta of two bodies are equal, their velocities will be in the inverse proportion of their quantities of matter.*—The meaning of which is, that the velocity of the lesser body will be just so much greater than the velocity of the greater as the quantity of matter in the latter is greater than the quantity of matter in the former.

A ball of cork which strikes upon a plank of wood, even with the greatest velocity, will scarcely produce an indentation of its surface. A ball of lead striking on the same plank with the same velocity will penetrate it. The momentum of the latter will be greater than the former, in the same proportion as lead is heavier than cork. But the effect produced in this case does not solely depend upon the momentum of the moving bodies. This will be seen further on, in the chapter on 'Work and Energy.'

CHAPTER VIII.

THE COMMUNICATION OF MOMENTUM BETWEEN BODY AND BODY.

93. **Effects of collision.**—When a body in motion encounters another body, certain changes ensue in the motion and in the momentum of both bodies. These changes are in general of a complicated kind, depending on the degree of elasticity of the bodies, their form, weight, and other physical circumstances.

To simplify the question, however, at present, we shall consider the bodies completely devoid of elasticity, and so constituted that after the one impinges on the other they shall coalesce and move as one body in some determinate direction and with some definite speed.

Although these conditions be not strictly fulfilled in practice, they lead to conclusions of the greatest practical utility, which approximate more or less to the results of experience, which

results, however, are modified by various physical and mechanical conditions, the consideration of which is for the present omitted.

If we suppose a mass, M (*fig.* 37), moving in a certain direction, A B, with a velocity V, to encounter another mass M′, at rest, and that after the impact the two masses shall coalesce and move together ; let it be required to ascertain what will be the direction, velocity, and force of the united mass after such impact.

The mass M′, being supposed to be previously at rest, can have no motion save what it may receive from the mass M ; and consequently, after the impact, it must move in the same direc-

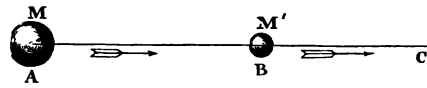

Fig. 37.

tion A B in which the mass M moved before the impact. Whatever momentum M′ may obtain by its coalition with M must be lost by the mass M. This is an immediate and very important consequence of the quality of inertia, which has been already fully explained.

Bodies cannot generate motion in themselves, nor can they destroy motion which they have independent of any external agency. Whatever motion, therefore, the mass M′ may acquire after the coalition of the two bodies, must be lost by the mass M ; and consequently the total momentum of the united masses, after the impact, must be exactly equal to the momentum of the mass M before impact. Now, according to what has been already explained, the momentum of the mass M before impact was M × V. If v be taken to express the velocity of the united mass after such coalition, then, since the quantity of matter in the combined mass is M + M′, its momentum after coalition will be expressed by (M + M′) × v. But this momentum cannot be greater or less than the momentum of M before impact ; for to suppose it greater would be to assume that the bodies have a power of creating motion in themselves with which they were not endued by any external agency ; and to suppose it less would be to suppose them capable of destroying a motion which they had independently of any external agency.

Both of these suppositions, however, are equally incompatible with the quality of inertia, which implies the absence in bodies of all power to create or to destroy, independently of any external agency, any motion in themselves. It follows, therefore, that

the momentum of the mass M before impact, is equal to the momentum of the united mass after impact, which is thus expressed in algebraical symbols :—

$$M \times V = (M + M') \times v.$$

That is to say, the mass M, multiplied by its previous velocity, is equal to the united masses multiplied by their subsequent velocity.

To find, therefore, the velocity with which the united mass will move, it is only necessary to divide the momentum of the mass M before impact, by the sum of the united masses M and M'.

94. **Examples.**—As an example of this, let us suppose a boat, connected with a ship by a tow-rope, the latter being slack, to be propelled by rowers, at the rate of ten miles an hour, the boat weighing 5 cwt., and the ship weighing 250 tons, being 1,000 times the weight of the boat. The moment the rope becomes tight, or, as seamen call it, *taut*, the momentum of the boat is divided between the mass of itself and of the ship, and the common velocity of the two will be determined by the principles above explained. To find it, therefore, it will be necessary to diminish the previous velocity of the boat in the proportion of the weight of the united masses of the ship and boat to that of the boat, that is, in the ratio of 1,001 to 1.

The velocity of the boat having been ten miles an hour, or about $14\frac{1}{2}$ feet per second, the velocity with which the ship would be towed will be, therefore, the $\frac{14\frac{5}{10}}{1001}$th part of a foot per second. Thus, the force which was sufficient to propel the towing-boat alone through 145 feet in ten seconds, will only tow the ship through little more than one-sixth of an inch in the same time.

95. **Collision of two bodies moving in the same direction.**—We have here taken the case in which the body M is at rest. Let us now suppose that it has a motion in the same direction A B, as that of the mass M, but with a velocity V' less than V, the velocity of M, so that the body M shall overtake the body M', and that after impact they shall move together with a common speed ; what will this common speed be ? It follows, as a consequence of the quality of inertia, that, after their collision, the united masses cannot have a momentum either greater or less than they had before their union ; consequently, the momentum of the coalesced bodies will be found by adding together the two momenta which they had before their collision. These two momenta, according to what has been explained, being expressed by $M \times V$ and $M' \times V'$, the momenta of the coalesced

masses will be $M \times V + M' \times V'$. But if v express the velocity of the united mass, its momentum must also be expressed by this velocity v, multiplied by the total mass, that is, by $M + M'$; and, consequently, we have

$$M \times V + M' \times V' = (M + M') \times v.$$

It follows, therefore, from this, that if we divide the sum of the momenta of the two bodies, before their union, by the sum of their masses, the quotient will be the velocity of the united mass after their union; the momentum of the united mass being equivalent to the sum of their momenta before their union.

It is evident that this common velocity, which the combined bodies will have after their union, will be less than the speed of the body M, which overtakes the other, and greater than that of the body M', which is overtaken. The one gains and the other loses velocity; and, consequently, it is evident that the one must gain and the other lose momentum. Now, this gain and loss, on the one side and on the other, is always precisely equal. Whatever momentum the body struck receives from the body striking, the latter loses, and neither more nor less. There is, therefore, a precisely equal change of momentum.

96. **Action and reaction.**—This general principle, which, as we have seen, is a direct consequence of the quality of inertia, and which, if not established, would lead to the conclusion that mere matter has a power in itself to produce or destroy its motion, is usually enunciated under the following form as a mechanical dogma or axiom :—

ACTION AND REACTION ARE EQUAL AND CONTRARY.

The term *action*, applied to the case above explained, is the power which the striking body has to give increased momentum to the body struck; and *reaction* expresses the correlative power which the body struck has of depriving the striking body of an exactly equal quantity of momentum.

This celebrated law of the equality of action and reaction, therefore, means nothing more than the equal interchange of momentum, in contrary directions, between two bodies which come into collision, and, as such, is an immediate consequence of the quality of inertia.

If a boat weighing a hundredweight, rowed at the rate of fifteen feet per second, be suddenly connected by a towing-line with another boat which is rowed in the same direction at the rate of ten feet per second, and which weighs two hundred-

weight, then, the momentum of the first being expressed by 15, and that of the second by 20, their combined momenta will be 35, which, being divided by their united weights 3, gives the quotient $11\frac{2}{3}$, or 11 ft. 8 in., as the space per second through which they will be rowed together.

97. **Collision of bodies moving in opposite directions.**—Let us next consider the case in which bodies M, M′, *fig.* 38, are moving in contrary directions. Let M be supposed to be moving from A towards C, and M′ from B towards C; and let C be the point at which they will impinge upon one another, the momentum of M being supposed to be greater than that of M′.

When impact takes place, then the momentum of M′ will destroy just so much of the momentum of M as is equal to its own amount; for it is evident that equal and contrary forces must destroy each other. After the union of the two bodies,

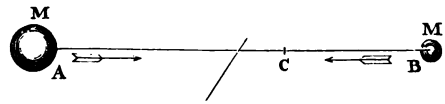

Fig. 38.

the momentum, therefore, that will remain undestroyed, will be the excess of the momentum of M over that of M′; and this excess will involve motion of both bodies in the direction C B of the progressive motion of the body M. But, as we have seen, the momentum of the body M is expressed by $M \times V$; and the momentum of the body M′ is expressed by $M' \times V'$. It therefore follows that the momentum which the united mass will have in the direction A B will be $M \times V - M' \times V'$. But it is also apparent that the momentum of the united mass will be expressed by the magnitude of the united mass multiplied by its velocity. If we express, as before, this velocity by v, then we shall have

$$M \times V - M' \times V' = (M + M') \times v.$$

This statement implies nothing more than that the momentum of the masses, after their coalition in the direction of the greater force, will be equal to the difference between their momenta in contrary directions before coalition.

It follows, also, from this, that the velocity of the united masses in the direction of that which had the greater momentum before impact, will be found by dividing the difference between their momenta by the sum of their masses.

This case affords another example of the application of the

axiom that action and reaction are equal and contrary. The effect which takes place when the bodies coalesce may be thus explained.

When the two bodies M and M′ meet at C (*fig.* 38) the action of the body M upon the body M′ destroys the entire momentum of the latter in the direction B C, and, in addition to that, imparts to it a certain momentum in the contrary direction C B, which will be expressed by the mass M′, multiplied by the velocity which that mass has in common with M, in the direction C B, after their coalition. This velocity is expressed by v; and, consequently, the momentum imparted to M′, in the direction C B, after impact by the action of the body M, will be $M' \times v$. The total action, therefore, of the body M upon the body M′ will be made up of the momentum of M′, which is destroyed in the direction B C, and which is expressed by $M' \times V'$, and the momentum which is imparted to it in the contrary direction C B, and which is expressed by $M' \times v$. Therefore, the entire action of M upon M′ will be expressed by

$$M' \times V' + M' \times v.$$

Such, then, being the effect of the action of M upon M′, let us now consider what will be the effect of the reaction of M′ upon M. Upon the collision at C, when M′ loses its entire momentum in the direction C A, it will destroy in the body M an exactly equal amount of momentum; that is to say, the body M will lose, in the direction A C, a momentum equal to $M' \times V'$, which is lost by the body M′. But M also imparts, by its action to M′, a momentum in the direction of C B, which is expressed by $M' \times v$. Now, the reaction of M′ upon M, in receiving this momentum in the direction of C B, must deprive M of exactly the same momentum in that direction; that is, it must deprive it of a momentum in the direction C B expressed by $M' \times v$. Hence it follows that the total amount of momentum lost by reason of the reaction of M′ upon M is expressed by

$$M' \times V' + M' \times v,$$

which is precisely equal to the momentum gained by M′. It appears, then, that in this case the law of equal action and reaction is still fulfilled; the action of M upon M′ being precisely equal to the reaction of M′ upon M.

When two equal bodies meet, moving with equal velocities in opposite directions, their shock will immediately destroy each other's momentum; for in this case the momenta, being equal *and contrary, will be* mutually destroyed. The effect of the

impulse produced by the two bodies in this case will be equal to the impulse which either, being at rest, would sustain, if struck by the other moving with double the velocity; for the action and reaction being equal, each of the two will sustain as much impulse from reaction as from action.

98. **Examples.**—If two railway trains, moving in contrary directions at twenty miles an hour, sustain a collision, the shock will be the same as if one of them, being at rest, were struck by the other moving at forty miles an hour.

If two steam-boats, of equal weight, approach each other, one moving at twelve miles an hour, and the other fifteen miles an hour, each will suffer a shock from the collision, the same as if it were struck by the other moving at twenty-seven miles an hour.

In the combats of pugilists, the most severe blows are those struck by fist against fist, for the force suffered by each in such case is equal to the sum of the forces exerted by either arm. Skilful pugilists, therefore, avoid such collisions, since both suffer equally and more severely.

99. **Oblique impact.**—In what precedes we have limited our observations to the cases in which the bodies which impinge one on another are moving either in the same or opposite directions in the same straight line. Let us consider now the case in which they move in different straight lines before their collision.

Let a body, M (*fig.* 39), be supposed to move in the line A B, and from A towards B. At some intermediate point, C, suppose it to be struck by another body, M′, moving in the line A′ B′ from A′ towards B′, and suppose that at the moment M arrives at C, M′ also arrives at that point and coalesces with it, and that after their union the bodies move together.

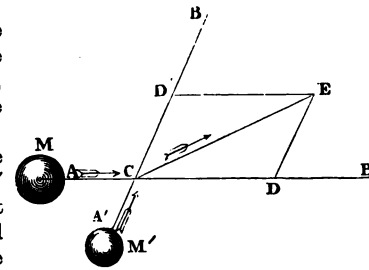

Fig. 39.

The question is, in what direction, with what force, and with what velocity they will be moved. This question is easily solved by the application of the principle of the parallelogram of forces already explained.

Let the velocity with which M moves in the direction A B be expressed by V, and let the velocity with which M′ moves in the direction A′ B′ be expressed by V′; then M × V will be the

momentum of the body M directed from C towards B, and $M' \times V'$ will be the momentum of the body M′ directed from C towards B′. Let the distance C D represent the quantity $M \times V$, and let C D′ represent $M' \times V'$, and complete the parallelogram C D E D′. Draw its diagonal C E. This diagonal will then represent the direction and the quantity of the momentum which the combined masses M and M′ will have after their coalition. If we would find the velocity, it will only be necessary to divide the number expressed by this diagonal C E by the number expressing the sum of the masses M and M′; the quotient will be the velocity with which the combined masses will move from C to E.

100. **Action and reaction modified by elasticity.**—When a body which strikes a hard surface is elastic, the effects of action and reaction are modified in a manner which it will be necessary to explain.

Let us suppose, for the simplicity of the explanation, that the form of the body is that of a sphere or globe. When it strikes a hard surface with any force, it will be momentarily flattened at the point of impact, and will take an oval form; the force of the impact will compress it in the direction of the blow, and it will be elongated in a direction at right angles to the blow.

Thus, if its spherical form before the blow be represented in *fig.* 40, and if D C be the diameter which is in the direction of the impact, C being the point at which the impact takes place, and A B be the diameter at right angles to D C, then the body at the moment of receiving the force of impact will take an oval form represented in *fig.* 41, the diameter A B will be elongated,

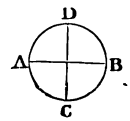

Fig. 40.

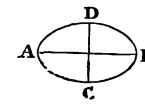

Fig. 41.

and D C contracted by the force of the impact. But the body, by force of its elasticity, will make an effort to recover its figure, and the point C will react upon the surface which it strikes; and by that effort the body will recoil, because the diameter D C, in its effort to recover its original length, will press the matter of the body against the hard surface at C, and this pressure being resisted will cause the body to rebound.

101. ***Perfect* and imperfect elasticity.**—If the elasticity of

the body be perfect, the force with which the spherical figure is restored will be equal to that with which it has been compressed into an ellipse, and this force being resisted by the surface, the body will rebound with the same force as that with which it struck it. But if the elasticity of the body be imperfect, then the restoring force will have less intensity than the compressing force, and the body will rebound with less force than that with which it struck the surface.

If an ivory ball, a substance which possesses elasticity in a high degree, be dropped upon any hard and smooth surface which is level, it will rise very nearly to the height from which it was dropped. It would rise exactly to that height, but for two causes: first, the want of perfect elasticity; and, secondly, the resistance of the air.

When an elastic body strikes a surface in a direction not perpendicular to that surface, it will be reflected in another direction, which will depend partly on the direction in which it strikes the surface, and partly on the degree of elasticity of the striking body. Let A B (*fig.* 42) be the surface; and let the body be supposed to strike it at C, having moved in the direction D C; and let us first suppose that the body is perfectly elastic. The force of the impact at C, being represented by C E, may be resolved into two forces, C F perpendicular to the surface A B, and C G parallel to it. We may therefore suppose the ball at C to be affected by two such forces. Now, since the ball is supposed perfectly elastic, the component C F will cause a rebound in the direction of C F′ equal to the force C F. If, therefore, we take C F′ equal to C F, the body will, after the impact, be affected by two forces, C G in the direction of the surface, and C F′ perpendicular to it; and it will accordingly move in the diagonal C E′ of the parallelogram of which C F′ and C G are sides; but this parallelogram being in every respect equal to the parallelogram C F E G, the angle formed by C E′ with the surface C B will be equal to the angle formed by C E and C D with the same surface. Hence it appears that in this case the body will be reflected from the surface which it strikes at the same angle

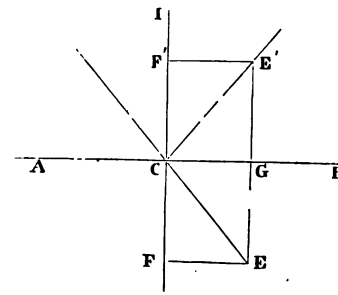

Fig. 42.

as that with which it strikes it ; that is to say, the angle D C A will be equal to the angle E′ C B.

This principle is usually enunciated thus : When a perfectly elastic body strikes a hard surface and rebounds from it, the angle of incidence will be equal to the angle of reflection, and these angles will be in the same plane.

By the angle of incidence is understood the angle which the direction, D C, of the original motion of the ball forms with the perpendicular, C I, to the surface struck ; and by the angle of reflection is understood that which the direction C E′, in which the body recoils, forms with the same perpendicular.

But if the body which struck the hard surface be imperfectly elastic, then the recoil produced by the component C F (*fig.* 43),

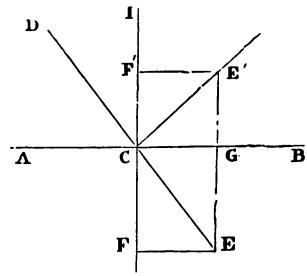

Fig. 43.

perpendicular to the surface C B, will be less than the force with which the body strikes the surface at C. The line C F′, therefore, which represents this force of recoil, will be less than C F, and the parallelogram C F′ E′ G, while it has the side C G in common with the parallelogram C F E G, has the side C F′ less than C F. Consequently it is obvious that the angle which C E′ makes with the perpendicular C I will be greater than the angle which D C makes with the same perpendicular.

This principle is enunciated thus : When a body imperfectly elastic strikes a hard surface, the angle of incidence will be less than the angle of reflection, and the difference between these angles will be greater, the more imperfect is the elasticity of the body.

The preceding facts may be summarised in algebraical formulæ thus : —

(*a*) M_1, M_2 are two masses; v_1, v_2 their velocities. They move in the same straight line and the same direction. Let $v_1 > v_2$. Let, after impact, their common velocity be v. Then M_1 has lost momentum, which is $= M_1 (v_1 - v)$, and M_2 has gained momentum $= M_2 (v - v_2)$, and both are equal.

$$\therefore v = \frac{M_1 v_1 + M_2 v_2}{M_1 + M_2}.$$

(*b*) If the direction of motion was opposite, then—

$$v = \frac{M_1 v_1 - M_2 v_2}{M_1 + M_2}.$$

(*c*) Let R be a measure of the effect of action and reaction at the moment of impact. Then the momentum of M_1 after impact is $M_1V_1 - R$, and that of M_2 is $M_2V_2 + R$, and the velocities of M_1 and M_2 are respectively $V_1 - \frac{R}{M_1}$, and $V_2 + \frac{R}{M_2}$, and these are equal—

$$\therefore R = \frac{M_1M_2}{M_1 + M_2}(V_1 - V_2)$$

(*d*) If the balls are elastic, then M_1 will be receiving an additional momentum backwards, and M_2 an equal momentum forwards, which is a fraction of R. The value of this constant fraction for two substances depends on their nature, and is the *modulus of elasticity.*

The mass M_1, therefore, will have afterwards a momentum

$$M_1V_o = M_1V_1 - (1 + e)\frac{M_1M_2}{M_1 + M_2}(V_1 - V_2),$$

and M_2 will have—

$$M_2V_{oo} = M_2V_2 + (1 + e)\frac{M_1M_2}{M_1 + M_2}(V_1 - V_2),$$

in which expressions e denotes the modulus of elasticity, and V_o, V_{oo}, the velocities after impact.

102. **Apparatus to illustrate collision experimentally.** —That generally found in collections for public instruction, shown in *figs.* 44, 45, 46, consists of balls, inelastic or elastic, suspended by diverging threads, the ends of which are attached to two parallel bars, and which are so adjusted that the balls, when quiescent, rest in a horizontal line and in mutual contact. Let us first suppose that two equal inelastic balls are suspended as shown in *fig.* 44. If one of them, being drawn aside, as in *fig.* 45, is allowed to fall against the other, it will strike it with a force proportionate to the distance, B A, from which it fell. After the collision, this force being divided equally between the balls, they will move together to a distance from their position of rest equal to half that from which the ball A fell. Thus, if the distance B A be 12 inches, the two balls, after collision, will move together to the distance of 6 inches, and will then swing together from side to side until they are brought to rest by the resistance of the air.

If three equal inelastic balls had been thus suspended, one being, as before, drawn aside to the distance of 12 inches, the force, after collision, being equally distributed, all three would move together to the distance of 4 inches.

In general, if any number be suspended, as in *fig.* 46, the

force of the single ball will be shared equally among all, after collision, and the whole will move to a distance, which will be found by dividing that from which the striking ball fell by the entire number of balls.

All these effects will be seen to be in complete accordance with the principles explained in the preceding paragraphs. Balls of putty or moist clay are sufficiently inelastic for such an illustration.

If the balls be elastic, the effects are different and very

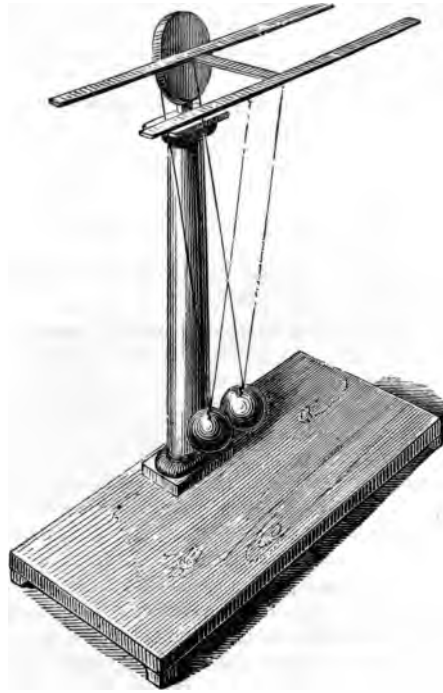

Fig. 44.

remarkable. Let two equal ivory balls, for example, be suspended, as in *fig.* 44. When one, A, is drawn aside, as in *fig.* 45, and let fall against the other, B, the immediate effect of the collision, as in the case of inelastic balls, is, that A loses half its force, which it imparts to B. But B being compressed with this force recovers its figure with equal force, and reacts against A

so as to deprive A of the other half of its force and to reduce it to rest. At the same time A being compressed by the reaction of B, recovers its figure with equal force, and reacting upon B imparts to it the other half of its force, so that B moves forward with the full force which A had at the moment of collision, while A remains at rest. The ball B, therefore, will move on to a distance equal to that from which A had descended. Thus, if A fall from 12 inches, it will remain at rest after the collision, and B will rise to 12 inches, from which it will descend and strike A, which in its turn will rise, B remaining at rest, and so on.

This alternate motion would continue indefinitely, but for

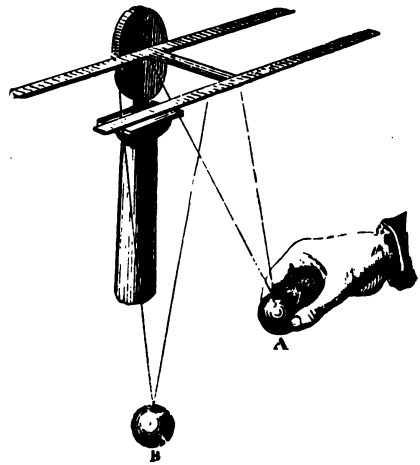

Fig. 45

the resistance of the air, by which the range of the vibration is gradually diminished.

If several ivory balls be suspended, as in *fig.* 46, and one be drawn aside and let fall, the effect of the collision will be transmitted through the series to the last ball, which will be affected exactly as if it were immediately acted upon by the first. Each of the intermediate balls in this case, being equally affected in opposite directions by the reaction of the contiguous balls in recovering their figures, will remain at rest, and the extreme balls alone will alternately rise and fall until reduced to rest by the resistance of the air.

103. **The laws of motion.**—Before the true principles of

inductive science were so well understood and so generally admitted as they are at present, the exposition of the property of inertia, and of its most important consequences, was embodied by Newton in three formularies, called by him *the laws of motion*, which have attained great celebrity in the history of mechanical science. Although these mechanical maxims have lost much of their importance by the more general diffusion of

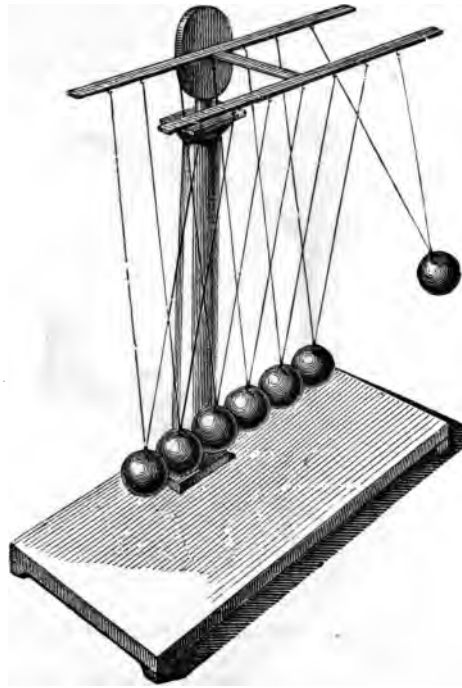

Fig. 46

correct principles of physical science, they are, nevertheless, entitled to notice, and ought to be registered in the memory of all students, were it only for their illustrious origin, and to commemorate the difficulty which the true principles of induction had to struggle against, in the extermination of the errors of the old philosophy. These laws of motion are enunciated as *follows* :—

FIRST LAW.

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare.

Every body must continue in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled by impressed forces to change that state.

SECOND LAW.

Mutationem motûs proportionalem esse vi motrici impressæ, et fieri secundum lineam rectam quâ vis illa imprimitur.

Change of motion is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.

THIRD LAW.

Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones inter se mutuo semper esse æquales et in partes contrarias dirigi.

To every action there is always an equal and contrary reaction: or, the mutual actions of any two bodies are always equal and oppositely directed.

The first of these propositions is little more than a definition of the quality of inertia. The terms in which the second is expressed require qualification and explanation, without which they might be subject to erroneous interpretation.

If a body be at rest, it is true that every motion it receives must be proportional to and in the direction of the force impressed, that is, of the force which produces the motion; but if a body be already in motion in one direction, and receive a force in another direction, then the new direction which the body takes will not be in the direction of the force impressed, nor proportional to it, but will be in the direction of the diagonal of a parallelogram, one side of which represents the force with which the body previously moved, and the other the new force which is impressed upon it. This has been already fully explained.

In the third law, the word action means the momentum which one body receives when another acts upon it, and the word reaction means the momentum which the latter loses in consequence of communicating momentum to the former.

The equality of action and reaction is, therefore, subject to

the bodies around it, but is attracted by them; and that the momentum which is imparted to them is balanced by a momentum impressed by them upon the terrestrial globe itself. In fact, in the *attraction of gravitation*, as this physical agent is called, the equality of action and reaction is verified as completely as it is in the mutual impact of bodies.

For the present, however, it will not be necessary to dwell on this point. What more immediately concerns us is the explanation of those phenomena which are exhibited in the effects of gravity acting on bodies at or near the surface of the earth.

105. **All bodies fall with the same velocity.**—Gravity acts equally and independently on all the particles composing a body, and therefore has a tendency to make all these particles move with equal velocities, and in parallel lines perpendicular to the surface. It is easily conceived that if two leaden balls of equal magnitude be placed side by side at the same height, they will fall together with the same velocity to the surface, and strike the earth at the same moment side by side. Now, if the matter of these two balls be moulded into a single ball, the effect will remain the same, since their form cannot affect the operation of gravity upon them.

In the same manner, if ten or a hundred leaden balls of equal magnitude be disengaged together, they will fall together; and if they be moulded into one ball of great magnitude, it will still fall in the same manner. Hence it follows that masses of matter, however they may vary in magnitude and weight, will descend to the surface of the earth with the same velocity, and if they fall from the same height will arrive at the surface of the earth in the same time, provided they be affected by no other force but that of gravity.

There are some circumstances involved in the fall of bodies, and the effects of the resistance of the air upon them, which are apparently incompatible with what has been just stated. If a feather and a leaden ball suspended at the same height be disengaged, it is evident that they will not fall with the same velocity. The leaden ball will be propelled with a rapidity much greater than that which affects the feather. But in this case the operation of gravitation is modified by the resistance of the air, which is much greater upon the feather than upon the leaden ball. That two such bodies would descend with the same velocity if relieved from the interference of the air may be shown by the experiment which is familiarly known as that of *the guinea and feather.*

Let a glass tube (*fig.* 47), of wide bore—as, for example, three or four inches, and five or six feet in length, be closed

Fig. 47.

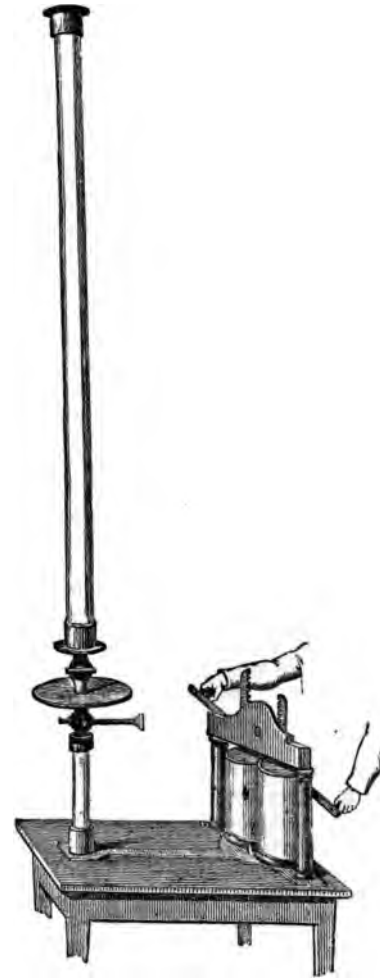

Fig. 48.

at one end, and supplied with an air-tight cap and stop-cock at the other end. The cap being unscrewed, let small pieces of

metal, cork, paper, and feathers be put into it, the cap screwed on, and the stop-cock closed. Let the tube be rapidly inverted, so as to let the objects included fall from end to end of it. It will be found that the heavier objects, such as the metal, will fall with greater, and the lighter with less speed, as might be expected. But that this difference of velocity in falling is due, not to any difference in the operation of gravity, but to the resistance of the air, is proved in the following manner. Let the stop-cock be screwed upon the plate of an air-pump (*fig.* 48), the cock being open, and let the tube be exhausted. Let the cock then be closed, and unscrewed from the plate. On rapidly inverting the tube, it will then be found that the feathers will be precipitated from end to end as rapidly as the metal, and that, in short, all the objects within the exhausted tube will fall together with a common velocity.

106. **Weight of bodies proportional to quantity of matter.**—Since the attraction of the earth acts equally on all the component parts of bodies, and since the aggregate forces produced by such attraction constitute what is called the *weight* of the body, it is clear that the weights of bodies must be in the exact proportion of the number of particles composing them, or of their quantity of matter.

Hence, in the common affairs of commerce, the quantities of definite masses, that is, of bodies, are estimated by their weights.

It will appear, hereafter, that the weight of a body, or the force with which it is attracted to the surface, is slightly different in different places upon the earth ; but this is a point which need not be insisted on at present.

At the same place the weights are invariably and exactly proportional to the quantities of matter composing the bodies. If one body have double or triple the weight of another, it will have double or triple the quantity of matter in the other.

107. **Motion of a falling body.**—It is not enough for the purposes of science to know merely the direction of the motion which gravity impresses upon bodies ; we require to know whether the motion be one having a uniform velocity ; or, if not, in what manner does the velocity of the falling body vary ?

If a man leap from a chair or table he will strike the ground without injury. If the same man leap from a house-top he will probably be destroyed by the fall. These, and innumerable similar effects, indicate that the momentum with which a body strikes the ground is augmented with the height from *which it falls.* Now, as this momentum depends on the velocity

of the body at the moment it touches the ground, it follows that the velocity of the fall is augmented with the height.

In short, when a body is disengaged and allowed to descend in obedience to gravity, its velocity is gradually accelerated as it descends. Meteoric stones, which descend from the upper regions of the atmosphere, strike the earth with such force that they are often known to penetrate the soil to a considerable depth.

It might be naturally enough conjectured that the force with which a body strikes the earth is proportional to the height from which it has fallen, and such an assumption has been accordingly used sometimes; but this is erroneous. The force of the fall is not, as we shall now show, proportional to the height from which the body has descended. A body falling from a double height does not strike the ground with a double momentum.

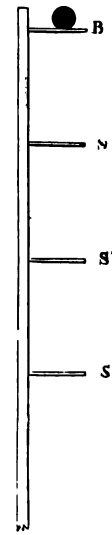

Fig. 49.

When a body, such, for example, as a leaden ball, is disengaged at any height, and delivered to the action of gravitation, the effect of this force upon it is to impart to it a certain velocity. Now it is evident that the quantity of velocity which the attraction of the earth gives to the ball in one second of time must be equal to the force which it would give to it in another second of time. Let us suppose, for example, that a moveable stage s (*fig.* 49) is attached to a wall or pillar, and is so adjusted that the ball disengaged at B shall arrive upon the stage s precisely at the termination of one second. The body will then strike the stage with a certain momentum.

Let another stage s′ be placed at the same distance from s as s is from B. If the ball, having been brought to rest by the stage s, is again disengaged, it will strike the stage s′ at the end of another second, and with the same momentum; and if the stage s″ be fixed at an equal distance below s′, the ball, having been brought to rest at s′, and then disengaged, will strike the stage s″ at the end of the third second, and with equal momentum.

In this case we have supposed that while the ball descends, the velocity it has acquired at the end of each successive second is destroyed by the resistance of the stages s, s′, and s″, &c. But suppose that on arriving at s, at the end of the first second, the body was not deprived of the velocity it had ac-

quired, but allowed to retain it in its descent, the retention of this velocity would not in the slightest degree prevent the action of gravity in imparting to it an equal quantity of velocity in the second second; therefore at the end of the second second the body would have the velocity with which it struck the stage s, *in addition to* the velocity which it had acquired during the second second. In the second second, therefore, the body would descend through a much greater space than s s′, and at its termination would have a velocity double that which it had at the end of the first second. In like manner, if the velocity acquired in the second second were not destroyed by the stage s′, the body would at the end of the third second possess this velocity, in addition to the velocity which would be imparted to it by the action of gravity in the third second.

The action of gravity imparts thus to a falling body a certain definite velocity in every successive second of time during its action, and, consequently, the velocity which a falling body has at the end of any number, say n, seconds, is exactly n times the velocity it had at the end of one second. In other words :—

The velocity acquired by a body in descending by the force of gravity increases in proportion to the time of the fall.

108. **Accelerated motion.**—A motion, the velocity of which is thus augmented in proportion to the time counted from its commencement, is called *uniformly accelerated motion*, and the force which produces such a motion is called *uniformly accelerating force.*

Gravity, therefore, acting on bodies near the surface of the earth, is a uniformly accelerating force.

Since a body in falling moves with a velocity gradually and uniformly accelerated, its average or mean velocity will be that which it had precisely at the middle point of the interval which elapses during its fall. Thus, if a body fall during ten seconds, the average speed will be that which it had at the end of the fifth second. This is evident, inasmuch as, the speed imparted in each successive second being the same, the average of all the speeds at the end of each number of seconds, counted from the commencement, will necessarily be that which it had at the middle point of the time.

It follows from this also, that the final speed acquired by a body at the end of any time will be double the average speed counted from the commencement of its fall. This is evident, since, the velocity being proportioned to the time, the final speed is necessarily double that which is acquired in half the *time,* which is, as has been just shown, the average speed.

It follows from this also, that if a body were to move with its final velocity continued uniformly, it would, in a time equal to that of the fall, move over a space equal to double that through which it had fallen; for, the final speed being double the average speed, the space described with the former will be double the space described with the latter in the same time.

To obtain a more exact estimate of the manner in which the descent of a heavy body is accelerated, it will be useful to investigate the spaces through which a body moves in its descent during every successive second of time.

109. **Space passed over in successive seconds.**—Let us express by H the height through which a body falls from a state of rest in one second. At the end of such second, the body has acquired a velocity, in virtue of which it would, in another second, without the further action of gravity, move through a space 2 H; but during the next second gravity would cause the body to descend through another space equal to H, supposing it to move from a state of rest. Therefore, during the next second the body is moved through a space equal to three times H; that is to say, twice H in virtue of the velocity acquired at the end of the first second, and a space H in virtue of the action of gravity upon it during the next second.

Let us now consider the motion of the body during the third second. At the end of the second second, the body, having fallen through a height expressed by 4 H, has acquired a velocity in virtue of which, without any further action of gravity, it would move through a space equal to 8 times H in two seconds, and 4 times H in one second; but, in addition to this, gravity also, in the third second, would move it through a space H; and, from these two effects combined, the body in the third second would descend through a space expressed by 5 H. But we have seen that in the first two seconds it has fallen through a space expressed by 4 H, and therefore at the end of the third second it will have fallen through a height from the state of rest expressed by 9 H.

Pursuing its course further, we find that it begins its motion during the fourth second with a velocity such as would make it, in three seconds, without the further aid of gravity, move through a space equal to double that which it had fallen through from a state of rest, that is to say, 18 H; consequently, with this velocity, it would move in the fourth second through a space equal to 6 H; but, in addition to this, the action of gravity carries it in the fourth second through the space H, and by these combined effects it must move in this second through a

space equal to 7 H. In the same manner, it may be shown that the space through which it moves in the fifth second is 9 H, while the space through which it moves in the first five seconds is 25 H; and the space through which it moves in the sixth second is 11 H, while the space through which it descends from a state of rest in the first six seconds is 36 H; and so on.

In the following table is expressed, in the first column, the number of seconds, or other equivalent intervals of time, counted from the commencement of the fall. In the second column is exhibited the space through which the falling body moves in each successive interval, the unit being understood to express the space through which a body falls in the first second of time. In the third column is expressed the velocity which the body has acquired at the end of each interval, counted from the commencement of the fall, and expressed by the space which, if such velocity continued uniformly, the body would describe in one second. In the fourth column is expressed the total heights from which the body falls from a state of rest to the end of the time expressed in the first column.

Number of Seconds in the Fall, counted from a State of Rest.	Spaces fallen through in each successive Second.	Velocities acquired at the End of Number of Seconds expressed in First Column.	Total Height fallen through from Rest in the Number of Seconds expressed in First Column.
1	1	2	1
2	3	4	4
3	5	6	9
4	7	8	16
5	9	10	25
6	11	12	36
7	13	14	49
8	15	16	64
9	17	18	81
10	19	20	100

Although all the circumstances attending the descent of bodies falling freely are included with arithmetical precision in the above table, we may nevertheless render it more easy to obtain a clear conception of these important physical phenomena by the annexed diagram (*fig.* 50), in which the divided scale represents the vertical line along which the body is supposed to fall, o being the point from which it commences its descent. The points which it successively passes at the termi*nation of 1, 2, 3, 4, 5, 6,* and 7 seconds respectively are marked

I, II, III, IV, V, VI, VII. The figures of the scale indicate the total heights through which the body has fallen at the end of each successive second, the unit being the height through which the body falls in the first second. The spaces included between brackets on the right of the diagram are those through which the body falls in each successive second. It will thus be apparent, first, that the body is accelerated in its motion, inasmuch as the spaces through which it falls in each successive second are evidently increasing; secondly, that the space through which it falls in any number of seconds is expressed by the square of this number, the unit being the space fallen through in the first second; thirdly, that the spaces fallen through in each successive second are expressed by the odd numbers with reference to the same unit.

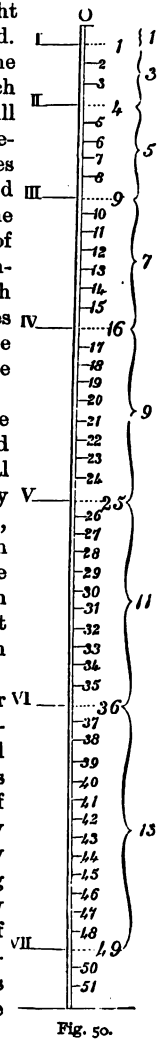

Fig. 50.

A direct experimental verification of the results exhibited in the preceding table and diagram would be attended with several practical difficulties. The heights through which a body falls by gravity, acting freely in several seconds, are considerable, and a great velocity is soon acquired. The resistance of the air disturbs the result, and some difficulty would be found in observing, with sufficient precision, the points at which the falling body would be found at each successive second of time.

110. **Atwood's machine.**—This and other practical difficulties have, however, been surmounted by a beautiful and useful experimental apparatus, called from its inventor 'Atwood's machine.' By this apparatus the intensity of the force of gravity can be diminished in any desired proportion without divesting it of any of its characters of a uniformly accelerating force. Thus we can make the falling body descend at so moderate a rate that the effect of the atmospheric resistance becomes imperceptible, and the height, and all the circumstances attending the fall, can be observed with the greatest precision.

This contrivance consists of two equal cylindrical weights,

w, w′ (*fig.* 51), connected by a fine silken thread, which passes in a groove over a nicely-constructed wheel R, turning on a horizontal axis, so as to be subject to an imperceptible friction. This wheel, and the stand which supports it, are placed upon a bracket A B, attached to a wall, or supported on a pillar, at a convenient height. Adjacent to the thread supporting one of the weights w, there is a divided vertical scale, by which the circumstances attending the descent of the weight can be noticed and measured. When the weight w is brought to the highest point of the scale, the weight w′ will be near the ground; but the weight of the thread is so insignificant that, though unequal portions of it hang on each side of the wheel R, the difference of their weights produces no perceptible defect, and, accordingly, the two equal weights w and w′ rest in equilibrium.

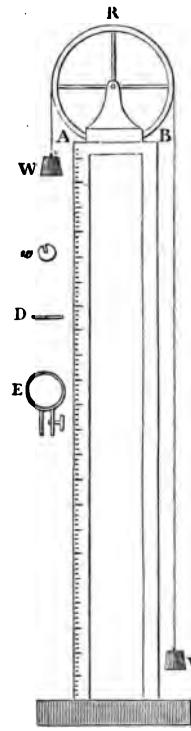

Fig. 51.

Now, if a small additional weight *w* be placed upon the top of the cylindrical weight w, it will cause the weight w to descend, and the weight w′ to rise; and this descent will have all the characters of a uniformly accelerated motion, for the force of gravity impresses on the preponderating weight *w* the same momentum which it would impress upon it if it were free; but this momentum is, by the very condition of the apparatus, shared by *w* and the equal weights w and w′, so that, instead of imparting to *w* the velocity which such weight would have were it free, the velocity of the augmented moving mass, consisting of *w*, w, and w′, will be diminished in precisely the same proportion as the mass moved is increased; therefore, the weight w, bearing upon it *w*, will fall with a velocity so much less than that with which *w* would fall, were it free, as the combined weights w′, w, and *w* are greater than *w* alone. But the other circumstances attending the descending motion will be precisely similar to those which attend the descending motion of any falling body. The machine will, in fact, present a miniature representation of the phenomena of *falling bodies;* the effects will be the same as though the

attraction of the earth upon a falling body were diminished to such an extent that the velocity of the descent would be reduced to that with which the weight w actually falls.

Now, as we can adopt a preponderating weight w as small as may be desired, it is clear that we may reduce the velocity of the fall in so great a degree that all the circumstances attending the motion during the descent can be deliberately and accurately observed.

Let us suppose, for example, that the weights w and w′ are each twenty-four ounces, and that the preponderating weight w, placed upon w to produce its descent, is a quarter of an ounce. The total mass moved, therefore, by the action of gravity impressed upon the weight w, will be 193 times the weight w, for the weights w and w′, taken together, are forty-eight ounces, that is to say, 192 quarters of an ounce ; and the weight w, which is one quarter of an ounce, being added to this, will make a total of 193 quarters of an ounce.

The attraction of gravity, therefore, instead of imparting velocity to one quarter of an ounce, has to move 193 quarters of an ounce, and, consequently, the velocity it imparts per second will be 193 times less.

We have here, with a view to simplify the explanation, avoided all reference to the motion imparted to the wheel over which the strings pass ; but it will be evident that the force impressed by gravity on the preponderating weight w must be shared with the matter of the wheel, as well as with the weights w and w′. If the matter of the wheel were all collected at its edge it would then be moved with the same velocity as the weights w and w′, and in this case it would be only necessary to consider the weight of the wheel as forming part of the masses w and w′, and therefore to diminish the latter so that the total weight of w and w′ and the wheel should make up forty-eight ounces.

But as the mass of the wheel is not all collected at its edge, it does not all receive the same velocity, but, on the contrary, its different parts are moved with less velocities the nearer they are to its centre. This difference of moving force, imparted to different parts of the wheel, requires to be allowed for, by calculating how much matter collected at the edge of the wheel, would have an equal moving force. Such a calculation, though presenting no difficulty, and subject to no inaccuracy or doubt, would involve mathematical principles and operations which cannot be conveniently introduced here ; and we may therefore assume that the momentum imparted to the wheel is

represented by an equivalent portion of the forty-eight ounces assigned to the weights w and w′, and that, in fact, the real weights of these must be a little less than those assigned to them, the difference being represented by the effect of the wheel.

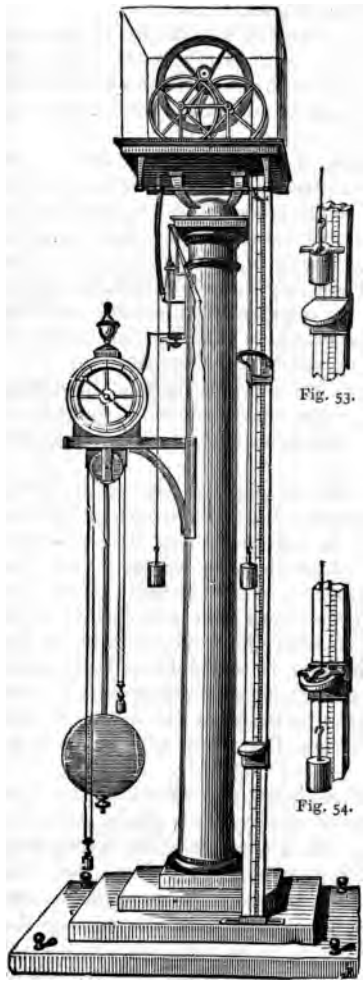

Fig. 52.

Fig. 53.

Fig. 54.

Being provided with a pendulum beating seconds in an audible manner, and taking the thread which sustains the weight w, between the fingers, let the weight w be elevated until its upper surface coincides with the zero of the scale. Listening attentively to the beats of the pendulum, let the thread be disengaged at the moment of any one beat.

It will be found that, at the moment of the next beat, the weight w will have fallen precisely *one* inch. During the second beat it will have fallen through precisely *three* inches more ; during the third beat it will have fallen through *five* inches ; during the fourth beat it will have fallen through *seven* inches ; during the fifth beat it will have fallen through *nine* inches; and so on. Now, if these distances be compared with those given in the second column of the preceding table, they will be found to correspond ; the spaces through which the weight w *descends* in successive seconds being, as shown in that table, *expressed by the odd* numbers.

In the same manner, it will appear that the height through which the weight W falls during the first second, being one inch, the height through which it falls during the first two seconds will be four inches, the height through which it falls during the first three seconds will be nine inches, the height through which it falls during the first four seconds will be sixteen inches, the height through which it falls during the first five seconds will be twenty-five inches, and so forth.

These numbers correspond with and verify those given in the fourth column of the preceding table.

The principles of Atwood's machine being explained in *fig.* 51, its form, as actually constructed, with all its accessories, is shown in *figs.* 52, 53, 54.

111. **Morin's apparatus.**—Another apparatus for the experimental illustration of the laws which regulate the descent of bodies by gravity has recently been invented and constructed by M. Morin, the director of the *Conservatoire des Arts et Métiers* at Paris. It consists of a cylinder or drum A A, *fig.* 55, mounted on a vertical axis, on which it is moved uniformly by a train of clock-work B, impelled by a weight C, rendered uniform by a fly *a*.

A small cylindrical weight D, carries a pencil, the point of which is pressed gently against the surface of the cylinder. This weight is guided in its fall between two vertical wires, passing through holes in a plate fixed upon it, which projects from it at either side, so that in its fall it keeps constantly parallel to the cylinder and at the same distance from it. An apparatus for detaching this weight is fixed above the cylinder. By merely pulling a cord, like a bell-pull, shown in the figure beside the weight D, this weight can be let fall. Now, if the cylinder did not revolve, the pencil carried by the weight would evidently trace a vertical line upon its surface. If, on the other hand, the weight were stationary and the cylinder revolved, the pencil would trace a horizontal circle around it. But, if while the cylinder turns uniformly round its axis the weight falls, the pencil will trace around the surface of the cylinder a curved line, such as is shown in *fig.* 56. To facilitate the experiment, the surface of the cylinder is divided into equal parts vertically by the parallel lines *r r*, *s s*, *t t*, &c., &c.; and since the motion of the cylinder is uniform, the intervals between the moments at which these parallels pass under the pencil are equal. The vertical space through which the weight falls in the first interval will be $n' n$; in the first two intervals $p p$; in the first three $q' q$, and so on.

It is evident, therefore, that the circumstances of the de-

scent will be indicated exactly by this means. In Atwood's

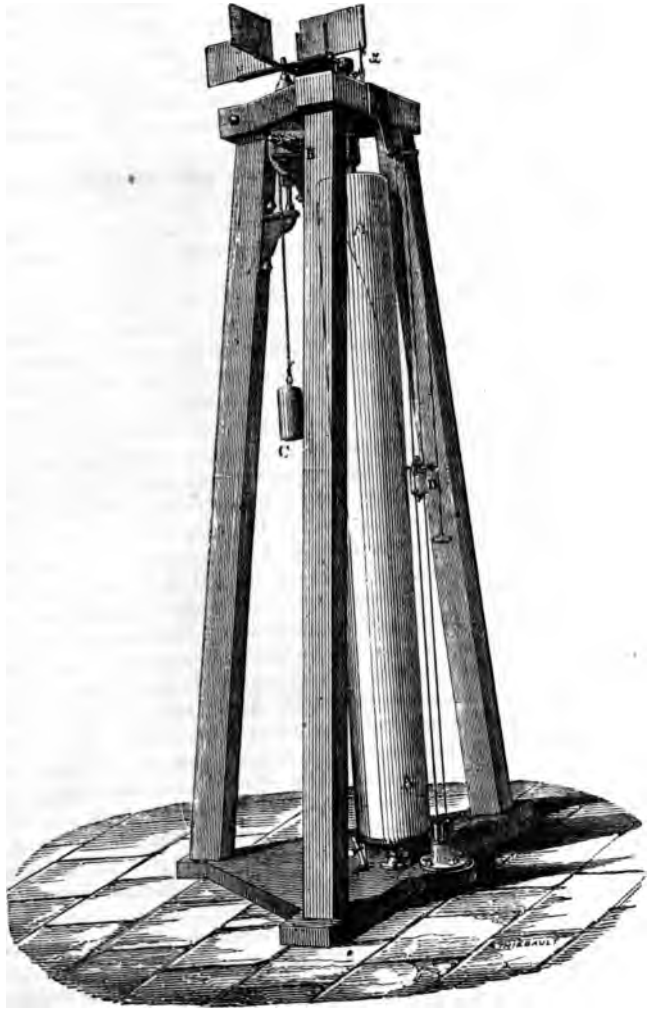

Fig. 55.

machine it is not, properly speaking, a body falling freely

which is observed, but one the rate of whose fall is diminished in a known proportion. Here, however, it is the actual descent which is exhibited and analysed.

112. **Law of free descent.**—It is evident that the numbers, which express the heights through which the bodies fall in any number of seconds, counted from the commencement of the motion, are the squares of the numbers of seconds ; and hence we have the following general principle :—

When a body is moved by a uniformly accelerating force, such as gravity, the spaces through which it moves, counted from the commencement of the motion, will be proportional to the squares of the times, and the spaces through which it moves in equal intervals of time will be as the odd numbers.

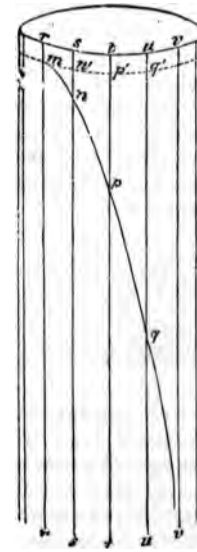

Fig. 56.

These rules, which are of the highest importance, may be conveniently reduced to algebraical symbols. Let us express by $\frac{1}{2}g$ the space through which a body, urged by a uniformly accelerating force from a state of rest, would move in one second, a space which, in the case of gravity, is 16 ft. 1 in., or 193 inches. Then it is evident, from what has been stated, that we shall find the space which the body would move through in any given number of seconds, counted from the commencement of its motion, by multiplying $\frac{1}{2}g$ by the square of this number of seconds.

113. **Formulæ for the motion of falling bodies.**—Hence, in general, if T express the number of seconds during which the body has been moving from a state of rest, $T^2 \times \frac{1}{2}g$ will express the entire space through which the body has moved in the number of seconds expressed by T. If this space, then, be expressed by H, we shall have

$$H = T^2 \times \tfrac{1}{2}g.$$

In like manner, since it has been established that the velocity which is gained in falling during one second is such that in each second the body would with that velocity move through a space equal to twice that through which it had fallen, it follows that the velocity acquired in one second is g; in other words, it is such that a body moving with that uniform

velocity would move through a space expressed by g in each second.

But it has also been shown that the velocity augments in proportion to the time, and that the velocity in two, three, four, and five seconds is two, three, four, and five times the velocity in one second. To find, therefore, the velocity acquired in any number of seconds, we shall only have to multiply g by that number of seconds. If, then, T express the number of seconds during which the body has been falling, and V the velocity which it has gained in the time T, we shall have

$$V = T \times g.$$

The two preceding formulæ include the whole theory of falling bodies *in vacuo*. From these may be deduced the following formula, by which the velocity which is acquired in falling through any given height is known :—

$$V^2 = 2H \times g.$$

It remains now to show that by Atwood's machine the numbers given in the third column of the preceding table may be verified ; that is to say, to demonstrate, by direct experiment, that the velocity imparted to the body in its descent increases in the proportion of the time of the fall.

To accomplish this, the following arrangements are made. The preponderating weight used to produce the descent of W has the form of a long narrow bar D (*fig.* 51), which is laid across the upper surface of the cylindrical weight W. A ring E, large enough to allow the weight W to pass through it, but not large enough to allow the bar resting on the weight to pass, is attached to the scale at the division marked 1. If the weight be now brought to such a position that its upper surface shall coincide with the zero of the scale, and if it be let fall at a moment corresponding with one beat of the pendulum, its upper surface will arrive at the ring E at the moment of the next beat, and the ring which allows the weight W to pass freely through will catch the bar, which will rest upon it. After the top of the weight, therefore, has passed the ring, the weight W being relieved from the bar, by whose preponderance its motion was accelerated, will continue to move downwards without further acceleration, with the velocity it had acquired at the end of the first second, such velocity being now continued uniform. If, then, the descent of this weight uniformly downwards be compared with the beats of the pendulum, it will be found to move *uniformly at the rate* of two inches per second.

Thus, we infer, first, that the velocity imparted at the end of the first second is such as to make the weight w move uniformly in one second through double the space through which it has fallen, and that such velocity is at the rate of two inches per second.

Let the ring be now moved to the fourth division of the scale and the bar being put upon the weight w, let the experiment be repeated. It will be found that at the end of two seconds the bar will strike the ring and the weight will pass below it, moving with a uniform velocity ; and by comparing its motion along the scale with the beats of the pendulum it will be found that this velocity is at the rate of four inches per second.

Again, let the position of the ring be fixed at the ninth division of the scale, and, replacing the bar, let the experiment be once more repeated. It will be found that the bar will strike the ring at the end of the third second, and that the weight w, when disengaged from the bar, will continue to descend with the uniform velocity of six inches per second. The same experiment may be repeated for as many seconds as the height of the scale may admit, and like results will be obtained. We may thus obtain a complete verification of the numbers contained in the third column of the table, p. 102.

114. **Calculation of motion due to gravity.**—From these experiments we are enabled to calculate the height through which a body would fall in one second of time by the effect of the force of gravity, and independently of any influence from the resistance of the air.

It appears from what has been stated, that when the magnitude of the weights w′, w, and w was so adjusted that the height of the descent was 193 times less than the height with which w would fall freely, the height through which it fell was one inch. It consequently follows, that if w were submitted to the unimpeded action of gravity it would fall through 193 inches, or 16 ft. 1 in., in the first second.

The table at page 102, compared with this result, will show all the circumstances attending the descent of bodies falling freely, 16 ft. 1 in. being the unit of the table. Thus, if we desire to ascertain the height from which a body would fall in five seconds, we take the number in the fourth column of the table opposite 5 seconds, which is 25, and multiply it by 16 ft. 1 in.; the product, which is 402 ft. 1 in., will be the height required.

In the same manner, if it be required to determine what space a falling body would descend through in the fifth second

of its motion, we take, in the second column of the table, the number opposite 5 seconds, which is 9; we multiply 16 ft. 1 in. by this number, and find the product, which is 144 ft. 9 in., which is the space required.

In like manner, if it be required to determine with what velocity a body would strike the ground after falling during an interval of five seconds, we take the number in the third column of the table opposite 5 seconds, which we find to be 10, and we multiply 16 ft. 1 in. by this number. The product, which is 160 ft. 10 in., will be the velocity required; and we infer that the body thus falling would have, when it strikes the ground, a velocity of 160 ft. 10 in. per second.

It will be observed that the numbers in the first column of the table now referred to, and which express the time of the fall, are the square roots of the numbers in the fourth column, which express the height from which the body falls. We have, therefore, this general principle of uniformly-accelerated motion:

When a body is moved by a uniformly accelerating force, the times required to move through any given space are proportional to the square roots of those spaces.

By the aid of this rule, and the results already obtained, we are enabled to ascertain the time which a body would take to fall from any given height. Thus, if a body be supposed to fall from a height of 10,000 feet: Find the number of times which 16 ft. 1 in. are contained in 10,000 feet, which is done by dividing 10,000 by $16\frac{1}{12}$. The quotient is 621·76.

This number is then the square of the number of seconds in the time of the fall. The square root of this obtained from a table of square roots being 24·9, we infer that the time a body would take to fall through the height of 10,000 feet is 24·9 seconds.

In the same manner it follows that, since the velocity acquired by a body in its fall is proportional to the time of the fall, and since the time of the fall itself is proportional to the square root of the height, the velocity acquired must also be proportional to the square root of the height.

If we would, therefore, determine the velocity with which a body falling from a given height would strike the ground, independently of the effect of the resistance of the air, we are enabled to do so by these principles.

Thus, let it be required to determine the velocity with which a body falling from the height of 10,000 feet would strike the ground. *It has* been just shown that the time of the fall would

be 24·9 seconds, and it has been already demonstrated that the velocity acquired by the body is such that it would move uniformly over a space equal to double the height through which it has fallen, and in the same time. Therefore, the velocity in this case would be such that in 24·9 seconds the body would move uniformly through 20,000 feet; and consequently, by dividing 20,000 by 24·9, we shall obtain the velocity in feet per second, which appears, therefore, to be 803 feet per second.

The velocity with which a falling body strikes the ground increases thus in a much less proportion than the height from which it falls. If the height be augmented in a fourfold proportion, the velocity at the instant of reaching the ground will only be doubled; and if the height be augmented nine times the velocity will only be three times greater, and so on.

115. **Retarded motion of bodies projected upwards.**—All the circumstances attending the accelerated descent of falling bodies, which have been explained in the present chapter, are exhibited in a reversed order when a body is projected upwards. Gravity then acts as a uniformly retarding, instead of a uniformly accelerating force, depriving the body so projected of equal quantities of velocity in equal times; and, further, it is apparent that the velocities which the force of gravity thus destroys in a body projected upwards in any given time are exactly equal to those which it would impart to a body in the same time when falling freely.

Thus, if a body be projected vertically upwards with the velocity which it would acquire in falling freely during one second, the body so projected will rise exactly to the height from which it would have fallen in one second, and at any point of its ascent it will have the velocity which it would have at the same point if it had descended.

To determine, therefore, the height to which a body will rise projected upwards with a given velocity, it is only necessary to determine the height from which it would fall to acquire the same velocity.

In like manner, to determine the time which a body would take to rise to a certain height when projected upwards, it is only necessary to determine the time which it would take to fall freely from the same height.

116. **Motion down an inclined plane.**—A plane and hard surface, which is neither in the vertical nor horizontal position, is called an inclined plane. In *fig.* 57, if the line W O be vertical, then W M will represent an inclined plane.

Bodies which descend upon inclined planes move under the

influence of a uniformly accelerating force similar to that of gravity, omitting, for the present, the consideration of friction, and, as usual, the resistance of the air.

Let W be a body placed upon the inclined plane. The force of gravity acts upon it in the vertical direction W O. Let this line W O, thus representing the force of gravity, be considered as the diagonal of a parallelogram, of which W N and W M are sides, the side W N being perpendicular to W M. The entire force of gravity, therefore, represented by W O, and acting on the body W, will, by the principle of composition of forces, be equal to the two forces represented by the sides of the parallelogram W M and W N. But W N, being perpendicular to the plane, is counteracted by it, and exhibits itself merely as a pressure upon it. The component W M, however, being in the direction of the plane and downwards, will cause the body to move down the plane.

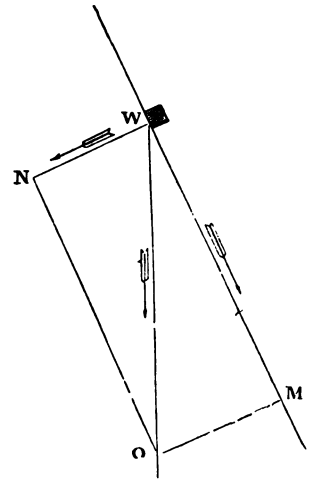

Fig. 57.

The proportion of this accelerating force down the plane to that of gravity acting freely in the vertical direction, will, therefore, be that of the lines W M to W O. If W O be the height through which a body would fall vertically in one second, then W M will be the distance through which the body would fall in the first second on the inclined plane. It is evident, therefore, that by taking W O equal to 193 inches, the distance W M will be actually that down which the body W, independently of friction, &c., would fall in the first second.

If it be desired to ascertain the force with which the body W presses on the inclined plane, let W O be taken so as to consist of as many inches as there are pounds in the weight W. Then W N will consist of as many inches as there are pounds in the pressure which W exerts on the plane.

The motion down an inclined plane, therefore, being uniformly accelerated, like that solely due to gravity, but only *modified in its* velocity in a certain ratio depending on the

inclination of the plane, all the circumstances which have been already explained in reference to the accelerated motion of bodies falling freely will be similarly exhibited in the motion down an inclined plane.

Let W M (*fig.* 58) be an inclined plane, and W O the vertical line, and let us suppose two bodies dismissed at the same moment from W, one falling down the vertical line W O, and the other down the line W M. Let I, II, III, IV, V, be the points upon the vertical line O, at which the body is found at the end of one, two, three, four, and five seconds.

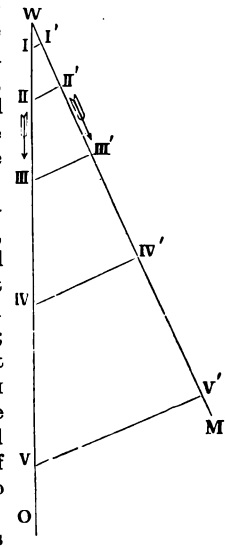

Fig. 58.

If from this point lines be drawn perpendicular to W M, the points I′, II′, III′, IV′, V′, where these perpendiculars will meet the inclined plane, will be those at which the body falling down such inclined plane will be found at the same instants; that is to say, at the end of the first second the one body will be found at I and the other at I′, at the end of the second second the bodies will be found respectively at II and II′, at the end of the third second at III and III′, and so on.

The force down the inclined plane is just so much less in intensity than the force of gravity as the spaces W I′, W II′, W III′, &c., are respectively less than W I, W II, W III, &c. Consequently, it is evident that these spaces, being in the proportion of the forces, will be described in the same time, as, indeed, has been already proved.

In this manner, therefore, the circumstances of the motion down an inclined plane may always be determined with reference to the circumstances of the motion down a vertical line.

If it be desired to ascertain the points at which a body falling down an inclined plane will acquire the same velocities which it acquired in one or more seconds in falling freely in the vertical direction, it is only necessary to consider that the more feeble force down the plane requires a proportionally greater space to produce a given velocity. If, then, W M and W O (*fig.* 59) represent, as before, an inclined plane and a vertical line,

and if, as before, I, II, III, IV, V represent the points at which the body, falling vertically, would be found at the end of one, two, three, four, and five seconds, then the points on the plane where the same velocity would be attained as the body had at the points I, II, III, IV, and V, will be determined by drawing lines from the points I, II, III, IV, and V respectively in the horizontal direction, because, by these means, the line W V on the plane will be just so much longer than the line W I as the force of gravity, acting freely, is more intense than the force down the inclined plane; consequently the velocity which will be acquired at V on the plane, will be the same as the velocity acquired at I in falling freely.

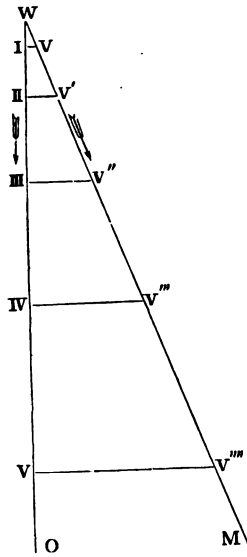

Fig. 59.

In the same manner, it will appear that the velocities acquired on the plane at the points v′, v″, v‴, v⁗ will be the same as the velocities acquired in falling freely at the points II, III, IV, and V.

117. **Projectiles.**—We have considered the case where a body, acted on freely by the force of gravity, is either allowed to fall vertically downwards, or is projected vertically upwards. We shall now consider the other cases, in which a body is projected in any other direction, not vertical, and then abandoned to the action of gravity,—a problem which forms the foundation of the doctrine of projectiles. The solution of this problem follows immediately from the principles which determine the motion of a body falling freely, as explained in the present chapter, and the composition of motion.

Let us first take the case in which a body W (*fig.* 60), as, for instance, a ball shot from a cannon, is projected in the horizontal direction W M. If the force of gravity did not act on it, it would move forwards towards M with the velocity of projection continued uniform, and, in virtue of such motion, would pass over equal spaces in equal times. Thus, if, by the velocity of projection, the body would move from W to I′ in the first second, it would move from I′ to II′ in the next second, from II′ to III′ *in the following* second, and so on, these successive spaces being *equal.*

But if, on the other hand, the body, without being projected at all, were disengaged at w, and left to the action of gravity alone, it would, as has been already explained, descend vertically, and would be found at the points I, II, III, IV, V at the end of the successive seconds, the distance being, as already explained, represented by the numbers 1, 4, 9, &c.

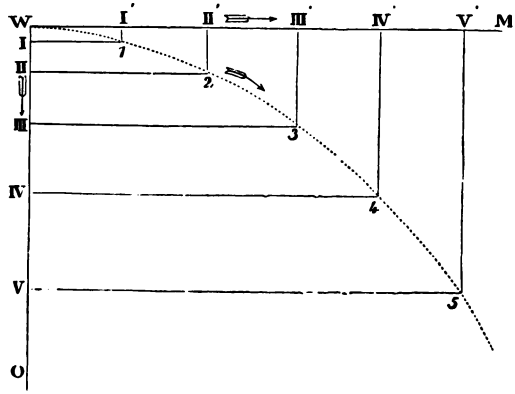

Fig. 60.

Now, the body leaving w, being submitted to both these forces simultaneously, will, by the composition of motion, be found at the end of each successive second at the extremities of the diagonals of a parallelogram whose sides represent these motions. Thus, at the end of the first second, the body will be found at the point I, being the extremity of the diagonal of a parallelogram whose sides are the space w I′, through which the body would move in virtue of the velocity of progression, and w I the space through which it would fall freely in the same time by gravity. If the force of gravity would have made it move over w I with a uniform motion, then the body, in moving from w, would follow exactly the diagonal of the parallelogram. But the force of gravity imparting to the body not a uniform, but an accelerated motion, first very slow and then more rapid, the body will pass from w to I, not by a strict diagonal course, but by a curved line, as represented in the figure.

In the same manner, at the end of two seconds, the body will be found at 2. For it is actuated at the same time by two motions ; first, the projectile motion, which, acting alone upon it, would carry it uniformly from w to II′, and, secondly, the

force of gravity, which, acting alone upon it, would cause it to to fall from W to II. At the end of two seconds it will therefore be found at the point 2, being the extremity of the diagonal.

But, as before, the motion from W to II not being uniform but accelerated, first slow but afterwards more rapid, the body will pass from W to 2, not along the diagonal, but over the curved line represented in the figure.

The same explanation will be applicable to its remaining course, and it will follow that the body will pursue the curved course from W to 5 in five seconds, in consequence of the combination of the velocity of projection imparted to it, and represented by W V′, with the descending motion imparted to it by gravity, and represented by W V.

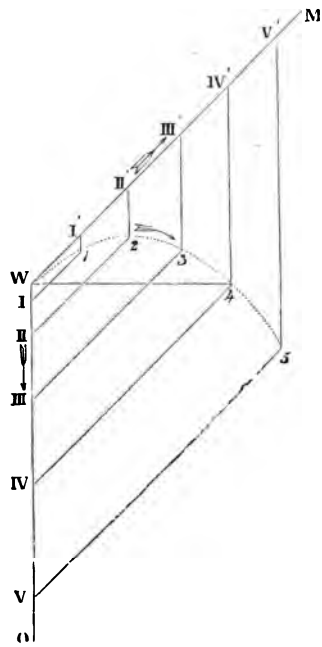

Fig. 61.

118. **Oblique projection.**—In this case we have supposed, for simplicity, the body to be projected in the horizontal direction; but the same principles will explain its motion, if projected in an oblique direction, such as W M (*fig.* 61).

As before, let the space which the body would move over in one second, in virtue of the projectile force alone, gravity being supposed not to act upon it, be W I′. It would move over the equal spaces terminating at I′, II″, III′, IV′, V′ in the successive seconds.

On the other hand, suppose the body to be acted on by gravity alone, independently of the projectile force. It would then, as before, moving in the vertical line W O, be found at the end of the successive seconds at the points I, II, III, IV, V.

Now, by the principle of the composition of motion, the body will actually be found, in consequence of the simultaneous effects of the two motions imparted to it by gravity and by the projectile force, at the end of the successive seconds, at the points 1, 2, 3, 4, 5, which are the extremities of the diagonals of

parallelograms, whose sides are respectively the spaces which the body would describe in virtue of the force of projection, and of gravity, each acting separately. The course of the body will be the curved line represented in the figure, and not the straight diagonal, for the reasons already explained.

119. **Motion in parabolic curves.**—The path which the projectile follows in this case is a curve, known in geometry as the *parabola*, the property of which is, that the sides of the parallelogram whose diagonal determines its successive points are related to each other as the successive whole numbers 1, 2, 3, 4, &c., and their squares 1, 4, 9, 16, &c. This property may be expressed algebraically by saying that the ratio of the square of the side WI′ to the side WI (figs. 60 and 61) is the same as that of the square WII′ to WII, and so on; in other words, the ratios

$$\frac{(\text{WI}')^2}{\text{WI}}, \quad \frac{(\text{WII})^2}{\text{WII}}, \quad \frac{(\text{WIII}')^2}{\text{WIII}}, \text{ etc.},$$

are constant for the same velocity and direction of projection.

120. **Resistance of the air.**—The conclusions and results at which we have arrived in the preceding articles rest upon the supposition that the body moves either *in vacuo* or is a medium which offers no resistance to it, so that the forces of gravity and of projection which are imparted to the body are in no way diminished during its motion. In reality, however, the motion of all projectiles takes place in the air, which exerts resistance to the motion of bodies, and, moreover, the resistance thus exerted is by no means uniformly exerted but varies constantly, increasing in general with the square of the velocity. The real path, therefore, of projectiles differs more or less from the parabola explained above. The deviation is not very considerable when the velocity of the moving body is not great; but when the projectile is driven with great velocity, as in the practice of gunnery, then the deviation from the parabolic path is so considerable that the above theory becomes altogether inapplicable.

121. **Application of projectiles in gunnery.**—According to what has been explained above, a ball projected from any missile will not follow the direction of the axis of the barrel, but will proceed in a curved line, concave downwards, to which the direction of the barrel is a tangent; thus, for example, if the barrel be directed horizontally in the line W M, *fig.* 60, the ball proceeding along the dotted curve will fall below the line of aim; and if that line be directed to the object the ball will miss it by passing too low; to hit an object, therefore, at any proposed distance, the gun must be aimed in a direction above that of the object, more or less according to the distance of the object and the force of the charge; thus, for example,

if the gun be discharged from W, *fig.* 61, and the object be at 4, the force of the charge being such as to cause the ball to move in the curve 1, 2, 3, 4, 5, it is evident that in order to hit the object at 4, the gun must be aimed in the direction W M. In practical gunnery, the force of the charge for each form and class of gun is known, and expedients are found by which the distance of objects aimed at may be determined with sufficient approximation for practical purposes. With these data the inclination at which aim is to be taken with each sort of weapon, in order to hit the object, is determined by simple rules, which can be, without difficulty, applied in the field.

Until recently the muskets placed in the hands of soldiers were usually aimed so that the line of sight was at once parallel to the barrel and directed to the object, as shown in *fig.* 62. But since the improvements which have recently taken place in musketry, and more especially by the introduction of the improved rifles, greater precision of aim has been attained. With

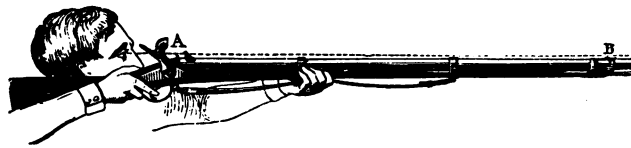

Fig. 62.

an aim directed, as in *fig.* 62, at the object, the ball must necessarily pass below it. So long as the range of a musket was of limited extent, and when great precision was not expected to be attained, this deviation was disregarded; but since, by the modern improvements, the range has been greatly augmented, the drop of the ball produced by the curvature of the projectile would become so considerable as to deprive the weapon of the necessary precision. On the modern guns, therefore, a double sight is provided, by which the elevation necessary to ensure point-

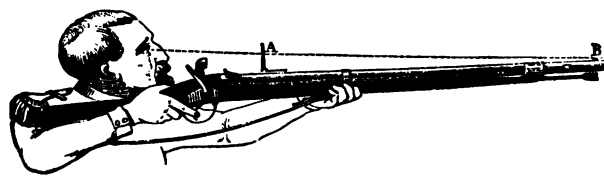

Fig. 63.

blank precision can always be given to the barrel; one of the *sights* B, *fig.* 63, is fixed in the usual manner on the extremity of

the barrel, while the other, A, is one which is graduated, and sometimes provided with an adjustment, by which it can be adapted to objects at different distances, so as to hit them point-blank.

122. **Effect of a hammer.**—When a nail is driven by the strokes of a hammer the resistance which the moving body has to overcome is the friction between the nail and the wood; the momentum of the hammer is, however, imparted to the wood, and if the latter have not sufficient resistance to prevent it from unduly yielding to the force, the nail will not penetrate. In such a case, for example, as that which is represented in *fig.* 64, where the nail is to be driven into a board, having no support behind it, and not thick enough itself to offer the necessary resistance, the blows of the hammer, if strong enough, would break the board, but would not drive in the nail. The object is attained by applying behind the board *fig.* 65, a block of wood, or, still better, a lump of lead, against which the blows of the hammer will be directed. It is not, however, as might be supposed, by any increased resistance thus opposed to the blows that in this case the object is attained. To comprehend the effect produced, it is necessary to consider that in each case the momentum of the hammer is equally imparted to the mass which it strikes; but in the one case (*fig.* 64) this momentum is received by the board alone, which, having little weight, is driven by it through so great a space as to produce a considerable flexure, or even fracture; but in the second case, the same momentum, being shared between the board and the block of wood or metal applied behind it, will produce a flexure of the former, less in the same proportion exactly as the weight of the board and block applied to it together is greater than the weight of the board alone.

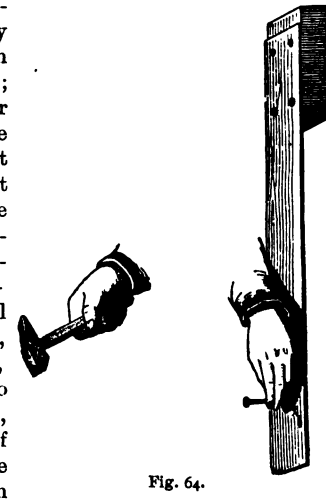

Fig. 64.

The same principle serves to explain a trick, or *tour de force*, sometimes shown by public exhibitors. The exhibitor, extending his body horizontally, his legs and shoulders being supported, causes a heavy anvil to be placed upon his chest and

abdomen ; men employed for the purpose then give successive blows of heavy sledge-hammers upon the anvil, without injury to the exhibitor ; blows which would speedily put an end to his exhibitions if they were received without the interposition of the anvil.

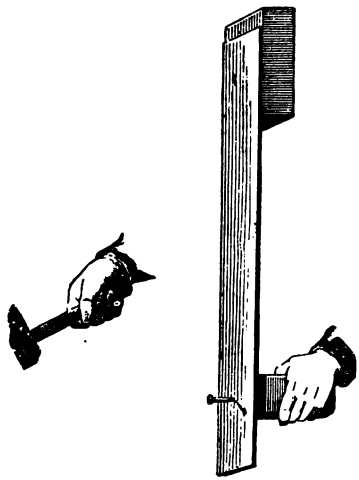

Fig. 65.

To explain this feat, it is only necessary to consider that the whole momentum of the sledge, being imparted to the anvil, will give the latter a downward motion just as much less than the motion of the sledge as the mass of the sledge is less than the mass of the anvil. Thus, if we suppose the weight of the sledge to be 100 times less than that of the anvil, the downward motion of the anvil upon the body of the exhibitor will be also 100 times less than the motion with which the sledge strikes it, and the body of the exhibitor easily yielding to so slight a displacement, he passes through the ordeal with complete impunity.

123. **Influence of time upon the effect of force.**—Some extraordinary and apparently unaccountable mechanical effects are explained by the fact that the effects of forces are greatly modified by the continuance of their action ; a resistance which will be sufficient to counteract a force whose action continues only for a few seconds will often yield to it if it act for as many minutes.

It is a fact familiar to all skaters that they may pass with impunity over thin ice, which would break under their weight if they moved over it more slowly, and still more if they stood still upon it.

The effect of forces acting upon solid bodies is transmitted from molecule to molecule through their dimensions, before it can affect the aggregate of their mass ; and if, from the nature of the force, the entire duration of its action be less than that which is necessary to propagate through the mass its effect, that effect can only be produced upon the part of the mass on which *it immediately* acts.

Let us suppose, for example, that a musket-ball be pressed with the hand against a pane of glass, it will break the glass to pieces—the fractures extending to its very edges. If the same ball be flung from the hand with some velocity against the same pane, it will pass through it, making a hole considerably larger than the ball, surrounded by diverging cracks. But if the ball be propelled from a gun against the same pane of glass, it will pierce it with a hole whose diameter is nearly equal to that of the ball, and the hole will have clean edges as if the glass were bored through with an auger. Now these effects are easily explained: when the ball is merely pressed against the glass, the force has such continuance of action that its effects have time to be propagated to the limits of the pane, which is accordingly broken in pieces; when the ball is projected against the glass with the hand, the action is more sudden and of shorter continuance, and consequently the fractures round the hole are much less extensive; but when, in fine, it is discharged from a gun, its action being instantaneous, there is no time for the propagation of the force, and it merely drives before it the portion of the glass which lies in its way.

If a cannon-ball be flung with the hand against the panel of a door, suspended on hinges, it will cause the door to turn on its hinges, yielding to its force; but if the same cannon-ball be projected against the door by a cannon, it will pass clean through the panel, making a hole equal in diameter to the ball, and without imparting any motion to the door.

In the former case, the action is slow enough to allow the force to be propagated to all parts of the door, and therefore to move it; but in the latter case, being instantaneous, it only affects the parts of the wood which lie immediately in its way.

CHAPTER X.

CENTRE OF GRAVITY.

124. **Weight.**—If a body be prevented from moving in obedience to the force of gravity by a fixed axis passing through it, a fixed point from which it is suspended, or a surface placed beneath it, the effect of gravity upon it will be manifested by a

pressure produced upon such axis, point of suspension, or surface. This pressure is called the *weight* of the body.

As gravity acts separately upon all the component particles of a body, the weight of such body is composed of the aggregate of the weights of all its particles. This, which is manifest from what has been already explained, may be rendered still more clear, from considering that if a body be divided into parts, no matter how minute and numerous, each of these parts will have a certain weight, and the aggregate amount of their several weights will be exactly equal to the weight of the body of which they are the fragments.

Such a division may be carried to the most extreme practical limit of comminution by pounding, grinding, filing, and other processes known in the arts, and the weight will still be divided as the matter is divided; nor is it possible, even in imagination, to conceive any degree of comminution so great that the same principle will not prevail; and it may, therefore, be considered as established that every individual atom which composes a body has weight, and that the weight of the mass is the sum of the weights of all its constituent atoms or molecules.

125. **Effect of cohesion.**—If the particles composing a body had no mutual coherence or other mechanical connection having a tendency to retain them in juxtaposition, each particle would obey the force of gravity independently of the others, and they would fall under the action of gravity like a mass of sand. But, if they be so connected by their mutual cohesion, as they are in fact in all solid bodies, this cohesion will resist any tendency to separate them; they will maintain their juxtaposition, the body will retain its form, and the several forces with which gravity affects them will become compounded, so as to produce a single force or pressure, which is the resultant of all the separate forces impressed upon the particles.

126. **Resultant of forces of gravity acting on molecules.**—As this resultant enters as a condition into every mechanical question affecting bodies, it is of the greatest importance to investigate the conditions by which in every case its intensity and the line of its direction may be determined.

It has been already shown that the weights of all the particles composing a body act in directions parallel to a plumb-line, or perpendicular to a level surface. But it has been also demonstrated that when any number of forces act in the same direction in parallel lines their resultant is a force acting in a line parallel to them, and in the same direction in this line, and *that its intensity* or magnitude is equal to the sum of these

forces. The resultant, therefore, of the forces of gravity affecting all the particles of any mass of matter is a single force acting vertically downwards, which is equal to the sum of all the forces affecting the particles severally, and therefore equal to the weight of the mass.

If, for example, A B, *fig.* 66, represent a mass of matter, and the small arrows pointing vertically downwards represent the direction of the gravitating forces of the particles composing such mass, then it follows, from what has been explained, that the resultant of all these forces, or a single force equal to them, will have a direction parallel to them, such as D E, and will, in its intensity, be equal to their sum.

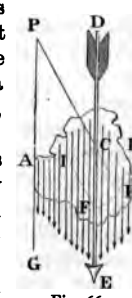

Fig. 66.

But this is not yet sufficient to indicate this resultant in a definite manner. We as yet only know that its direction is parallel to the common direction of the gravity of the particles; but innumerable lines may be imagined passing through the body vertically downwards, and the question still remains to be determined which of these lines is the direction of the resultant.

When the body in question has a determinate form and a uniform density, or even a density varying according to some known conditions, the principles of mathematical science supply methods by which the line of direction of the resultant may be determined; but we shall here adopt a more simple and generally intelligible method of explanation.

If we suppose the line represented by the great arrow D E (*fig.* 66) to be that of the resultant, then it is evident that if any point, such as C in that line, be supported, the body will remain at rest, because the resultant of all the forces acting upon the body having the direction D E will be expended in pressure on the fixed point C. The effect, therefore, will be that the whole weight of the body will press upon C, and the body will remain at rest.

The same would be true for any point whatever in the direction of the arrow D E. If, for example, D were a pin from which a thread was suspended, and this thread were attached to the body at any point in the line D C, then the body would still remain at rest, the whole weight being expended in pressure upon the pin at D; for, as before, the resultant of all the forces of gravity acting upon the component particles of the body would have the direction D E, and would therefore be supported by the fixed pin at D.

But if a point of support be selected which is not in the direction of the resultant D E, such as P, and a string be carried from P to any point of the body, such as C, then the body, although it will not be permitted to descend vertically, in obedience to gravity, will nevertheless not remain at rest.

If we suppose the weight of the body to be expressed by the line C F, let this line be taken as the diagonal of a parallelogram whose sides are C H and C I, one in the direction of the cord, and the other at right angles with it—that portion of the weight which is represented by C H, and which is in the direction of the string, will act upon the fixed point P, and produce pressure upon it. The portion of the weight which acts in the direction C I will cause the body to move towards the vertical line P G, which passes directly downwards from the point of suspension. The body will therefore move towards that vertical line. If the body had been on the other side of the vertical line P G, it would still have moved towards it, and therefore in a direction contrary to its present motion.

127. **Experimental method of determining the direction of the resultant.**—From what has just been stated, it follows that a body which is supported by a fixed point cannot remain at rest unless the resultant D E of all the parallel forces which gravity impresses upon its molecules passes through the point of suspension.

This result supplies us with a practical means of ascertaining the direction of the resultant of the weights of all the component parts of a body with reference to any given point taken upon it, since we have only to suspend the body by a string attached to the given point, and allow it to settle itself to rest. When thus at rest, the resultant of the weights of all its particles will be in the direction of the string by which it is suspended.

If the same body be suspended by different points upon it, the parallel directions of the gravitating forces of its particles will differ in reference to the body, although they are the same in reference to the direction of the suspending string, being always parallel to it. Thus, for example, if an egg be suspended with its length vertical, the parallel forces which gravity impresses on its particles will be parallel to its length ; but if it be suspended with its length horizontal, then the parallel directions of the gravity of its particles will be perpendicular to its length.

128. **Direction of resultant varies with point of suspension.**—Since in each case the resultant of these parallel *forces* will coincide with the direction of the string, it must in

the one case pass through the egg in the direction of its length, and in the other in a direction at right angles to its length. In like manner, the body being supported by any point whatever taken upon it, the direction of the string will be different for each such point; and consequently, there will be an infinite variety of resultants of the gravitating forces of the particles of the body, according to the different points by which it may be suspended.

Now, the question arises, whether there is any relation between this infinite variety of resultants; for if such be not the case, the determination of the resultant of the gravitating forces of a body would be a problem which would present itself under an infinite diversity of forms and conditions for every individual body.

129. **Point of intersection of resultants.**—This question may be solved by a very simple experiment; and its solution is attended with a remarkable and important result.

Take a solid body of any form, regular or irregular, and composed of a material which is easily perforated without diminishing its mass, or considerably deranging its structure. Take, for example, a mass of putty of any form. Let this mass be suspended by a thread attached to a fixed point, which it may easily be if previously surrounded by a thread forming a loop. When at rest, the resultant of the forces of gravity, acting upon all its particles, will be a vertical line penetrating its dimensions in the direction of the suspending thread. Take a needle, and pierce the putty in this direction. The hole which is thus made through it will represent the direction of the resultant of the gravitation of its particles.

Let the mass be now detached from the thread of suspension, and let it be again suspended, but in a different position, which may be easily accomplished by the loops of thread surrounding it. The mass will again settle itself into a position of rest, and, as before, the direction of the resultant of all its gravitating particles will be a vertical line in the exact direction of the suspending thread. Let the putty, as before, be thoroughly pierced in this direction with a needle.

Let the same experiment be repeated in three or four other different positions of the mass, so that we shall obtain several holes pierced through the body by the needle, representing the direction of the resultant of the gravitating forces, in the several positions in which the body was suspended.

Now a curious relation will be found to exist between the several directions in which the needle has pierced through the

putty. It will be found, in fact, that all these lines of perforation intersect, at a common point, within the dimensions of the body. This fact may be easily established.

Let a needle be inserted in any one of the perforations, and it will be found that another needle cannot pass through any of them, for its progress will be stopped by the needle already inserted. All the perforations, therefore, must intersect each other at a common point within the putty.

It appears from this experiment that there is a certain point, within the dimensions of the body, through which the resultant of all the gravitating forces of the particles of the mass must pass, no matter in what position the body may be placed.

130. **Centre of gravity.**—This result, which is of high importance, may be further illustrated and verified in the following manner :—

Let a flat, thin plate of metal, or a piece of card, of any form, however irregular, be pierced with small holes at several points, so that it may be suspended upon a horizontal pin, the plate itself being vertical. When so suspended, it can only remain at rest provided the resultant of the gravitating forces of its particles pass through the pin ; for otherwise, as has been already explained, the body would move, in one direction or other, round the pin on which it is suspended.

If a plumb-line be suspended from this pin, it is evident that when the plate is at rest the direction of the resultant of the gravitating forces must coincide with the direction of the plumb-line. Let a line then be traced upon the plate coinciding with the direction of the plumb-line. Let the body be then detached from the pin, and let the pin be inserted in another hole. The body will now hang in another position, the resultant of the gravitating forces of its particles again coinciding with the plumb-line. Let the direction of the plumb-line be traced upon the plate as before. In fine, let this experiment be repeated, with all the holes pierced in the plate, and it will be found that the lines traced upon the plate, indicating the various directions of the resultant of the gravitating forces of its particles, will intersect each other at one common point.

This common point, through which the resultants of the gravity of the particles of a body pass, is called its *centre of gravity.* A line drawn in the vertical direction through the centre of gravity of a body is called the *line of direction of the centre of gravity.*

If the centre of gravity of a body be supported on a point, *or axis, and the* body is free to turn round such axis, the body

will, in that case, remain at rest in any position in which it may be placed ; for, according to what has been already stated, the resultant of the gravitating forces of all its particles must be in the direction of a vertical line passing through the centre of gravity, and the whole weight of the body may be considered as acting in that line. But, if the centre of gravity be suspended by a pivot, or an axis, then the whole weight of the body will press upon such pivot or axis, no matter what be the position in which the body is placed. This may be easily verified by experiment.

Let the centre of gravity of any solid body be determined, by suspending it from different points, in the manner explained above, and let the body be placed upon a pivot or axis, passing through this point. It will be found to rest indifferently on such axis or pivot, in any position in which it may be placed. This experiment may be easily performed with a piece of card or pasteboard. The centre of gravity being determined, let a pin be passed through it, and it will be found that the card will rest in any position upon the pin.

131. **Case of regular figures.**—If a body, being of uniform density, have any regular figure, its centre of gravity will coincide with its geometrical centre ; for the matter composing the body will, in such case, be symmetrically arranged round that point, so that it is self-evident that if this point be supported the body will have no tendency to turn in any direction round it.

Thus, for example, it is evident, without experiment, that a ball or sphere of uniform density, such as a billiard-ball, has its centre of gravity at the geometrical centre. In like manner, a cube has its centre of gravity at the point where straight lines joining its opposite corners would intersect each other—that is to say, at its geometrical centre.

If the figure of a body be such that the matter composing it is uniformly distributed round any line passing through it, its centre of gravity must lie in that line, because, if it be suspended by a string in the direction of that line, it will remain at rest ; since the gravity of its particles, acting equally on every side of such line, will have no tendency to move it, it will equilibrate. Thus, it is evident that the centre of gravity of a cone, being of uniform density, must be situate in its axis, that is, in a straight line drawn from the point of the cone to the middle of its base.

It may be shown in the same manner that the centre of gravity of a regular oval solid will be in the axis of the oval ; that

of a cylinder will be at the middle point of its axis; and the centre of gravity of a straight rod of uniform thickness will be at the middle point of its length and at the geometrical centre of a section across it.

132. **Centre of gravity not within the body.**—It sometimes happens that bodies are of such a form that their centre of gravity is not placed within their substance. Thus, for example, the centre of gravity of a hoop is at its centre, an immaterial point, which does not constitute any part of the body in question. In like manner, in all hollow bodies the centre of gravity is an immaterial point. Thus it is in the centre of a hollow sphere. The centre of gravity of an empty box or cask is within it, at an immaterial point.

If a piece of wire, which when straight has its centre of gravity at its middle point, be bent into a curved form, its centre of gravity will be some immaterial point within the concave part of the curve. In like manner, if the wire be bent into the form of a V, the centre of gravity will be a point within the angle of the V.

These conclusions may be verified, and the centre of gravity in all such cases found, by suspending the body in different positions in the manner already explained.

Although the centre of gravity in such cases be not a material point, and not included within the matter of the body, it nevertheless still possesses those properties which it would possess were it actually included within the mass of the body.

To verify this by experiment, let us suppose a bar of metal A B (*fig.* 67), bent into a curved form. Let its centre of gravity be determined by suspension. When supported by the point A, let A C be the direction of the plumb-line, and when supported by the point B, let B D be the direction of the plumb-line. It follows, therefore, that the point O within the concavity of the curve where these two lines intersect, will be the centre of gravity. Let a light silk cord be attached to the points A and C, and stretched tight between them, and let another silk cord be stretched between the points B and D in the same manner. Now the point O, where these two points cross each other, will be the centre of gravity.

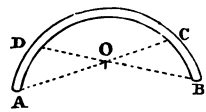

Fig. 67.

Let a cord be tied to the junction of the string at O, and let the upper extremity of this cord be attached to a fixed point, so that the wire may be thus suspended. It will be *found that* in this case the hoop of wire will rest in equilibrium

in any position in which it may be placed. In this case, the weight of the silk string being insignificant in comparison with the weight of the wire, does not disturb the position of the centre of gravity, which still remains at o.

133. **Properties of the centre of gravity.**—If a body, without being absolutely fixed in its position, so as to be immoveable, be nevertheless partially restrained, so as to be capable of moving only under certain conditions or within certain limits, then the centre of gravity will have always a tendency to move to the lowest position which the conditions under which the body is placed will admit of; and in all cases the body can never remain at rest unless its line of direction of the centre of gravity—that is to say, a vertical line passing through it—should pass through a point of support.

It may therefore be assumed, as a principle of the highest generality, that, in all cases in which a body is at rest, a vertical line passing through its centre of gravity must also pass through a point of support. If the point of support through which this line passes be placed above the centre of gravity, the body is said to be suspended; if it be placed below, it is said to be supported.

If a body be suspended from a fixed point by a string it will remain at rest, as has been already explained, provided its centre of gravity be placed in a vertical line under the point of support. But if the body be drawn out of that position, so that the centre of gravity will be on either side of such vertical line, then the body when disengaged will fall from such position to the vertical line, and in consequence of its inertia will continue its motion beyond the vertical line until it comes to rest; it will then return to the vertical line, and thus oscillate from side to side.

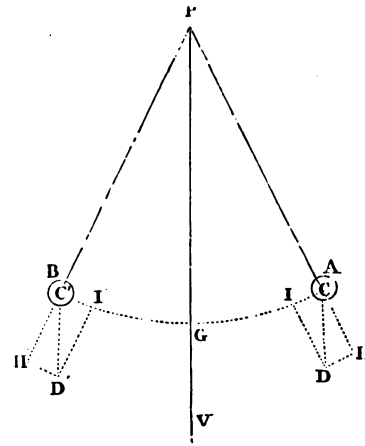

Fig. 68.

134. **Pendulum.**—Such a body constitutes what is called a *pendulum*.

Let P, *fig.* 68, be the point of suspension. Let P A represent the string, and C be the centre of gravity of the body. Let the weight of the body be represented by the vertical line C D. Let this be taken as the diagonal of a parallelogram, one of whose sides C H is in the direction of the string, and the other C I at right angles to it. The weight represented by the diagonal will thus, by the resolution of forces, be equal to two forces, one represented by C H and the other by C I. That which is represented by C H expends itself in pressure on the point of suspension ; the other, represented by C I, will cause the body to move towards the vertical line P V, and in so moving the centre of gravity will describe the circular arc CG. When the centre of gravity arrives at G, it will be in the vertical line P V, passing through the point of suspension ; and if the body were at rest, it would remain there ; but on arriving at G the body has a certain velocity and hence momentum, which it will retain in virtue of its inertia, until deprived of it by some external agency. It will therefore continue to move to the left of G, and the centre of gravity will describe the circular arc G C′. In ascending this circular arc the force of gravity has a tendency to destroy its velocity.

Let the weight of the body, as before, be represented by the the vertical line C′D′: it will be equivalent to the two forces represented by C′ H′ and C′ I′. The force C′ H′ is expended in pressure upon the point of suspension, P ; the other, C′ I′, has a tendency to carry the centre of gravity, C′, back to the point G, along the circular arc C′ G. This component of gravity, while the body moves from G to C, gradually deprives it of its momentum, and if the momentum be entirely destroyed at the point C′, then this same component of gravity, C′I′, will cause the body to return along the circular arc to the point G. In this manner the body would oscillate continually from side to side of the vertical line P V, the centre of gravity describing alternately equal arcs, G C and G C′. But the resistance of the air and other impediments have a tendency continually to diminish the length of the arcs, by which it departs from the vertical line, until at length the body ceases to vibrate and settles itself in such a position that the centre of gravity C will be quiescent in the vertical line P V.

135. **Stability.**—The stability of a body resting in any position is estimated by the magnitude of the force required to disturb and overturn it, and therefore will depend on the position of its centre of gravity with respect to the base. If its [illegible]ition can be disturbed or deranged without raising its centre [illegible]*ravity, then* the slightest force will be sufficient to move it ;

but if the position of the body cannot be changed without causing its centre of gravity to rise to a higher position, then a force will be necessary which would be sufficient to raise the entire weight of the body through the height to which its centre of gravity must be elevated; for, according to what has been already explained, the whole weight of the body may be considered concentrated at its centre of gravity.

Let B A C, *fig.* 69, represent a pyramid, the centre of gravity of which is G. To turn this over the edge B, the centre of gravity must be carried through the arc G E, and must therefore be raised through the height H E. If, however, the pyramid were

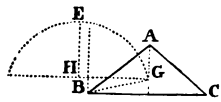

Fig. 69.

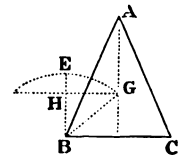

Fig. 70.

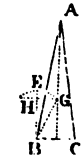

Fig. 71.

taller relative to its base, as in *fig.* 70, the height H E, through which the centre of gravity would have to be elevated, would be proportionably less; and if the base were still smaller in reference to the height, as in *fig.* 71, the height H E would be still less, and so small that a very slight force would throw the pyramid over the edge B. It is evident, from examining these diagrams, that the principle may be generalised, and that it may be stated that the stability of any body depends, other things being the same, upon the distance of the line of direction of its centre of gravity from the edges of its base. The nearer this direction is to one edge of the base, the more easily will the body be turned over this edge.

136. **Line of direction.**—If the line of direction of the centre of gravity fall outside the edge, as in *fig.* 72, then the weight of the body concentrated at G will be unsupported, and the body will fall over its edge.

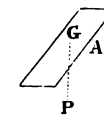

Fig. 72.

This will always take place, if the body be not attached to the ground at its base; but it happens, in some cases, that the body is so rooted to the ground at its base, that it will resist the tendency of its weight to make it fall, even though the line of direction of its centre of gravity should fall a little outside its base. Thus, we see trees not unfrequently leaning in such a position that their centre of gravity obviously falls outside the limits of their trunk. Yet the trees nevertheless remain standing, the tenacity

of the roots and their hold upon the soil being sufficient to resist the effect of their weight acting at the centre of gravity.

In the case of the celebrated leaning towers of Pisa and Bologna, although they are inclined considerably from the perpendicular, the lines of direction of their centres of gravity still fall within their bases. The tower of Pisa is 315 feet high; and it is inclined so that if a plumb-line hang from the side towards which the inclination takes place, it will meet the ground at 12 ft. 4 in. from the base. The tower of Bologna is 134 feet high, and a plumb-line similarly suspended would fall at 9 ft. 2 in. from the base. Nevertheless, these structures have stood, and will probably stand, as permanently as if they were erected in the true perpendicular; for the line of direction of their centre of gravity falls sufficiently within the base to render their overthrow impossible by any common force to which they will be exposed.

If the line of direction of the centre of gravity, however, fall upon the edge, as in *fig.* 73, then the body will still stand; but it will be in a condition in which the slightest possible force will turn it over, as in this case it can be overturned without causing the centre of gravity to rise.

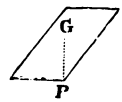

Fig. 73.

In *figs.* 74, 75, and 76 the line of direction falls within the base; and the position is stable.

Fig. 74.

Fig. 75.

Fig. 76.

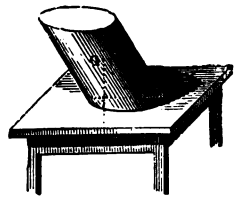

Fig. 77.

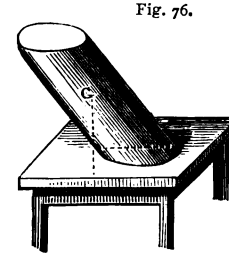

Fig. 78.

In *fig.* 77 it falls within, and in *fig.* 78 outside the base. The position is stable in the former case; and in the latter the ***body falls.***

Hence appears the principle upon which the stability of loaded carriages or waggons depends. When the load is placed at a considerable elevation above the wheels, the centre of gravity is elevated, and the carriage becomes proportionally unstable. In coaches for the conveyance of passengers, the luggage is therefore very unsafely placed when collected on the roof, as is generally done. It would be more secure to pack the heavier luggage in the lower parts of the coach, placing light parcels on the top, for in such a case the centre of gravity of the loaded vehicle would be in a lower position.

Drays for the carriage of heavy loads are often constructed in such a manner that the load can be placed below the axle of the wheels. If a waggon or cart, loaded in such a manner that its centre of gravity shall be in an elevated position, pass over an inclined road, so that the line of direction of gravity would fall outside the wheels it would cause the vehicle to be overturned.

The same waggon will have a greater stability when loaded with a heavy substance which occupies a small space, such as metal, than when it carries the same weight of a lighter substance, such as hay, because the centre of gravity of the latter will be much more elevated.

If a large table be placed upon a single leg in its centre, it will be impracticable to make it stand firm; but if the pillar on which it rests terminate in a tripod, it will have the same stability as if it had three legs attached to the points directly over the places where the feet of the tripod rest.

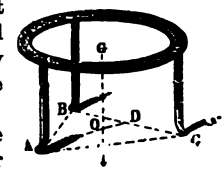

Fig. 79.

It will be still more stable if it be supported on three legs placed further apart as in *fig.* 79.

When a solid body is supported by more points than one, it is not necessary for its stability that the line of direction should fall on one of these points. If there be only two points of support, the line of direction must fall between them. The body is in this case supported as effectually as if it rested on an edge coinciding with a straight line drawn from one point of support to the other. If there be three points of support, which are not ranged in the same straight line, the body will be supported in the same manner as it would be by a base coinciding with the triangle formed by straight lines joining the three points of support. In the same manner, whatever be the number of points on which the body

rests, its virtual base will be found by supposing straight lines drawn, joining the several points of support. When the line of direction falls within this base, the body will always stand firm; but otherwise not. The degree of stability is determined in the same manner as if the base were a continued surface.

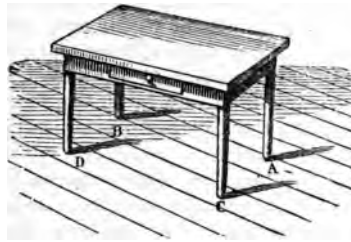

Fig. 80.

If there be four or more legs, the table may be unstable, even though the centre of gravity fall within the base, and will be so if the ends (A, B, C, D, *fig.* 80) of the legs be not in the same plane; for in that case they cannot all rest together on a level floor.

137. **Animal locomotion governed by the position of centre of gravity.**—All the attitudes, gestures, and movements of animals are governed with reference to the centre of gravity of their bodies. When a man stands, the line of direction of his weight must fall within the base formed by his feet. If A B, C D (*fig.* 81), be the feet, this base is the space A B D C. It is evident that the more his toes are turned outwards, the more contracted the base will become in the direction E F, and the more liable he will be to fall backwards or forwards. Also, the closer his feet are together, the more contracted the base will become in the direction G H, and the more liable he will be to fall towards either side.

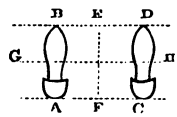

Fig. 81.

When a man walks, the legs are alternately lifted from the ground, and the centre of gravity is either unsupported or thrown from the one side or the other. The body is also thrown a little forward, in order that the tendency of the centre of gravity to fall in the direction of the toes may assist the muscular action in propelling the body. This forward inclination of the body increases with the speed of the motion.

But for the flexibility of the knee-joints, the labour of walking would be much greater than it is, for the centre of gravity would be more elevated by each step. The line of motion of the centre of gravity in walking is represented by *fig.* 82, and deviates but little from a regular horizontal line, so that the elevation of the centre of gravity is subject to very slight variation.

But if there were no knee-joints, as when a man has wooden *legs, the centre of* gravity would move as in *fig.* 83, so that at

each step the weight of the body would be lifted through a more considerable height, and therefore the labour of walking would

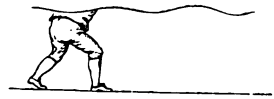

Fig. 82.

Fig. 83.

be much increased. If a man stand on one leg, the line of direction of his weight must fall within the space on which his foot treads. The smallness of this space, compared with the height of the centre of gravity, accounts for the difficulty of maintaining the body in this position for any length of time.

The position of the centre of gravity of the body changes with the posture and position of the limbs. If the arm be extended from one side, the centre of gravity is brought nearer to that side than it was when the arm hung perpendicularly. When dancers, standing on one leg, extend the other at right angles to it, they must incline the body in the direction opposite to that in which the leg is extended, in order to bring the centre of gravity over the foot which supports them.

When a porter carries a load, his position must be regulated by the centre of gravity of his body and the load taken together. If, standing upright, he bore the load on his back, the line of direction would pass beyond his heels, and he would fall backwards. To bring the centre of gravity over his feet, he accordingly leans forward (*fig.* 84). If a nurse carry a child in her

Fig. 84.

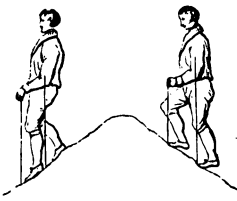

Fig. 85.

arms, she leans back for a like reason. When a load is carried on the head, the bearer stands upright, that the centre of gravity may be over his feet.

In ascending a hill we appear to incline forward, and in descending to lean backward; but, in truth, we are standing upright with respect to a level plane. This is necessary to keep

the line of direction between the feet, as is evident from *fig.* 85.

A person sitting on a chair cannot rise from it without either stooping forward to bring the centre of gravity over the feet, or drawing back the feet to bring them under the centre of gravity.

If a person stand with his side close against a wall, his feet being close together, he will find it impracticable to raise the outside foot; for if he did, the line of direction of the centre of gravity of his body would fall outside the inner foot and he would be unsupported.

When a quadruped stands with his four feet on the ground, the centre of gravity of his body is over a point found by drawing the two diagonals of the quadrilateral formed by his feet; that is to say, if a line be drawn, joining his right fore foot with his left hind foot, and another joining his left fore-foot with his right hind-foot, then the centre of gravity of his body will be very nearly over the point where these lines cross each other. Strictly speaking, it will generally be a little nearer to his fore-feet than this point. It will, however, still be very nearly on the centre of the quadrilateral base formed by his four feet, and, therefore, in a position to give complete stability to the animal.

When a quadruped walks, he raises his right fore and left hind foot (the former leaving the ground a little before the latter), the diagonal line joining his left fore-foot and right hind-foot supporting his weight. The centre of gravity of his body is a little in advance of this line, and his weight, therefore, assists his forward motion. The left fore-foot is raised a moment before the left hind-foot is brought to the ground, and, in like manner, the right hind-foot is raised immediately after the right fore-foot comes to the ground. The effect of these motions is, that the weight of the animal is thrown alternately upon the two diagonal lines joining the right fore and left hind foot, and the left fore and right hind foot.

When a quadruped trots, he also raises his legs from the ground, alternately, by pairs, placed diagonally; but in this case the two feet leave the ground and return to it precisely together, and each pair springs from the ground a moment before the other pair returns to it; so that there are short intervals between the successive returns of the feet, by pairs, to the ground, during which the entire body is unsupported. The weight is, in these intervals, projected upwards by the spring of the legs, *so* that the centre of gravity of the body describes a succession

of arcs concave towards the ground. It is this motion of the body which produces the action sustained by the rider of a horse in trotting.

When a quadruped gallops, he raises simultaneously his two fore-legs, and by the muscular action of his hind-legs he projects his weight forwards. During the spring the centre of gravity is unsupported, but is thrown forward, describing a circular arc concave towards the ground. When this arc has been completed, the fore-legs reach the ground, and immediately afterwards the hind-legs; and the centre of gravity is again momentarily supported, and the animal is in an attitude to repeat the same action.

138. **Different examples of stability.**—If a cylindrical body of uniform density be placed upon a horizontal plane, A B, fig. 86, its centre of gravity being its geometrical centre, C, the line of its direction, C P, will necessarily meet the plane at the point where the cylinder touches it, and the body will consequently remain at rest. If the cylinder be rolled upon the plane, the centre of gravity will be carried in a horizontal line, parallel to the plane represented by the dotted line in the figure. Since, therefore, in this motion the centre of gravity does not rise, any force applied to the body, however slight, will cause it to move, since no elevation of its weight is required. But, on the other hand, the body if left to itself will have no tendency to change its position, because the centre of gravity is not only supported, but because by no change of the body can it assume a lower position.

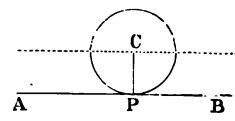

Fig. 86.

If a board of uniform density be cut into the form of an ellipse, A B, fig. 87, and be placed upon a level surface, D E, with the longer axis, A B, of the ellipse parallel to the surface D E, the centre of gravity, C, will then be vertically over the point P, at which the board touches the surface, and the body will be supported at rest. If the body be disturbed slightly from this position, the end A being depressed, and the end B elevated, then the centre of gravity, C, will be elevated towards the point O, and if, on the other hand, the end B be depressed, and the end A elevated, then the centre of gravity will be raised towards the point O'. In either case, this elevation of the centre of gravity will require the application of such a force as would be sufficient to raise the body through that height, whatever it be, through

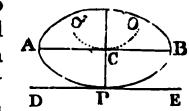

Fig. 87.

which the centre of gravity has been elevated ; and if, after such elevation, the body be disengaged, and left to the free action of gravity, the centre of gravity will descend to the lowest possible position, that is to say, to the position represented in the figure, and will oscillate from side to side of this position until the vibrating motion be destroyed by the resistance of the air and by friction. The centre of gravity will then rest in the position represented in the figure ; that is, at C, over P.

Let us now suppose that the same board is placed on the horizontal plane D E, with its longer axis vertical, as represented in *fig.* 88. The line of direction, C P, of the centre of gravity will now pass through the point of support of the body, and consequently the board will be supported. But if in this case the body be slightly turned from its position to the right or to the left, the centre of gravity will descend towards o or towards o′, and cannot resume the original position at C until a force be applied to it which would be sufficient to raise the weight of the body through the height to which the centre of gravity has fallen. It is evident, therefore, that in this case, if the position of equilibrium C P, be disturbed in the slightest degree by inclining the body a little either to the right or the left, the centre of gravity will move downwards, and the body will fall until it take the position represented in *fig.* 87, with its long axis horizontal.

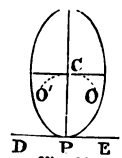

Fig. 88.

139. **Stable, unstable, and neutral equilibrium.**—Now, it will be observed—that in each of the three cases represented in *figs.* 86, 87, and 88, there is equilibrium ; but this equilibrium is characterised in each case by particular conditions.

In the case represented in *fig.* 87 the equilibrium is called *stable*, because, if the body be displaced either to the right or to the left, the body will of itself return to its former position, and settle definitively into it, after some oscillations, the centre of gravity resuming its position over the point P. This position of stable equilibrium is determined by the condition that no change of position can take place in the body without causing an elevation in the centre of gravity ; or, what is the same, it is that position in which the centre of gravity is at the lowest point it is capable of assuming consistently with the conditions under which the body is placed.

The state of equilibrium represented in *fig.* 88 is called *unstable*, or tottering equilibrium. It is such a state of equilibrium that if the slightest derangement takes place in the position of

the centre of gravity it will not return to the same point, but the body will assume another position, in which the centre of gravity will be in a state of stable equilibrium as represented in *fig.* 87.

Unstable equilibrium, then, is characterized by the quality that the centre of gravity is at the highest point which it can assume compatibly with the conditions in which the body is placed; and although it is vertically over the point of support, it is nevertheless in such a condition that the slightest derangement will cause it to descend, and the body to be overturned.

The case represented in *fig.* 86 is an intermediate condition between these two extremes, and is called the state of *neutral* equilibrium. It is neither stable nor unstable. It is not stable, because the slightest force applied to the body will permanently change its position. It is not unstable, because the body will not be overturned by the action of its own weight, however its position may be changed.

Another case of neutral equilibrium is shown in *fig.* 89.

140. **Examples.**—The effects of a variety of children's toys are explained by this principle. The centre of gravity, by loading one extremity in a manner not perceptible to the eye, is

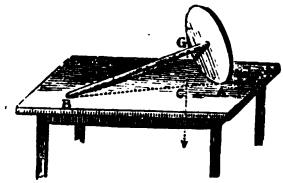

Fig. 89.

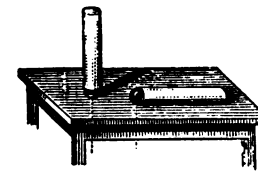

Fig. 90.

moved to a considerable distance from the centre of the figure. The object, therefore, will only stand when the point which is the real centre of gravity is in the lowest position.

Thus, a figure (*fig.* 90) made of some light substance, such as elder pith or cork, has a piece of lead attached to one of its extremities. If it be placed on the other extremity, the end being rounded, it will, apparently by a spontaneous movement, invert its position, and a sort of tumbler will be formed.

In *fig.* 91 a toy is represented, the principle of which will be easily understood from what has been explained. The figure rests steadily on the toe, because, by the weights at the lower ends of the rods, the centre of gravity is brought below the point of support.

141. **Familiar illustrations.**—Many of the feats exhibited by sleight-of-hand performers are explained by the principles of stable and unstable equilibrium. If any object, such as a sword, be supported on its point, it will be in unstable equilibrium so long as its centre of gravity is directly over the point; but as it cannot long be maintained precisely so, the finger or other support of the point is moved slightly in one direction or another, so as to keep nearly under the centre of gravity, and thus to check the tendency of the sword to fall on the one side or on the other.

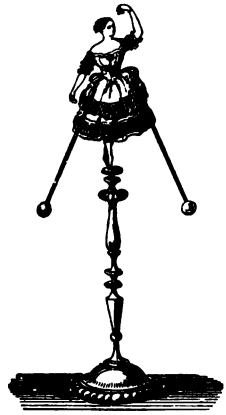

Fig. 91.

But these and similar feats are greatly facilitated if the object thus balanced is made to spin upon its point; for in that case the centre of gravity, though not vertically over the point of support, is continually revolving round a vertical line passing through the point of support, and the tendency which it has at one moment to make the body fall on one side is instantly counteracted by a contrary tendency when it passes to the opposite side.

It is in this manner that the common effect of a spinning-top is explained. It would be stable, if placed as in *fig.* 92, but

Fig. 92.

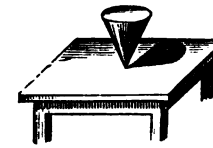

Fig. 93.

it would be quite impracticable to make it stand on its point, as in *fig.* 93, if it did not revolve, or if it revolved very slowly; but if it have a very rapid motion of gyration, then it will stand steadily on its point.

It may be asked how the rapidity of the gyration affects the question. This is easily explained. If the centre of gravity revolve round the line so slowly that the time taken in half a revolution is so considerable as to allow it to fall to any considerable depth, then it cannot recover itself when it passes to the

other side; but if the revolution be so rapid that half the time of one gyration is so small that the centre of gravity cannot fall through any sensible height, the top will maintain its position.

Public exhibitors place a circular plate on the point of a sword, the point being placed as near the centre of the plate as possible. But, however near the centre it may be placed, it is not easily possible to ensure its coincidence with the centre of gravity of the plate. If in this case the plate were at rest on the point of the sword, it would not be balanced, but would incline and fall on that side at which the centre of gravity would lie. The exhibitor, therefore, prevents this effect, by giving to the plate a rapid rotation on the point of the sword. The centre of gravity of the plate rapidly moves in a small circle round the point of support, and its tendency at one moment to fall down at one side is checked the next moment by a contrary tendency to fall down at the other side, and the plate accordingly remains balanced on the point.

In some cases the centre of gravity of a body apparently ascends; but this is always deceptive, and its real motion is invariably a descending one. Let a cylinder of wood, A B, *fig.* 94, be pierced by a hole, O, near its surface B, and let a cylinder of lead be inserted in this hole. The centre of gravity of the mass will then be, not at the centre of the figure, but between that point and the centre of the cylinder of lead which fills the hole O, and will not be far removed from the centre of the lead, in consequence of the great comparative weight of that substance.

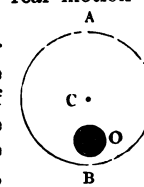

Fig. 94.

If such a cylinder as this be placed upon the foot of an in-

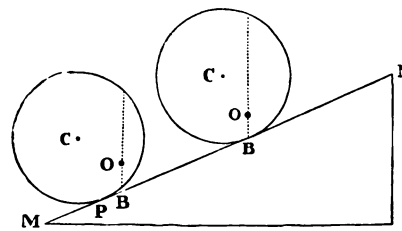

Fig. 95.

clined plane, M N, *fig.* 95, in such a position that the line of direction, O B, of the centre of gravity shall fall above the point

of contact, P, of the cylinder with the plane, the cylinder will roll up the plane, because its weight, concentrated at the centre of gravity, O, acting downwards in the line O B, will have a tendency to descend, towards the plane, and will so descend, causing the body to roll towards N; and it will continue to descend until the cylinder take such a position that the line of direction of the centre of gravity, O B, shall pass through the point of contact of the cylinder with the plane.

In these and similar cases, although the general mass of the body rises, the particular part occupied by its centre of gravity falls; that point is, in effect, simultaneously affected by two motions, one produced by the progressive motion of the cylinder up the plane from M to N, and the other by the motion of revolution of the cylinder round its centre, C. In virtue of the former, the centre of gravity would rise: and in virtue of the latter, it would fall. The effect of the latter predominates until the centre of gravity comes into the vertical line, passing through the point of contact of the cylinder with the plane.

A case of the ascent of the centre of gravity is sometimes produced by public exhibitors, the explanation of which may here be found instructive. The exhibitor places a sphere of wood upon an inclined plane, and standing upon it, he places his feet on that side of the centre which is towards the elevation of the plane. Immediately the globe begins to roll up the plane, and the exhibitor, by moving his feet so as to keep them near the highest point of the globe, but still on the side next the elevation of the plane, the globe continues to roll up the plane, the exhibitor dexterously maintaining his position as here described.

In this case there is a real ascent of the common centre of gravity of the globe and the body of the exhibitor. Now the question is, What force in this case produces this ascent? for it is evident that in the time during which it rolls to the top of the plane, the entire weight of the globe and the body of the exhibitor has been elevated through a perpendicular space equal to the height of the plane.

The force which accomplishes this is the muscular action of the feet of the exhibitor upon the surface of the globe. As the globe rolls up the plane, if the feet of the exhibitor pressed upon the same point of its surface, they would descend; and in that case the common centre of gravity of the globe and the body of the exhibitor, instead of ascending, would in fact descend, until the feet of the exhibitor would sink down to the *surface of the plane;* but this is prevented by the feet of the

exhibitor continually stepping backwards upon the surface of the globe, so as to stand near the top ; and thus, by moving his feet on the globe backwards continually towards the top of it, the exhibitor elevates the centre of gravity of his body, while the action of his feet upon the globe, causing it to roll up the plane, at the same time raises the centre of gravity of the globe.

142. **Magic Clock.**—Time-pieces, thus called, with a transparent glass dial and without any apparent works, often excite the wonder of the curious. The principle of their motion depends on the tendency of the centre of gravity to take the lowest possible position. In the centre of the dial is a small circular gilt plate of metal, which conceals a very short arm, A, of the hand A C B, *fig.* 96. This short arm carries a hollow ring, in the interior of which a small weight is moved by clockwork through the intervention of an arbor which passes through the centre of the dial, the works which turn the rod being at a distance behind the dial, so as not to be visible through it. This weight A, is thus made to move uniformly round the ring, and to change continually the position of the centre of gravity of the hand, so as to make it describe uniformly round the centre of the dial the small dotted circle C G in twelve hours, so that the successive hours are indicated by its extremity B. The ring and weight A being concealed, the hand seems to move without any apparent cause of propulsion ; and what adds to the illusion is, that the hand can be freely turned by applying the finger to it, but when left to itself it always returns to the point from which it was turned, so as still to indicate the time, and to be incapable of being set to a wrong hour.

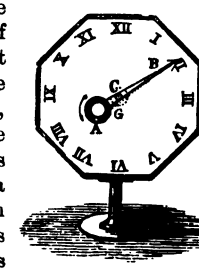

Fig. 96.

143. **Centre of gravity of fluids.**—In all that we have stated respecting the centre of gravity, we have supposed the body to be solid ; but this quality also plays an important part in the phenomena of fluids. The centre of gravity of a fluid mass is determined by the same conditions as if it were solid. It is that point which would have the properties already defined, if the fluid mass were supposed to be congealed.

Thus the centre of gravity of the water forming a lake is that point which would have the properties already explained, if the water of the lake were converted into a mass of ice. It will appear hereafter, however, to possess, in reference to fluid bodies, many important characters.

144. **Centre of gravity of two bodies.**—The centre of gravity of two separate and independent bodies is that point between them which would possess the characters already defined, if the two bodies were united by a straight rod which is itself devoid of weight. This point may be determined by a very simple mathematical process. Let the centres of gravity of the two bodies in question be conceived to be connected by a straight line, and let a point be found upon this straight line which shall divide it into two parts, which shall be in the inverse proportion of the weights of the two bodies. Then this point will be their common centre of gravity.

Thus let A and B, *fig.* 97, be the two bodies, and let *a*, *b*, be their centres of gravity. Draw the line *a b*, and take upon it a point C, so that *b* C shall bear to *a* C the same proportion as the weight of A bears to the weight of B. In this case, C will be the centre of gravity of the two bodies. Now if the line *a b* were a rigid rod devoid of weight, the point C would have all the properties which have been already explained as belonging to the centre of gravity; thus, the bodies would balance themselves on C in any position.

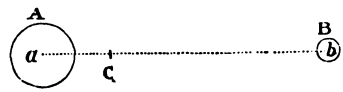

Fig. 97.

CHAPTER XI.

CENTRAL FORCES.

145. **Motion round a centre.**—If a ball of metal or other heavy substance, placed upon a smooth and level surface, be attached to the extremity of a string, the other extremity of which is fastened to a fixed point upon the surface, and then whirled round in a circle, it is known, by universal and constant experience, that the string will be stretched with a certain force, which will be augmented as the velocity of the whirling motion is increased, or as the string is lengthened. That in this experiment the action of gravity has no influence whatever on the tension of the string is evident, inasmuch as the level surface upon which the body moves supports its weight.

146. **Central force.**—The force which always acts upon

matter that is moved round a centre, in what manuer and under what form soever the motion be produced, has formerly been called *centrifugal force*, because it is manifested by a tendency of the matter which revolves to recede from the centre of revolution; this tendency in the case just mentioned being manifested by the force with which the string connecting the body with the fixed point is stretched. This tension resists the tendency of the ball to fly from the centre, and is therefore the measure of its centrifugal force, just as the reaction to a force may be used as a measure of its action.

That centrifugal force is a mere effect of reaction, may be easily shown. It has been already explained, that, in virtue of its inertia, a body, if in motion, can only move uniformly in a straight line. If, therefore, it be deflected from one straight line into another straight line, it must be by the action of some force impressed upon it at the moment of deflection; and if a body be continually deflected from a straight direction, which it must be if it move in a curve, then such body must be under the operation of a force continually acting upon it, producing such incessant change of direction by continually acting towards a centre. Hence the name of *central forces* has been given to forces acting in this manner.

Let P, *fig.* 98, be the fixed point to which the string is attached. Let A be the ball, and let A C F be the circle in which the ball is whirled round. Let A C be a small arc of this circle moved over in a given interval of time. Starting from A, the motion of the ball has the direction of the tangent A D to the circle, and it would move from A to D in the given interval of time, if it were not deflected from the rectilinear course; but it is deflected into the diagonal A C, and this diagonal, by the composition of forces, is equivalent to two forces represented by the sides A D, A B. But the motion A D is that which the body would have in virtue of its inertia: and therefore the force A B, directed towards the fixed point P, is that which is impressed upon it by the tension of the string, and which, combined with the motion A D, causes it to move in the diagonal A C.

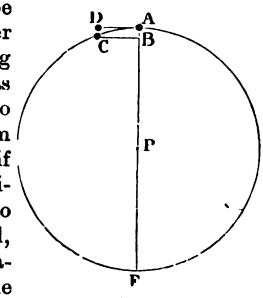

Fig. 98.

The tension of the string, therefore, is in fact a force directed to the centre, P, which continually deflects the body from

the tangent to the circle in which it has a constant tendency to move in virtue of its inertia.

147. **Calculation of effects of central force.**—It follows from the elementary principles of geometry, that the space A B, which is that which represents the force of the string upon the ball, or the central force, and which is in fact the space through which the body is moved in a given small interval of time by the tension of the string which measures the central force, is found by dividing the square of the number representing A C by the number representing the diameter A F of the circle, or twice the length of the string A P. But A C, being the space described in a given time by the revolving body, is its velocity.

If we would then compare the central force of the body with its weight, we have only to compare the space which the body would be moved through by the central force acting alone upon it, with the space which gravity would move the same body through acting equally alone upon it.

Let us then express the physical quantities involved in this question as follows:

W = the weight of the revolving body.
C = the central force.
V = its velocity in feet per second.
R = length of string in feet.
$\frac{1}{2}g = 16\frac{1}{12}$ feet, being the height through which W would fall freely in one second.

We shall have then the following proportion :

$$W : C :: \tfrac{1}{2}g : \frac{V^2}{2R};$$

and therefore we have

$$C = W \times \frac{V^2}{R \times g};$$

This formula, expressed in ordinary language, is as follows :

The central force acting on a body revolving in a circle is found by multiplying its weight by the square of the number of feet which it moves through in a second, and dividing the product by the number of feet in the radius of the circle it described, multiplied by $32\frac{1}{6}$.

But it is more convenient in practice to express the central force acting on a revolving body by reference to the number of revolutions it performs in a given time. Let us therefore express by N the number of revolutions, or fraction of a revolution, performed by the body in one second. The circumference of the circle which it describes, the length of the string being R, will be $6{\cdot}2832\ R$.

If, then, this be multiplied by N, we shall obtain the space through which the body moves in one second, or its velocity; and since the square of 6·2832 is 39·4786, we shall have

$$V^2 = 39{\cdot}4786\ R^2 \times N^2;$$

and therefore we shall have the central force expressed by

$$C = 1{\cdot}2273\ W \times R \times N^2.$$

In this formula it must be understood, however, that the length of the string must be expressed in feet or fractions of a foot, and that N must express the number of revolutions, or fraction of a revolution, made by the body in one second.

This formula, expressed in ordinary language, is as follows:

To find the central force acting on a revolving body, multiply its weight by the number 1·2273. *Multiply this product by the number of feet in its distance from the centre round which it turns, and finally multiply this product by the square of the number of revolutions, or fraction of a revolution, which it makes round that centre in one second of time.*

148. **Example.**—Let it be required to find with what force a body weighing 2lbs. would stretch a string 3 feet long, revolving four times per second. Multiply 2 lbs. by 1·2273, and we obtain 2·4546 lbs.; multiplying this by 3, the number of feet in the length of the string, we obtain 7·3638. In fine, multiply this by 16, which is the square of 4, the number of revolutions per second, and we have 117·8208. So that the central force with which the string is stretched would be $117\frac{8}{10}$ lbs. very nearly.

From the preceding conclusions it follows, that if two bodies of equal weights be whirled round their centres by strings or rods of the same length, the central forces acting upon them will be in proportion to the squares of the number of revolutions which they perform in a given time. Thus, if one of them make three revolutions while the other makes two, the central force acting on the former will be to that acting on the latter as 9 to 4.

Again, if two bodies of equal weight are attached to centres by strings of different lengths, but perform the same number of revolutions in a given time, the central forces will be in proportion to the lengths of the strings. Thus, if one be attached by a string of two feet, and the other by a string of three feet, the central force of the former will be to that acting on the latter as 2 to 3.

In general, if two bodies of equal weight be at different dis-

tances from the centres round which they revolve, and also make a different number of revolutions in the same time, then the central forces acting on them will be as the products found by multiplying their distance from the centre by the squares of the number of revolutions which they make in the same time.

149. **Whirling-table.**—These conclusions may be experimentally verified by an apparatus called a whirling-table, usually found in collections of physical apparatus.

A part of this instrument is represented in *fig.* 99, where C is a metallic bar, having two upright pieces f' f at its ends, in which a polished metal rod is fixed parallel to C. On this rod a ball g slides, being perforated by a hole corresponding to the rod. At the centre C is a vertical framework, which contains a number of thin circular weights h placed one above the other, and supported on a sliding stage, which is capable of rising and falling. At the centre of the top of this stage, carrying the weights, is a hook, to which two strings are attached, which are carried over grooves in the pulley k, and then pass over corresponding grooves in the lower pulley k', from which they are carried to the ball g to which they are attached. Now if the ball g be drawn along the rod of f f' towards f' with sufficient force, the weights h will be raised, and the force necessary to effect this may be increased or diminished by varying the number of weights at h. This apparatus has a square hole at the centre of its lower part C, by which it can be attached to a spindle, by which a regulated revolution can be imparted to it. The apparatus is provided with two such spindles, so that two instruments, like that represented in *fig.* 99, can be fixed upon the table and put in rotation with any required velocities, the number of revolutions which they make in a given time having any desired ratio to each other. By this apparatus, the central force which acts on the ball g can always be estimated by the weight which such central force is capable of raising at h. A rotatory motion is given to C, such that the central force of g is just sufficient to lift the weights h, but not to carry them to the top of the *frame.* When this takes place the central force of g will be equal to the weight.

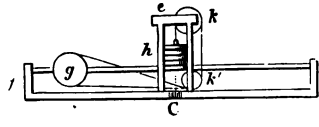

Fig. 99.

A variety of conditions affecting revolving bodies can be examined and determined by this apparatus. By varying the distance of the weights from the centres round which they revolve,

the central forces in circles with different radii can be determined; and by varying the velocities of rotation, the effects of different angular motions, or of a different number of revolutions in a given time, can be ascertained. By this apparatus, therefore, the general principles which have been already established respecting central force, if the motion is in a circle, can be verified.

Thus we can show:

1st. That when equal weights are at equal distances from the centres of revolution, the central forces will be proportional to the squares of the numbers of revolutions which they make in a given time.

2nd. When they revolve in the same time, then the central forces will be in the direct ratio of their distances from the centre.

3rd. When they are at different distances from the centre, and revolve in different times, then the central forces will be in the ratio of the products found by multiplying their distances from the centre by the squares of the numbers of revolutions which they make in a given time.

4th. But if the weights be unequal, then let the proportion of the central forces which would act upon them if they were equal be first found, and let the numbers expressing them be multiplied by the numbers expressing the weights; the product will then express the proportion of the central forces.

These propositions involve the whole theory of a central force acting on a body moving in a circle.

150. **Motion of bodies revolving round common centre of gravity.**—If two bodies revolve round a common centre in the same time, but at different distances from it, the central forces acting upon them would be in the proportion of these distances if the weights of the bodies were equal, the body at the greater distance being acted on by a greater proportional central force. But if the body at the lesser distance be increased in weight, so as to exceed the other in the same ratio as the distance of the other from the centre of rotation is greater, then the central forces will be equal; for what the central force acting on the lesser gains by distance, that of the greater gains by weight. Thus, if one of the bodies weigh three ounces, and the other five ounces, and if, further, the latter be at three inches from the centre, the other being at five inches from it, they will be acted on by the same central force, provided they revolve in the same time. This, which is an important proposition, may be experimentally proved by the whirling-table.

Let A and B, *fig.* 100, be two bodies connected by a wire, and let a point, C, be taken upon this wire, in such a position that B C shall bear to A C the same proportion as the weight of A bears to the weight of B; then let the wire be attached to the spindle of the whirling-table at C, so that the balls shall be made to whirl round C. It will be found that the wire will maintain its position, although free to move in the direction of its own length; showing that the central force acting on the greater ball, A, along the wire in the direction C A, is equal to the central force acting on the lesser ball, B, along the wire in the direction C B. The force acting on the lesser ball, B, therefore, gains as much by its greater radius, B C, as the greater, A, gains by its superior weight.

Fig. 100.

The point C, which divides the distance between the balls in the inverse ratio of their weights, is, as has already been shown, their common centre of gravity; and it therefore follows that if two bodies revolve round their common centre of gravity in the same time, they will be acted on by equal central forces.

The same important principle may be illustrated experimentally by the apparatus shown in *fig.* 101, which will be readily understood on inspection. The ball D is fixed, while E, sliding on the wire, re-acts on the spiral spring, C E. The central force causes E to compress the spring until it retires to such a distance from D that the centre of gravity coincides with that of *rotation.*

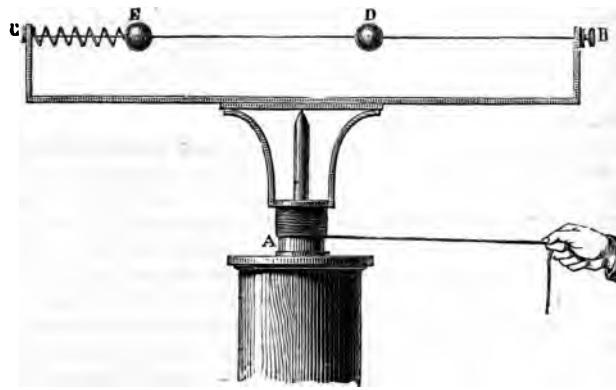

Fig. 101.

151. **Examples.**—Examples are presented of the effects of central force in almost all the motions which fall within our daily observation.

A horseman, or a pedestrian passing round a corner, moves in a curve, and consequently is under the action of a central force directed towards the centre of the curve, which increases with his velocity, and which impresses on his body a reaction directed from the corner. He resists this latter force by inclining his body towards the corner. An animal made to move in a ring, as is customary in training horses, inclines his body towards the centre of the ring.

In all the equestrian feats exhibited in the circus, it will be observed, that not only the horse, but the rider, inclines his body towards the centre, *fig.* 102, and according as the speed of

Fig. 102.

the horse round the ring is increased, this inclination becomes more considerable. When the horse walks slowly round a large ring, the inclination of his body is imperceptible ; if he trot, there is a visible inclination inwards ; if he gallop, he inclines still more ; and when urged to full speed, almost lies down upon his side, his feet acting against the partition which separates the circus from the adjacent parts of the theatre.

In all these cases, the facts are explained by considering that the central force and the weight of the horse are compounded together, and form a resultant which is directed upon the ground, and is represented by the pressure of the horse's feet.

The actual amount of the central force, and the proportion which it bears to the weight of the animal or other body which is moved in the circle, can be determined by the principles

already explained, if the radius of the circle and the velocity of the body moving in it are known ; and from these may be calculated the inclination which the body of the animal must assume in order to be whirled round the circle without falling outwards by the effect of the reaction due to the central force.

Let C (*fig.* 103) be the centre of the ring round which the animal moves. Let C F be its radius, F being therefore the point at which the feet of the animal would act. Take the line F A, perpendicular to F C, and consisting of as many inches as there are pounds weight in the animal. Take A B parallel to F C, consisting of as many inches as there are pounds weight in the central force. Then F B will represent the inclination which the animal must assume in order to prevent it from falling either outwards or inwards. A less inclination than this would cause him to fall outwards, and a greater inwards.

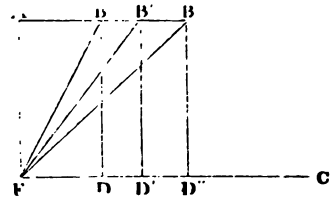

Fig. 103.

To demonstrate this, let the weight be conceived as acting at B, and to be represented by B D, which is equal to A F ; the reaction due to the central force is represented by B A, and these two forces combined will produce a resultant represented by the diagonal B F. If the body of the animal be inclined according to this line B F, then the resultant will press upon its feet ; but if it inclined at a less angle, the resultant will cause it to fall outwards ; and if at a greater angle, it would cause it to fall inwards.

If the central force be increased, as will be the case if the animal moves with increased speed, then it would be represented by A B′ ; and the resultant of it, and of the weight, would be represented by B′ F. Again, if the central force be further increased, and represented by A B″, the weight being represented by B″ D″, the resultant of these will be represented by B″ F, which line must be the inclination of the body of the animal. It is clear, then, that an increase of the central force, which arises from increased speed, will cause the body of the animal to incline more and more towards the centre of the circle.

For example, if a horse move in a ring of 60 feetdiameter, with a speed of 15 feet per second, the ratio of the central force to his weight will be that of the square of 15, or 225, to

60 × 32⅛, which is equal to 1,930; the ratio, therefore, of the central force to the weight is 1 to 8½ very nearly. We shall therefore find the inclination corresponding to this, by taking A F (*fig.* 103) equal to 8½ inches, and A B equal to 1 inch: the line F B would represent the inclination of the horse.

A carriage, not having voluntary motion, cannot make this compensation for the disturbing force which is called into existence by the gradual change of direction of the motion in turning a corner; consequently it will, under certain circumstances, be overturned, falling, of course, outwards, or *from* the corner. If A B be the carriage, and C (*fig.* 104) the place at which the weight is principally collected, this point C will be under the influence of two forces; the weight, which may be represented by the perpendicular C D; and the central force, which will be represented by a line C F, which shall have the same proportion to C D as the central force has to the weight. Now, the com-

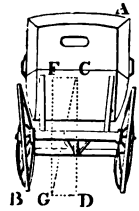

Fig. 104.

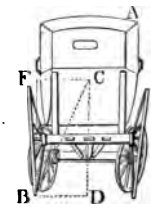

Fig. 105.

Fig. 106.

bined effect of these two forces will be the same as the effect of a single force represented by C G. Thus, the pressure of the carriage on the road is brought nearer to the outer wheel B. If the centrifugal force bear the same proportion to the weight as G F (or D B), *fig.* 105, bears to C D, the whole pressure is thrown upon the wheel B.

If the central force have to the weight a greater proportion than D B has to C D, then the line C F, which represents it (*fig.* 106), will be greater than D B. The diagonal C G, which represents the combined effects of the weight and central force, will, in this case, pass outside the wheel B, and therefore this resultant will be unresisted. To perceive how far it will tend to overthrow the carriage, let the force C G be resolved into two; one in the direction of C B, and the other, C K, perpendicular to C B. The former, C B, will be resisted by the road; but the latter C K, will tend to lift the carriage over the external wheel. If the velocity and the curvature of the course

be continued for a sufficient time to enable this force C K to elevate the weights so that the line of direction shall fall on B, the carriage will be overthrown.

It is evident, from what has been now stated, that the chances of overthrow, under these circumstances, depend on the propor-

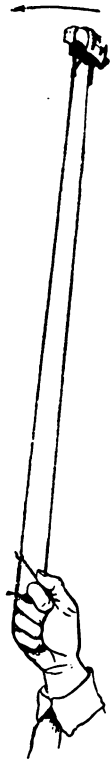

Fig. 107.

Fig. 108.

tion of B D to C D, or, what is to the same purport, of half the distance between the wheels to the height of the principal seat of the load. It was shown in the last chapter, that there is a certain point, called the centre of gravity, at which the entire weight of the vehicle and its load may be conceived to be concentrated. This is the point which, in the present investigation,

we have marked C. The security of the carriage, therefore, depends on the greatness of the distance between the wheels and the smallness of the elevation of the centre of gravity above the road; for either or both of these circumstances will increase the proportion of B D to C D.

If a stone or other weight be placed in a sling which is whirled round by the hand in a direction perpendicular to the ground (*fig.* 107), the stone will not fall out of the sling, even when it is at the top of its circuit, and consequently has no support beneath it. The central force, in this case acting from the hand, which is the centre of rotation, is greater than the weight of the body, and therefore prevents its fall.

In like manner, a bucket of water may be whirled so rapidly, that even when the mouth is presented downwards, the water will still be retained in it by the central force (*fig.* 108).

The tendency of the water to recede from the centre of ro-

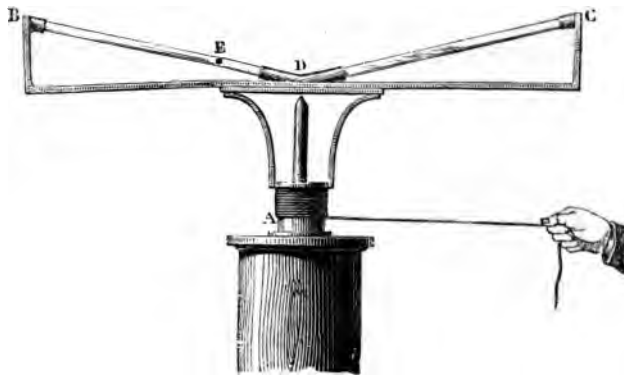

Fig. 109.

tation may be also shown by the apparatus represented in *fig.* 109.

152. **Action of a central force on liquids.**—If a glass of water be fixed upon the whirling-table, and be made to revolve rapidly round its central line A B (*fig.* 110), as an axis, the water will rise upon its sides and sink upon its centre, the surface assuming a concave form. This effect is due to the central force, which affects every particle of the water, and gives it a tendency to fly from the centre of revolution; it would

be possible to impart to the glass a revolution so rapid as to cause the water to flow over the sides.

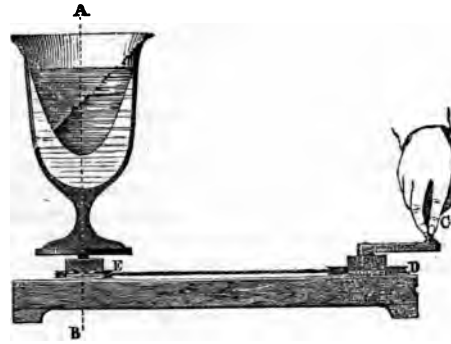

Fig. 110.

153. **Centrifugal drying-machine for laundries.** — The agency of a central force has been applied with great ingenuity in a machine for drying linen in some of the larger class

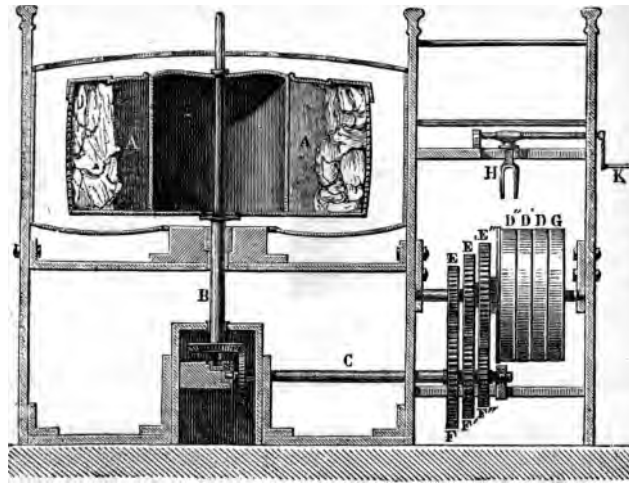

Fig. 111.

of laundries in France. This machine, which is represented in *fig.* 111, consists of a large hollow drum, within which there is a [illegible]ce left empty, through which the axle passes. The

space A A, shown in section, surrounding this vacant central space, is enclosed by two cylindrical surfaces, an exterior and an interior. This circular cylindrical chamber is closed by covers, by opening which the linen to be dried can be introduced. The bottom of this chamber is pierced with holes like a sieve, through which the water expressed from the linen can flow off; a rapid rotation being given to this cylinder, the linen, by the effect of the central force, is urged against the exterior surface of the cylinder, and is there squeezed with a force which increases with the rapidity of the rotation, by the effect of which the water is pressed out of it, and escapes through the holes in the bottom. A rotation so rapid as to produce 25 turns per second, or 1,500 per minute, is given to these drying cylinders, by which the linen, however moist it may be, is rendered so nearly dry that a few minutes' exposure in the air renders it perfectly so.

The mechanism by which this rapid rotation is produced consists of a horizontal shaft C, connected with the vertical shaft B by a pair of bevelled wheels; the shaft C receives its motion from the system of toothed wheels F E, F′ E′, F″ E″, which are themselves driven by a system of hoops D, D′, D″. These hoops receive their motion from a shaft driven by any moving power with which they are connected by an endless band.

As the very rapid rotation necessary to produce the desired effect could not be instantaneously or even very rapidly produced without injury to the mechanism, an ingenious expedient is adopted by which it is gradually imparted without any injurious shocks. The endless band, which imparts the motion, and which is shifted horizontally by the fork H, moved by the screw and winch K, is first placed upon the hoop D; this hoop is fixed upon the axle of the toothed wheel E, which drives the wheel F, the axle C, and the cylinder A A, with a moderate velocity; the wheels F′ F″, also fixed upon the axle C, are moved at the same rate, and they impart motion to the wheels E′ and E″, which are respectively fixed upon hollow axles, which pass one within the other, and which carry respectively the hoops D′ and D″.

When the apparatus is thus put in motion, with a certain moderate velocity, the endless band is transferred by the apparatus from the hoop D to the hoop D′, and the motion is then conveyed to the shaft C, through the wheels E′ and F′, instead of E and F; and the proportion of these wheels is so arranged that the same velocity of the hoop D′ will impart an increased velocity to the shaft C and the cylinder A A.

In the same manner exactly, by transferring the band from

D′ to D″, the velocity is still further increased, and the full effect produced.

When the motion of the machine is suspended, to remove the dried linen and recharge the cylinder, the band is transferred to the hoop G, which turns on the axle without being attached to it.

154. **Motion down a convex curve.**—If a body B (*fig.* 112) move down a curved surface G F, whose centre is at C, it may acquire such a velocity that the central force will cause it to leave the surface, and to be projected forwards to the ground. The conditions under which this would take place are easily explained.

Let the weight of the body be expressed by B A, and the central force acting upon it by B E, and let the parallelogram A B E D be completed. Then the diagonal B D will be the resultant of these two forces, and will be the line in which the body has a tendency to move. So long as this diagonal forms an acute angle with B C, the body will remain on the curved surface; but so soon as it forms a right angle with B C, then it will become a tangent to the surface, and the body will fall off.

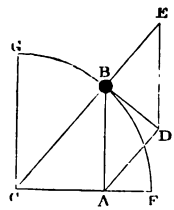

Fig. 112.

155. **Body revolving on a fixed axis.**—The most important classes of problems in mechanical science in which the principles determining the action of central forces are practically applied, are those which relate to solid bodies revolving on an axis. If a solid body be pierced by a straight and round hole in which a cylindrical rod is inserted, and the body be made to turn rapidly round this rod as an axis, each particle of matter composing the body will revolve in a circle round such axis; all these circles will be described in the same time, and consequently the central forces acting on the particles exerted upon the axis will be in proportion to their distances from the axis, such distances being the radii of the circles which they describe respectively round it.

All the particles of the body which are at the same distance from the axis will therefore be acted on by equal central forces, and those which are at greater distances by central forces proportionally greater than those at less distances. The particles distributed round the axis will be acted on by forces directed towards the axis in the direction of perpendiculars connecting them with the axes respectively.

Now it is evident that as many different forces will thus be *exerted in* the direction of the axis as there are different *particles of matter* composing the mass of the revolving body.

Three cases are presented which may arise in the combination of these forces.

1st. They may be in equilibrium ; that is to say, the central forces acting upon all the particles composing the body on the axis may neutralise each other. In this case the axis would suffer neither pressure nor strain in consequence of the central forces, and the body would spin round it without producing any effect upon it.

In such a case, if the axis were withdrawn from the hole in which it is inserted, the body would still continue to spin as before ; because, since the axis suffered no pressure or strain from the revolving matter, its presence or absence can make no difference in the motion.

2nd. The central forces acting on the particles of the revolving mass may be such as to be capable of being represented by a single force applied at some point of the axis, and at right angles to it. In this case the axis will suffer a corresponding pressure or strain at this point. If this point could be fixed, the remainder of the axis might in that case be withdrawn ; because, since no other point of it suffers any strain from the effect of the motion, its presence or absence can produce no difference in the motion.

3rd. The central forces may be such that their combined effect cannot be represented by a single force, but may be represented by two equal and parallel forces acting on two different points of the axis, and in contrary directions. This combination is what has been already called a *couple*, and its effect is to twist the axis round some point intermediate between the two contrary forces. In this case the axis could not be withdrawn unless two fixed points were provided, representing the points to which the opposite forces of the couple are applied.

The combination of forces acting on the revolving matter may be such as to be incapable of being represented either by a single force or by a couple. In this case it can be proved that their combined effects will be represented by a single force and a couple taken together. The effect, in such a case, is a pressure upon the axis at right angles to its length at the point where the single force is placed, and a tension or twist produced at the same time by the couple.

These are all the possible cases which can be presented by a solid body revolving on a fixed axis.

156. **Examples.**—Their complete analysis and demonstration would require the use of the principles and formulæ of the higher parts of mathematical science, the introduction of which

would not be suitable to the purpose of the present treatise. We shall therefore limit ourselves to some examples which will convey a sufficiently clear notion of the general effects produced by the rotation of solid bodies on fixed axes.

If a series of particles of matter placed in the circumference of a circle are made to revolve by a common motion round an axis, passing through such circle, and perpendicular to its plane, their central forces will be evidently in equilibrium, and no pressure on the axis will be produced. A circular series of such particles is represented in *fig.* 113, the radii represent the direction of the central forces, which are all equal, because the particles are equal and the distances from the centre are equal.

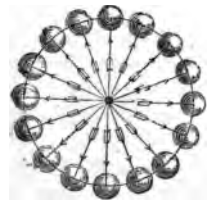

Fig. 113.

It is evident on inspection that these forces equilibrate round the centre, and that the central point, therefore, would suffer no pressure in one direction rather than in another.

A flat circular plate of uniform thickness and density may be considered as consisting of a series of concentrical rings of such particles. If such a plate revolve round an axis passing through its centre, and perpendicular to its plane, the central forces acting on the particles will be in equilibrium, and no pressure will be produced on the axis.

A cylinder may be considered as composed of a number of such circular plates placed one upon the other, and the axis of the cylinder will be the line formed by the centres of these plates. If such a cylinder, therefore, revolve round its axis the central forces acting on its mass would be in equilibrium, because each separate plate being in equilibrium, the entire pile would necessarily also be in equilibrium.

157. **Solids of revolution.**—There is an extensive and important class of solid bodies having a geometrical form, which gives to their axes this property. They are called *solids of revolution.*

Let A B D (*fig.* 114) be a triangle, with a right angle at A. If this be supposed to revolve round the side A B as an axis, it will by its revolution generate a solid called a right cone ; that is to say, the space through which it would pass as it revolves, and which it would include within its sides in revolving if filled by any solid matter, would form a cone.

If a right-angled parallelogram (*fig.* 115) revolve round its *side A B, it would in* the same way generate a cylinder.

If a triangle A B D (*fig.* 116) revolve round a side A B not adjacent to a right angle, it would generate the double cone.

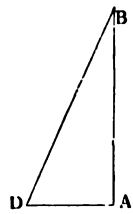

Fig. 114.

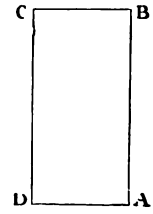

Fig. 115.

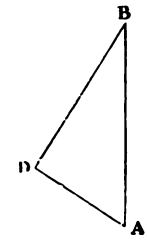

Fig. 116.

If a semicircle (*fig.* 117) revolve round its diameter A B, it would generate a sphere or globe.

If a semi-ellipse (*fig.* 118) revolve round its shorter axis A B, it would generate an *oblate* spheroid, being a figure resembling an orange.

If a semi-ellipse (*fig.* 119) revolve round its longer axis, it would generate a *prolate* spheroid, being a figure resembling an egg, only that its ends are similar.

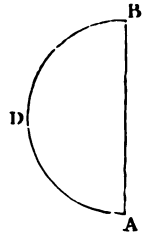

Fig. 117.

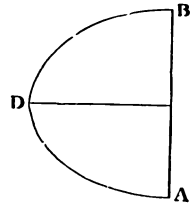

Fig. 118.

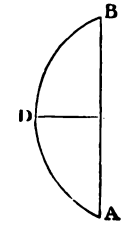

Fig. 119.

Now it is evident, that in these and all solid bodies whose figures are determined by the same principle, all sections made by planes perpendicular to their axes of revolution are circles through the centre of which such axes pass. Since the central forces of the particles of matter composing each of such sections are in equilibrium, the central forces of the entire mass of the body are necessarily also in equilibrium, such mass being composed of these several sections. The body, in short, may be considered to be made up of a number of circular plates laid one upon another, varying in their diameter according to the form of the body.

If a solid of revolution, therefore, be made to revolve upon its geometrical axis, the centrifugal forces of its mass will be in equilibrium, and no pressure or strain whatever will take place upon its axis.

If a globe or sphere be composed of any materials capable of a change of figure, it will be converted into a flattened elliptical spheroid by the central force produced by its rotation.

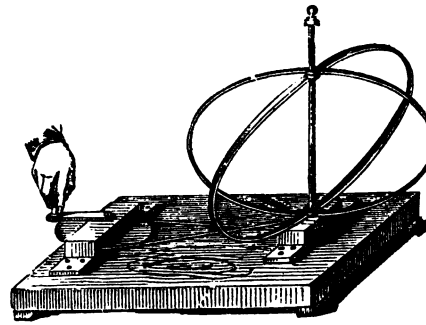

Fig. 120.

This fact is shown experimentally by an apparatus composed of elastic hoops put in rotation by the whirling-table, as shown in *fig.* 120.

The same reasoning will be applicable to all solids which have an axis round which the particles of such section made by a perpendicular plane are so arranged that every particle of matter at one side has a corresponding particle at an equal distance at the other side ; for in this case every pair of such equidistant particles will exert equal and opposite central forces on the axis.

A great number of solid bodies, including the class of solids of revolution described above, fulfil this condition.

If the sections of a solid be all equal, and have any regular geometrical figure having a point within it forming its geometrical centre, and through which all lines drawn are bisected, then such a solid participates in the above property, and the central forces of its mass, when revolving round such an axis, will neutralise each other.

A column formed by laying one upon another equal flat plates of the same figure has this property. Such a column is *called in geometry,* a rectangular prism.

A pyramid formed by laying upon each other similar plates not equal, but diminishing gradually in magnitude, will have the same property.

It follows, therefore, that the axes of the rectangular prisms and pyramids whose bases have a centre of magnitude share the above property.

158. **Axis of revolutions through centre of gravity.**—It will be observed that the axis round which the central force equilibrates in the examples given above will pass through the centre of gravity of the bodies in question. It may therefore be asked, whether such property belongs to all lines whatever passing through the centre of gravity of a body; that is to say, whether, if a body is made to revolve upon an axis passing through its centre of gravity, the central forces will be in equilibrium, and whether such axis will be free from strain or pressure. It is easy to show that this will not be the case in general, and that only certain lines passing through the centre of gravity will have the property above mentioned.

If a rectangular plate having unequal sides, A B and B C (*fig.* 121), be made to revolve round a line M N, passing through the middle points of the opposite sides A B and D C, the central forces will be in equilibrium; because every point on the one side of M N has a corresponding point at an equal distance on the other side.

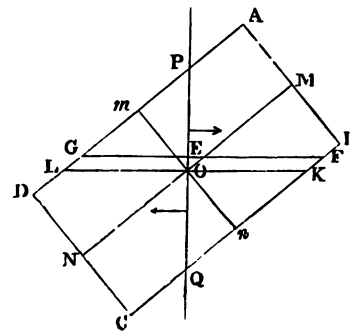

Fig. 121.

In like manner, the line *m n*, passing through the middle points of the sides A D and B C, would possess a like property. But if any other line, such as P Q, be drawn through the centre of gravity, O, and the plate be made to revolve round such line, then it will be evident that the central forces will not be in equilibrium. For through a point such as E, let a line F G be drawn perpendicular to the axis P Q. This line F G will be divided into unequal parts at E, and the total central force acting on the particles between E and F will be greater than the central force acting on the particles between E and G. The same will be true of all lines drawn perpendicular to O P,

and the combined effect of all the central forces acting in that part of the axis between O and P will be to produce a greater strain on the side of the angle A than on the side of the angle D, and the central forces will have a resultant directed towards the side A. By the same reasoning it may be shown that the central forces of that part of the plate which is below K L will have a resultant directed to the side of the angle C, and this resultant will be equal to the resultant of the central forces above K L, directed to the side of the angle A.

It follows, therefore, that the axis P Q round which the plate revolves will be affected by forces having two resultants parallel to each other and in opposite directions, one perpendicular to O P, and directed towards the side of the angle A, and the other perpendicular to O Q, and directed towards the side of the angle C. These two forces form a couple, and have a tendency to turn the axis of revolution towards the position M N, as represented by the arrows in the diagram.

This property is general, although it cannot be demonstrated in all its universality without the aid of the language and principles of the higher analysis. It may be stated thus :—In all bodies whatever, there are three lines passing through the centre of gravity which are at right angles to each other, each of which is so placed, in reference to the mass of the body, that the central forces produced by the revolution of the body round them respectively will be in equilibrium ; and such lines, when the body revolves round them, will suffer no strain or pressure.

But if the body be made to revolve round any other line passing through the centre of gravity, the central forces will produce a strain, which will be represented by two equal opposite and parallel forces acting upon the axis at opposite sides of the centre of gravity, and having a tendency to turn the axis in the position of one or other of the three axes of equilibrium here mentioned.

159. **Principal axes.**—An axis round which the central forces equilibrate is called a *principal axis* ; and from what has been explained, it appears that there are three principal axes through the centre of gravity at right angles to each other.

There are some particular cases in which every line passing through the centre of gravity is a principal axis ; such, for example, is the case with a sphere or globe of uniform density. Such a solid, whatever diameter it may revolve round, will be a solid of revolution, and the sections perpendicular to such *diameter will* be circles. The same principle is true of all the

regular solids. All lines, for example, passing through the centre of a cube are principal axes.

If a solid be made to revolve round an axis which does not pass through the centre of gravity, but which is parallel to one or other of the principal axes passing through that point, the central forces will not equilibrate round such axis, but they will be represented by a single force perpendicular to it, and passing through the centre of gravity.

Such an axis is called a principal axis, and in general there are three such axes corresponding with such points taken in the body, which are parallel respectively to the principal axes passing through the centre of gravity.

It may therefore be stated, in general, that if any point in a body be taken different from the centre of gravity, there are three lines passing through it at right angles to each other, round each of which, if the body is made to revolve, an effect will be produced by the central forces which can be represented by a single force, perpendicular to the circumference, and passing through the centre of gravity.

If the body be made to revolve round any line passing through a given point in it which is not a principal axis in the sense just referred to, then the central forces produced by such revolution cannot be represented either by a single force or by a pair of equal and opposite parallel forces, but will be represented by both of these together. This, therefore, is the character of all axes, round which a body would move, which do not pass through the centre of gravity, and are not parallel to either of the principal axes passing through that point.

160. **Experimental illustrations.**—Many of these important properties admit of being experimentally illustrated in a very striking manner by a simple apparatus.

Let a body be suspended by a thread, or, better still, by a skein composed of several threads, from a fixed point. Let this fixed point be so arranged, that a rapid rotatory motion may be given to it. This rotatory motion will soon be imparted to the body suspended to the threads by the twisting of the skein, and after a time the rotation of the body will become extremely rapid. It will first take place round the line, passing vertically through it in the direction of the skein, when it hangs quiescent. If this line happen to be one of the principal axes passing through its centre of gravity, it will continue to revolve round it; but if it be not one of these principal axes, the central force, as has been already explained, will produce an effect upon it, represented by two equal, opposite, and parallel forces tend-

ing to twist it. This effect will be soon rendered manifest in a remarkable manner : the body, when it revolves rapidly, will not continue in the same position which it had when it was quiescent. The line which was in the direction of the string will begin to take another direction, the point where the string is attached to it will not remain in its first position, and the body will throw itself into a position at a greater or lesser angle to the string, and, in a word, will at length, after the revolving motion has become sufficiently rapid and continued, assume such a position, that a principal axis, passing through its centre of gravity, will become vertical, and the body will spin round it.

It is evident, therefore, that this effect takes place in spite of the opposing influence of the gravity of the body, for in the new position which the body assumes, the line of direction of its centre of gravity ceases to be represented by the skein. This experiment may be varied in a great variety of ways, exhibiting most instructive and surprising effects.

Fig. 122.

Fig. 123.

Fig. 124.

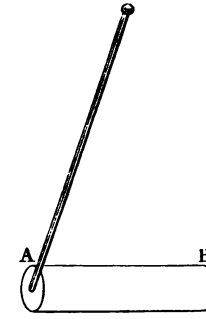

Fig. 125.

If a metallic ring be suspended by the skein by being attached to a point in its side, the ring, when quiescent, will hang with its plane vertical ; but when the rotation becomes rapid, the ring will throw itself into a horizontal position, and will spin round a vertical axis through its centre, and perpendicular to [redacted] *plane.*

If the experiment be made with an oblate spheroid suspended by a point P (*fig.* 122) in its equator P Q, its lesser axis A B being horizontal, it will, when it acquires a rapid motion, take the position represented in *fig.* 123, in which the shorter axis A B is vertical, and the equator P Q horizontal, and it will spin in this position round the principal axis A B, although the centre of gravity is unsupported by the string.

If a cylindrical rod A B (*fig.* 124) be suspended by a point at the centre of one of its ends, and a rapid revolution be imparted to it, it will not continue in this position, but will assume the position represented in *fig.* 125, in which its length will be horizontal, and it will revolve round an axis passing through its centre, and at right angles to the length.

If a metallic chain, the ends of which are united, as in a necklace or bracelet, be suspended from any point, and put in rapid revolution, the chain will at first make some irregular gyrations, but after a time it will gradually open, and settle itself into the form of a precise circle, the plane of which will be horizontal, and the string will then be attached to a point in this circle. In short, the chain will assume the form of a solid ring in a horizontal frame.

161. **Applications and Problems.**—The principles explained in the preceding chapters are applied in the following questions, which will serve to illustrate them still further:—

1. A stone is let fall; find velocity and space passed over after 10 seconds.

(α) $v = g t = 32{\cdot}2 \times 10 = 322$ feet per second.

(β) $s = \frac{1}{2} g t^2 = 16{\cdot}1 \times 10^2 = 1{,}610$ feet.

2. A stone is projected downwards with a velocity of 20 feet; find space and velocity after 5 seconds.

$v = c + g t = 20 + 32{\cdot}2 \times 5 = 181$ feet.

$s = c t + \frac{1}{2} g t^2 = 20 \times 5 + 16{\cdot}1 \times 5^2 = 100 + 402{\cdot}5 = 502{\cdot}5$ feet.

3. Prove generally that a body projected upwards has the same velocity and the same position in space at the same time before and after it attains the greatest height.

(α) The time in which a body projected upwards comes to rest is given by

$$0 = c - g t$$

$$v = c - g t$$

$$\therefore \frac{c}{g} = t.$$

Now let it be n seconds before t, then the body has been on its way $t - n$ seconds, and its velocity is then—

$$v = c - (t-n)\,g\,;$$

$$\text{but } t = \frac{c}{g} \therefore v = c - \left(\frac{c}{g} - n\right) g = n\,g.$$

Again, after attaining its position of rest, and coming back n seconds, its velocity is clearly also $n\,g$, $\therefore$ the velocity is the same, n seconds before and n seconds after attaining its position of rest.

(β) The space described n seconds before reaching the position of rest is—

$$\begin{aligned} s &= c\,(t-n) - \tfrac{1}{2}\,g\,(t-n)^2 \\ &= c\,t - c\,n - \tfrac{1}{2}\,g\,t^2 + g\,t\,n - \tfrac{1}{2}\,g\,n^2\,; \end{aligned}$$

but $c\,t - \frac{1}{2}\,g\,t^2 =$ whole space $= s_\infty$,

and $g\,t\,n = c\,n$, because $c = g\,t$, when the body comes to rest,

$$\therefore s = s_\infty - c\,n + c\,n - \tfrac{1}{2}\,g\,n^2 = s_\infty - \tfrac{1}{2}\,g\,n^2,$$

$\therefore$ the distance from the point of rest is $s_\infty - s = \frac{1}{2}\,g\,n^2$.

But $\frac{1}{2}\,g\,n^2$ represents the space passed over in n seconds, and this space is the same as that which the body would have still to pass over, *i.e.* its position in space is the same.

4. If w_1, w_2, be the weights in Atwood's machine, find the velocity and space for the time t in terms of w_1, w_2, and g.

From the general formula $f = \dfrac{\mathrm{P}}{\mathrm{W}}\,g$,

we have, since $\mathrm{P} = w_1 - w_2$, $\mathrm{W} = w_1 + w_2$,

$$\text{acceleration} = \frac{w_1 - w_2}{w_1 + w_2}\,g.$$

$$\therefore \text{Velocity after } t \text{ seconds} = \frac{w_1 - w_2}{w_1 + w_2}\,g\,t.$$

$$\text{Space after } t \text{ seconds} = \tfrac{1}{2}\,\frac{w_1 - w_2}{w_1 + w_2}\,g\,t^2.$$

5. How may the inclined plane and Atwood's machine be used for finding approximately the value of g?

(1) If the space were observed in Atwood's machine through which the weights in a number of seconds pass, we have—

$$s = \frac{w_1 - w_2}{2\,(w_1 + w_2)}\,g\,t^2,$$

$$\text{from which } g = \frac{2\,s\,(w_1 + w_2)}{(w_1 - w_2)\,t^2}.$$

(2) Similarly in the inclined plane, let the length and height be measured, and the time of descent of a body,

$$\text{then since } s = \frac{t^2\,g\,h}{2\,l},$$

$$\frac{2ls}{t^2h} = g, \text{ or } \frac{2l^2}{t^2h} = g,$$

if $l = s$, that is, if we observe the time it takes to descend from the top to the foot of the plane.

6. A stone is thrown vertically upwards with a velocity $3g$: find at what times its height will be $4g$, and find its velocity at these times.

Let u be the initial velocity.

(α) From $s = ut - \frac{1}{2}gt^2$, we have $4g = 3gt - \frac{1}{2}gt^2$, or,

$$8 = 6t - t^2 \therefore t^2 - 6t + 9 = 1 \therefore t = 4 \text{ or } 2.$$

(β) From $v^2 = u^2 - 2fs$, we have $v^2 = (3g)^2 - 2g \times 4g$

$$\therefore v^2 = g^2 \text{ or } v = \pm g.$$

7. A body is projected vertically upwards with a velocity which will carry it to a height $2g$: find after what interval the body will be descending with the velocity g.

From $v^2 = u^2 - 2fs$, we have $0 = u^2 - 2g \cdot 2g$ or $u = \pm 2g$. The positive value alone is clearly available.

Hence, the initial velocity being $2g$, we have for the time of ascent . . . . . $2g = gt$ or $t = 2$ secs.
But in order to attain a velocity g the body must descend 1 ,,
$\therefore$ Whole interval = 3 secs.

8. A body moves over 20 feet in the first second of time during which it is observed, over 84 feet during the third second, and over 148 feet during the fifth second. Is this consistent with the supposition of uniform acceleration?

The formula $u + \frac{f}{2}(2n - 1)$ gives the spaces passed over in—

1st second $u + \frac{f}{2}$, supposing u an original velocity, and f a constant force.

3rd second $u + \frac{5f}{2}$

5th second $u + \frac{9f}{2}$

$$\left.\begin{aligned} \text{Hence } 20 &= u + \frac{f}{2} \\ 84 &= u + \frac{5f}{2} \end{aligned}\right\} \therefore \begin{aligned} u &= 4 \\ f &= 32 \end{aligned}$$

Putting in the following equation these values—

$$u + \frac{9f}{2} = 148 = 4 + 9 \times 16,$$

which is correct, hence the spaces passed over agree with the supposition of a constant force = 32.

9. Similarly solved is the following: A particle uniformly accelerated describes 180 feet, and 140 feet is the 5th and 7th second of its motion. Find the initial velocity and the numerical measure of the acceleration.

$$\left.\begin{aligned} u + \frac{9f}{2} &= 108 \\ u + \frac{13f}{2} &= 140 \end{aligned}\right\} 2f = 32 \quad \begin{aligned} f &= 16 \\ u &= 36 \end{aligned}$$

10. What is the velocity acquired by a body when it strikes the earth, having been dropped from a height of 450 feet? If the body weighs 10 stones, what is the momentum acquired?

Here $s = 450$

$g = 32{\cdot}2$, and $v = \sqrt{2sg} = \sqrt{900 \times 32{\cdot}2}$

$= 170{\cdot}2$ feet per second.

The momentum is, therefore—

$$170{\cdot}2 \times 10 \times 14 = 23828.$$

11. A body has fallen through a height of 440 yards. What was the space described by it in the last second of its fall?

$s = 1320$ feet.

The whole time $= t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 1320}{32{\cdot}2}} = 9{\cdot}05$ seconds.

$\therefore$ space during last second $= \frac{g}{2}(2 \times 9{\cdot}05 - 1) = 275{\cdot}4$ feet.

12. A body in falling has described one-third of the altitude from which it fell, during the last second. Find the altitude and the whole time of descent.

Let t = time of descent. Then altitude $= \frac{1}{2} g t^2$,

and space in $(t-1)$ seconds $= \frac{1}{2} g (t-1)^2$.

,, in last second $= \frac{g}{2}(2t - 1)$.

By the question we have—

$$\tfrac{1}{2} g t^2 = 3 \times \frac{g}{2}(2t - 1);$$

from which equation—

$$t = 5{\cdot}449 \text{ seconds},$$

and

$$\text{altitude} = \tfrac{1}{2} g t^2 = 477{\cdot}9 \text{ feet}.$$

13. What is the measure of the force of gravity, when half a second is taken as unit of time; also, when the unit of space is 1 metre (1 metre $= 3{\cdot}28$ feet)?

In the first case the measure is—

$$(\tfrac{1}{2})^2 g = \frac{32{\cdot}2}{4} = 8{\cdot}05 \text{ feet}.$$

In the second case the measure is—

$$\frac{32{\cdot}2}{3{\cdot}28} = 9{\cdot}81 \text{ metres.}$$

14. Also when the unit of time is ten seconds, and that of space one yard.

$$\text{Acceleration} = \frac{10^2 \cdot g}{3} = 33{\cdot}3 \times 32{\cdot}2.$$

15. Also when the unit of time is a quarter of a second and the unit of space ½ yard.

$$\text{Here acceleration} = \frac{32{\cdot}2 \times (\frac{1}{4})^2}{1\frac{1}{2}} = \frac{2 \times 2}{3} = \quad \text{nearly.}$$

16. If f be the measure of an acceleration when m seconds is the unit of time and n feet the unit of length, find the measure of acceleration when a second or a foot are the units.

Obviously, if m seconds of time was the unit, it will be $\frac{1}{m^2} \cdot f$ if 1 second be taken as unit,

and $\frac{n}{m^2} \cdot f$ if 1 second and 1 foot.

17. If f be the measure of an acceleration when m seconds is the unit of time, and n feet the unit of length, find the measure of the acceleration when μ seconds is the unit of time, and ν feet the unit of length.

By the previous problem, the acceleration f, reduced to 1 second or 1 foot $= f \cdot \frac{n}{m^2}$;

and reduced again to μ seconds and ν feet it is—

$$f \cdot \frac{n}{\nu} \cdot \frac{\mu^2}{m^2}.$$

18. A body of given weight is suspended by a rope and allowed to descend with a given acceleration f. Find the tension of the string.

$$\text{Moving force} = W - T$$

$$\therefore \frac{W-T}{W} = \frac{f}{g} \text{ or } W - T = \frac{f \cdot W}{g}.$$

$$W\left(1 - \frac{f}{g}\right) = T$$

19. A body of weight W is suspended by a rope to the top of a beam which moves in a horizontal direction. If the string places itself in a position inclined ϕ degrees to the vertical, what is the acceleration of the beam?

From *fig.* 126 we see that the moving force is T sin ϕ. Since there is no motion downwards $W = T \cos \phi$

$$\frac{T \sin \phi}{W} = \frac{f}{g} \text{ or } \frac{T \sin \phi}{T \cos \phi} = \frac{f}{g} \therefore f = g \tan \phi.$$

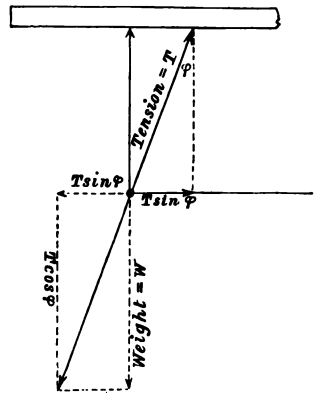

Fig. 126.

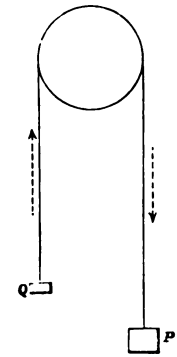

Fig. 127.

20. If two weights, P, Q, *fig.* 127, hang over a pulley, and motion takes place, the moving forces are P − T, and T − Q, T being the tension of the string.

Hence the acceleration

$$\text{downward } \frac{P-T}{P} g, \text{ upward } \frac{T-Q}{g};$$

but both are equal

$$\therefore T = \frac{2PQ}{P+Q}.$$

The acceleration is therefore—

$$\frac{P-T}{P} g = \left(1 - \frac{2Q}{P+Q}\right) g = \frac{P-Q}{P+Q} g,$$

the velocity after t seconds is—

$$\frac{P-Q}{P+Q} g t,$$

and the space passed over is—

$$\frac{1}{2} \frac{P-Q}{P+Q} g t^2.$$

21. In the position represented in *fig.* 128 the moving force, if P descends, is $P - Q \sin \alpha$; hence, the weights moved being $P + Q$—

$$\frac{f}{g} = \frac{P - Q \sin \alpha}{P + Q}.$$

To find T we have the moving force on $P = P - T$

$$\therefore \frac{P - T}{P} = \frac{f}{g} \text{ or } P - T = \frac{f}{g} P$$

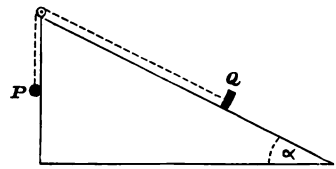

Fig. 128.

$$T = \left(1 - \frac{f}{g}\right) P$$

$$= \left(1 - \frac{P - Q \sin \alpha}{P + Q}\right) P$$

$$= \frac{P Q (1 + \sin \alpha)}{P + Q}.$$

22. In the case represented in *fig.* 129, we have by similar reasoning—

$$\frac{P \sin \beta - Q \sin \alpha}{P + Q} = \frac{f}{g},$$

and for the tension—

$$\frac{T - Q \sin \alpha}{Q} = \frac{f}{g}$$

$$\frac{P \sin \beta - T}{P} = \frac{f}{g}$$

$$\therefore T = \frac{(\sin \alpha + \sin \beta) P Q}{P + Q}.$$

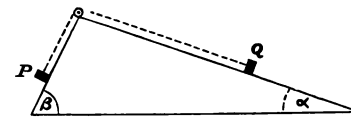

Fig. 129.

23. The speed of a railway train increases uniformly for the first three minutes after starting, and during this time it travels over one mile. What speed (in miles per hour) has it now gained, and what space did it describe in the first 2 minutes?

$$s = \tfrac{1}{2} f t^2$$

$$1 = \tfrac{1}{2} f \cdot 9, f = \frac{2}{9} \text{ of a mile.}$$

$$v = f\,t = \frac{2}{9}\,3 = \frac{6}{9} \text{ per minute,}$$

$$\text{and per hour} = \frac{6+60}{9} = 40 \text{ miles per hour.}$$

$$s \text{ in two minutes} = \tfrac{1}{2} \cdot \frac{2}{9} \cdot 4 = \frac{4}{9} \text{ miles.}$$

24. A body is projected horizontally with a velocity of 50 feet per second from the top of a tower 160 feet high. At what distance from the base of the tower will the body strike the ground?

$$\text{time of descent} = t = \sqrt{\frac{2 \times 160}{32}} = 3{\cdot}1.$$

$$50 \times 3{\cdot}1 = \text{distance from the foot of the tower.}$$

25. How far will a body fall in vacuo during the time in which its velocity increases from 40·25 feet per second to 88·55 feet per second?

$$s = \frac{88{\cdot}55^2 - 40{\cdot}25^2}{2 \times 32{\cdot}2}$$

$$= \frac{128{\cdot}8 \times 48{\cdot}3}{64{\cdot}4} = \underline{\underline{96{\cdot}6.}}$$

26. A weight of 20 lbs. is resting on an inclined plane without friction. When attached by a string which passes over a pulley to a weight of 4 lbs. hanging down perpendicularly, determine the motion and the tension of the string if the gradient of the plane is 1 in 13.

If T the tension, we have, if i = inclination of plane, the 20 lbs. urged down by a force which would give them an acceleration

$$\text{of } g \sin i$$

$$= g\,\frac{1}{13};$$

but this is counteracted by the acceleration produced by the tension,

$$\text{that is by } g\,\frac{T}{20};$$

hence the motion of the weight is—

$$g\,\frac{T}{20} - g\,\frac{1}{13}$$

$$= g\left(\frac{13\,T - 20}{13 \times 20}\right).$$

Similarly for the 4 lbs. the acceleration is—

$$g - g\frac{T}{4} = g\left(\frac{4-T}{4}\right),$$

$$\text{and } \therefore \frac{13T-20}{13\times 20} = \frac{4-T}{4}$$

$$\therefore 13T-20 = (4-T)5 \cdot 13$$

$$\therefore 13T-20 = 20 \cdot 13 \cdot 65T$$

$$78T = 14 \cdot 20$$

$$T = \frac{14 \cdot 20}{78} = \frac{7 \cdot 20}{39} = \frac{140}{39} = 3\frac{23}{39}.$$

$$\text{Acceleration} = \frac{32\left(4 - \frac{140}{39}\right)}{4} = 32 - \frac{8 \cdot 40}{39} = 32 - 28\frac{28}{39} = 3\frac{11}{39}.$$

27. A body, A, is projected vertically downwards from the top of a tower with the velocity V, and one second afterwards another body B is let fall from a window a feet below the top of the tower: in what time will A overtake B?

If x be the time, the space moved over by $A = Vx + \frac{1}{2}gx^2$

by $B = (x-1)^2\frac{1}{2}g$.

The difference of the two spaces is a—

$$\therefore Vx + \tfrac{1}{2}gx^2 - \tfrac{1}{2}g(x-1)^2 = a$$

$$Vx + \tfrac{1}{2}gx^2 - \tfrac{1}{2}gx^2 + gx - \tfrac{1}{2}g = a$$

$$\therefore x = \frac{2a+g}{2(V+g)}.$$

162. **Summary of facts and formulæ for motion.**—The following brief statements will be useful to the reader as a general summary and recapitulation of the principal primary facts regarding free motion:—

1. A body is said to be at rest when, according to the judgment of our senses, it does not change its position relative to surrounding objects; it is in motion when it changes its position.

2. These observed motions express, however, only *differences* of motion. An object moving on board ship opposite to the motion of the ship, with equal velocity, is at rest with reference to a fixed point on the shore. An object at rest on a ship moves with it. In a fog, the boat on a river appears at rest to those in it, objects near it move with the same velocity. In a balloon all seems at rest.

3. Our opinions as to rest or motion are always founded upon comparison with a point which is assumed as resting. But no rest exists in nature. Sun, earth, &c., move, and all

bodies upon them. Motion is the only real state existing. Rest is that state of motion of a body which it shares with other bodies, maintaining their relative position.

4. The motion of a body is independent of the rotation of the earth. A stone drops in a train as if the observer were outside on the ground; a stone thrown out falls in the same manner as if thrown while the carriage is at rest. Hence bodies participate in motions independently of those they had before, and each may be investigated as if none existed previously. *Mechanics* investigates the phenomena of motion.

5. Bodies are considered to be made up of small particles. The different parts of a body may have different motions, and a knowledge of the motion of the molecules is necessary for judging of the motion of the whole.

6. The successive *places* of a moving particle form its path. A falling body describes a continuous straight line. A point at the circumference of a wheel describes a curve; similarly, the hand of a watch describes with its extremity the circumference of a circle.

7. Motion in space involves duration and hence time. Time is the measure for the duration of a motion. Time and space determine motion. Both are continuous.

8. If s = length of the path or space passed over in the time t, an infinite number of relations may exist between them, but all relations may be classified into three groups :—

(*a*) The ratio of space and time is constant during the motion. (Uniform motion.)

(*b*) The space increases more rapidly than the time. (Accelerated motion.)

(*c*) The space decreases in equal successive times. (Retarded motion.)

9. In uniform motion the ratio $\frac{s}{t}$ is independent of the absolute value of s and t. The ratio $\frac{s}{t}$ is called the *velocity*, and velocity denotes in this case the space travelled over in the unit of time. If the time is less than one second, the velocity means the space which would have been traversed if the motion has lasted one second. (*Examples:* Rotation of the earth. Hand of a watch. Light.)

10. Velocities not strictly uniform may often be considered as uniform by introducing an *average velocity*. It is the velocity required by a particle for moving over the same space in the *same* time, if the motion were uniform. (*Examples:* Revolu-

tion of the Earth. Mill-wheel, water in a river, a train, wind, fly-wheel of an engine.)

11. The average or mean velocity approximates to the true motion the more, the shorter is the space and the time taken, from which it is determined.

12. Variable motions are those in which the velocity is variable. Such a velocity may thus be represented geometrically. Let, in *fig.* 131, the line *o t* represent the time of

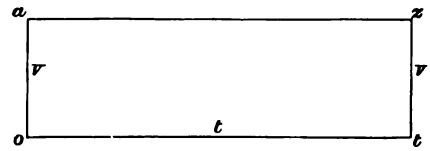

Fig. 130.

motion. Let a line perpendicular to *o t*, *o a*, represent the velocity at the beginning of the motion, and let a series of such perpendiculars represent the velocity at any instant, then *a z* will exhibit the variations of velocity during the time *t*, and is called the 'curve of velocity.'

13. If a small element of the path = *d s*, and *d t* the corresponding element of time, then since $\frac{d\,s}{d\,t} = v$, $d\,s = v\,d\,t$. Hence *d s* is determined by multiplying *v* by *d t*.

14. The area of the figure *n m q p* gives the space passed over in the time from *n* to *m*. The whole space is *o a t z*, that is, the area of *o a t z*.

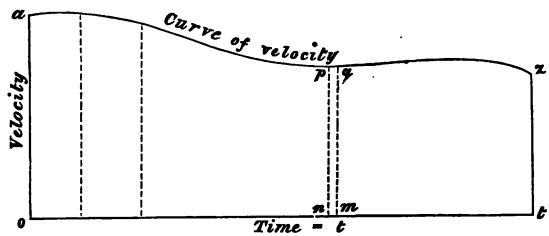

Fig. 131.

15. The curve of velocity in uniform motion is a straight line, as in *fig.* 130, and the area of the rectangle $o\,t\,a\,z = o\,t \times o\,a = V\,t$ = space.

16. If the curve of velocity is an inclined straight line,

as in *fig.* 132, forming with t the angle $z\,a\,n$, then the motion increases with the time. If $o\,a$ = the initial velocity = v_i, then the final velocity, that is,

$$t\,z = V_f = v_i + f\,t \quad . \quad . \quad . \quad . \quad (I.)$$

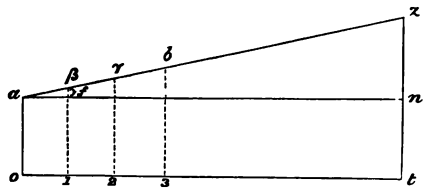

Fig. 132.

This is *uniformly accelerated* motion. The area of the figure is a trapezium, hence—

$$s = \frac{o\,a + t\,z}{2}\,o\,t = \frac{v_i + v_f}{2}\,t \quad . \quad . \quad . \quad (II.)$$

Putting into (II.) for V_f its value $v_i + f\,t$, we have—

$$s = \frac{v_i + v_i + f\,t}{2}\,t = v_i\,t + \frac{f\,t^2}{2} \quad . \quad . \quad . \quad (III.)$$

$$\text{Since } \frac{V_f - v_i}{f} = t \therefore s = \frac{V_f + v_i}{2} \times \frac{V_f - v_i}{f} = \frac{V_f^2 - v_i^2}{2\,f} \quad . \quad (IV.)$$

$$\text{From which } V_f = \sqrt{2\,f\,s + v_i^2} \quad . \quad . \quad (V.)$$

17. When the initial velocity is = ○, these formulæ become respectively—

$$V = f\,t,\ s = \frac{V}{2}\,t = \frac{f}{2}\,t^2 = \frac{v^2}{2f},\ V = \sqrt{2\,f\,s},$$

$$\text{or } t = \frac{V}{f},\ \text{or } t = \sqrt{\frac{2\,s}{f}}.$$

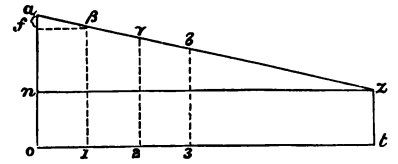

Fig. 133.

18. When the curve of velocity is a straight line inclined, as in *fig.* 133, downwards, the motion is uniformly retarded, and the formulæ become—

$$v = V - ft \text{ or } t = \frac{V - v}{f} \quad . \quad . \quad . \quad \text{(I.)}$$

$$s = \frac{V + v}{2} t \quad . \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

$$= V t - \frac{f}{2} t^2 \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

$$= \frac{V^2 - v^2}{2f} \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

$$v = \sqrt{V^2 - 2fs} \quad . \quad . \quad . \quad . \quad \text{(V.)}$$

19. The fall of a body is a uniformly accelerated motion; the ascent of a body thrown upwards is uniformly retarded. The acceleration of falling bodies is usually denoted by g. It equals in London 32·18 feet. The retardation of bodies thrown upwards is equal to the acceleration of falling bodies.

20. The path of a point is a line, of which the form and direction depend on various relations. A ball moved in the direction of a ship moves with a velocity compounded of that of both; the resultant velocity is either the sum or the difference of both.

21. If the direction differs, the resultant direction of motion can neither be one nor the other. Let A b, b c, c d, in *fig.* 134, represent the velocity, that is, the spaces passed over by the ball

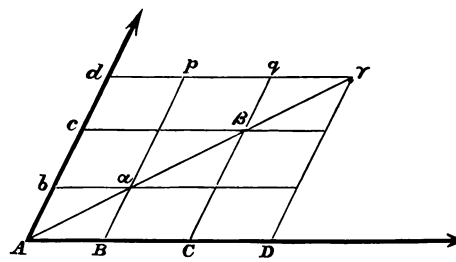

Fig. 134.

in successive equal times, and A B, B C, C D represent the spaces passed over by the ship. Now the whole path A b *moves*, hence the ball arrives at a, &c., and since $\frac{AB}{Ba} = \frac{AC}{C\beta}$, the path is a straight line, and A γ represents at a definite instant the *magnitude* and *direction*. Since the motion is uniform, the spaces A D, A d, A γ are as the velocities in the three directions, and since D γ parallel A d, A D parallel d γ, A γ is in direction and magni-

tude the diagonal of the parallelogram constructed of the magnitudes of the velocities and the directions of the two motions represented by adjacent sides.

22. The two sides represent the *component* velocities, the diagonal the *resultant* velocity. The law is the parallelogram of velocities. It is established by observation, and by the well-known fact upon which it rests, that bodies obey different motions at the same time.

23. The three quantities form a triangle, hence the solution of a triangle is in all cases required for determining the whole.

163. **Statical problems.** — The following problems are founded on the principles explained in the preceding chapters in reference to the resolution and composition of forces, and will serve to illustrate several other points in the solution of mechanical problems.

(1) A B, *fig.* 135, is a beam, B C a string; both having respectively the inclinations to the vertical = α, and θ. Find the pressure on A, the weight of the beam being given.

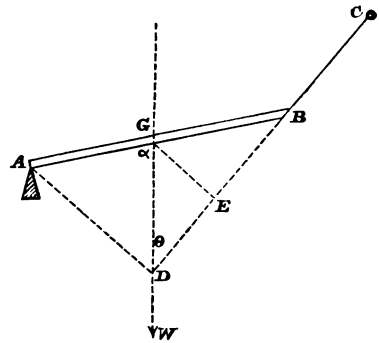

Fig. 135.

Calling the pressure P, and the weight W, the sides of the triangle G D E are parallel to the forces acting, viz.,

D E to the tension of the string,
G E to the pressure on A,
G D to the weight.

From the figure it follows that

$$P : W :: GE : GD :: \tfrac{1}{2} AD : GD$$

$$P = \frac{W \cdot AD}{2 \cdot GD}.$$

If G D $= x$, in the Δ B G D,

$$G B : x :: \sin\theta : \sin(\alpha-\theta)$$

$$G B = \frac{x \sin\theta}{\sin(\alpha-\theta)} = A G.$$

In Δ A G D,

$$A D^2 = A G^2 + G D^2 - A G \cdot G D \cos\alpha$$

$$= \frac{x^2 \sin^2\theta}{\sin^2(\alpha-\theta)} + x^2 - \frac{2 x^2 \sin\theta \cdot \cos\alpha}{\sin(\alpha-\theta)}.$$

$$A D = x \sqrt{\frac{\sin^2\theta}{\sin^2(\alpha-\theta)} + 1 - \frac{2 \sin\theta \cos\alpha}{\sin(\alpha-\theta)}}.$$

$$P = \frac{W}{2} \sqrt{\frac{\sin^2\theta}{\sin^2(\alpha-\theta)} + 1 - \frac{2 \sin\theta \cos\alpha}{\sin(\alpha-\theta)}}.$$

(2) M and N, in *fig.* 136, are two points in a horizontal line. To M a string is fastened equal to half the distance between

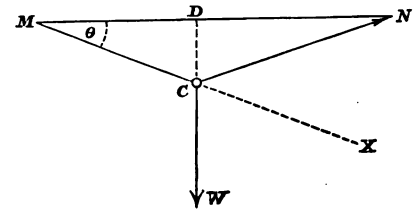

Fig. 136.

M and N. To N another string is fastened, passing through a ring, which is attached to the end of the other string, and carrying a weight, W, at its extremity. Find the condition of equilibrium.

The tension in C N must be = W. Hence M C produced bisects $\angle$ N C W.

Then N C X = X C W = D C M;

but M C N + N C X = 90° + θ + D C M. ∴ M C N = 90° + θ.

Similarly M N C + θ + M C N = 180°

∴ M N C = 180° − θ − M C N

= 180° − θ − (90° + θ). ∴ M N C = 90° − 2 θ;

but M N : L C :: 2 : 1 :: sin (90° + θ) : sin (90° − 2 θ) :: cos θ : cos 2 θ

∴ cos θ = 2 cos 2 θ = 2 (1 − 2 cos^2 θ)

∴ 2 = 4 cos^2 θ − cos θ, which gives θ;

and the angles being known, N C may be determined in terms of M C.

(3) Two equal weights are connected by a string passing over two fixed pulleys, situated in a horizontal line. Between

the two pulleys a weight, equal to P $\sqrt{3}$ is suspended to the string. Find the angle now formed by the string and the horizontal line drawn from one pulley to the other.

When there is equilibrium the tension in the two parts of the string is equal in each to P, and these two tensions are balanced by the weight P $\sqrt{3}$, which is their resultant.

But since $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\therefore$ here $(P\sqrt{3})^2 = 2P^2 + 2P^2\cos\theta$ or $3P^2 - 2P^2 = 2P^2\cos\theta$

$\therefore \cos\theta = \frac{1}{2}$, $\theta = 60°$.

(4) A weight, W, rests on an inclined plane, supported by three forces, each equal to $\frac{1}{3}$ W. These three forces act thus: one vertically upwards, one horizontally, one parallel to the plane. Find from this the inclination of the plane to the horizon.

Resolving all effects parallel to the plane, we have—

$\frac{2}{3}$ W . cos $(90 - a) = \frac{1}{3}$ W + $\frac{1}{3}$ W cos a, a being the inclination.

$\therefore$ 2 sin a = 1 + cos a, &c.

(5) Three particles, A, B, C, whose weights are proportional to 3, 2, 1, are placed so that AB = 5, BC = 4, CA = 2 feet. Find the distance of their common centre of gravity from C.

Let E be the centre of gravity of A, B.

$\therefore$ BE : AE :: 3 : 2

BE : $\underbrace{BE + AE}_{AB}$ :: 3 : 5 $\therefore$ BE = 3.

Now we have to find

CE from $CE^2 = BC^2 + BE^2 - 2BC \cdot BE\cos B$

$= 4^2 + 3^2 - 2 \cdot 7 \cdot 3 \cdot \left(\frac{4^2 + 5^2 - 2^2}{2 \cdot 4 \cdot 5}\right)$.

Having found CE, we find EG from CG : EG :: 5 : 1, &c.

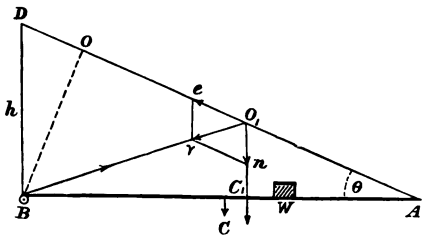

Fig. 137.

(6) A platform, AB, *fig.* 137, turning on a hinge at B, is supported at A by a chain AD, fixed at D vertically above B. A

weight W is placed on the platform at a given distance, q, from B. Find the tension of the cord, and the pressure on the hinge.

Let P = tension.

w = weight of platform.

a = length ,,

B O = perpendicular distance of B from A D. Let also B D = h.

Then we must have for equilibrium, if C the centre of gravity of platform—

$$P \times B O = W \times B W + w \times B C.$$

But $B C = \frac{1}{2} a$ and $B O : B D :: A B : A D$

$$B O : h :: a : \sqrt{a^2 + h^2}$$

$$B O = \frac{a h}{\sqrt{a^2 + h^2}}$$

$$\therefore P \times \frac{a h}{\sqrt{a^2 + h^2}} = W. q + w. \tfrac{1}{2} a,$$

$$\text{or } P = \frac{\sqrt{a^2 + h^2}}{2 a h} (2 W. q + w.a).$$

If θ given—

$$P. a. \sin\theta = W. q + w. \tfrac{1}{2} a$$

$$P = \frac{2 W. q + a. w}{2 a \sin\theta},$$

and if tension is given—

$$\sin\theta = \frac{2 W. q + a. w}{2 a P}.$$

(β) To find the pressure on the hinge. Let C_1 be the centre of gravity of weight W, and weight w of bar A B. Let $B C_1 = c$.

$O_1 \gamma$ is the direction and magnitude of this pressure if $O_1 n = W + w$.

To find $\angle A B O_1$ we have because $\Delta A B D \sim \Delta A C_1 O_1$

$$C_1 O_1 : A C_1 :: B D : A B$$

$$C_1 O_1 : a - c :: h : a. \therefore C_1 O_1 = \frac{(a-c) h}{a};$$

$$\tan A B O_1 = \frac{O_1 C_1}{B C_1} = \frac{(a-c) h}{a c}.$$

To find the magnitude of $O_1 \gamma$ we have—

$$B O_1 = \sqrt{B C_1^2 + O_1 C_1^2} = \sqrt{c^2 + \frac{(a-c)^2 h^2}{a^2}}$$

$$= \tfrac{1}{2} \sqrt{a^2 c^2 + (a-c)^2 h^2};$$

but the triangle $O_1 B D$ is similar to $\Delta O_1 \gamma e$,

$$\therefore O_1 \gamma : e \gamma :: O_1 B : B D$$

$$\therefore o_1\gamma = (W+w)\cdot\frac{I}{a}\frac{\sqrt{a^2c^2+(a-c)^2h^2}}{h}$$

$$= \frac{W+w}{a\,h}\sqrt{a^2c^2+(a-c)^2h^2}.$$

(7) A smooth beam rests upon a prop C, in *fig.* 138, and the extremity rests against a wall E F; the prop is I foot from the wall. How long must the beam be, to rest at an angle of 30° to the vertical?

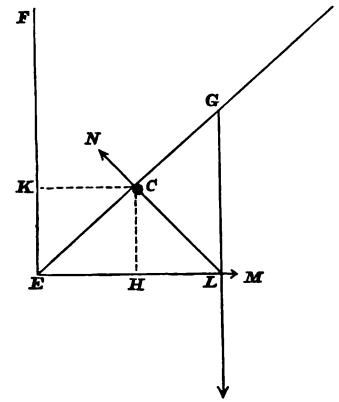

Fig. 138.

The resistance of the wall acts in the direction E M; that of the prop in the direction L N. Hence, the weight of the beam acting along G L, the point G must be perpendicular above L.

$$\text{Hence } \frac{EH}{EC}=\frac{EC}{EL}=\sin 30^\circ=\frac{EL}{EG},$$

$$\text{or } EH=EC\sin 30^\circ$$

$$EC=EL\sin 30^\circ.\therefore EH=EL\sin^2 30^\circ$$

$$EL=EG\sin 30^\circ.\therefore EH=EG\sin^3 30^\circ,$$

$$\text{or } EG=\frac{I}{\sin^3 30^\circ},\ \therefore \text{length of beam}=\frac{2}{\sin^3 30^\circ}.$$

(8) A B, *fig.* 139, is a beam of which one end A is held up by a string, and the other end B rests against a smooth vertical wall: required the direction in which the string must pull to keep the beam at rest.

Because E G is parallel to B C. $\therefore \frac{AG}{GB}=\frac{AE}{EC}$;

but G middle point of A B
$\therefore$ E ,, ,, A C,
and B C = 2 E G.

If θ = angle of beam with vertical
α = ,, string ,,

$$\therefore \tan\theta = \frac{EB}{EG},\ \tan\alpha = \frac{EB}{BC} = \frac{EB}{2EG}$$

$$\therefore \tan\theta = 2\tan\alpha.$$

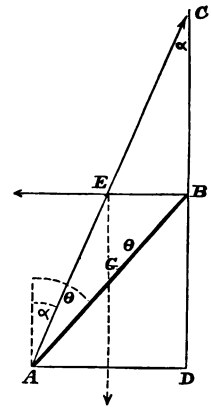

Fig. 139.

(9) A B, B C, *fig.* 140, are two smooth planes inclined to the horizon, E F a beam resting with its extremities on the planes; having given the inclination of one of the planes and of the beam, to find what the inclination of the other plane must be to keep the beam at rest.

The resistances of the two planes act along E H, F H, the weight of the beam along H G.

If the two angles were given, which the planes make with the *horizon*, then their complements are the angles which they make with the vertical; hence let α, β be the angles with the horizon, θ that of beam with the vertical. The relation of these angles follows thus:

In Δ E G H G H : G E :: $\sin(\theta-\alpha) : \sin\alpha$
In Δ G H F G H : G F :: $\sin(\theta+\beta) : \sin\beta$. But G E = G F

$$\therefore \frac{\sin(\theta-\alpha)}{\sin\alpha} = \frac{\sin(\theta+\beta)}{\sin\beta},$$

$$\text{or } \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\sin\alpha} = \frac{\sin\theta\cos\beta + \cos\theta\sin\beta}{\sin\beta}$$

$$\sin\theta\cot\alpha - \cos\theta = \sin\theta\cot\beta + \cos\theta$$

$$\sin\theta\cot\alpha - \sin\theta\cot\beta = 2\cos\theta$$

$$\sin\theta(\cot\alpha - \cot\beta) = 2\cos\theta$$

$$\tfrac{1}{2}(\cot\alpha - \cot\beta) = \cot\theta,$$

which expresses the relation.

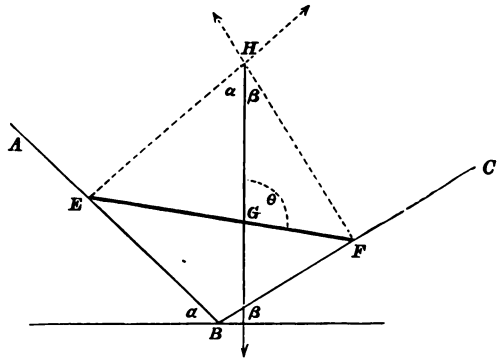

Fig. 140.

(10) A rectangular block, K B, *fig.* 141, is being overturned upon the edge A, by a force P applied at the opposite edge. Find

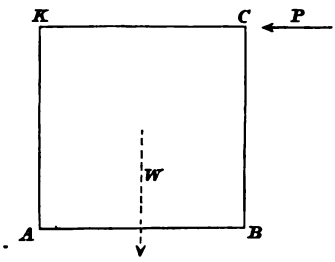

Fig. 141.

the magnitude of this force, if K A = 10 ft., A C = 4 ft., the length of the block = 20 ft., and one cubic foot of the material weighs 100 lbs.

The moment of the force which overturns, is P . K A. The moment of the weight is W. $\frac{1}{2}$ A B.

The block contains $20 \times 10 \times 4 = 800$ cubic feet.

$\therefore$ weight 80,000 lbs.

$\therefore$ P. $10 = 80{,}000 \times 2$

P $= 16{,}000$ lbs.

(11) A rope A D, *fig.* 142, 50 feet long, supports a uniform pole, O D, length 40 feet, resting on the ground at O, and carrying the weight W suspended from D. If A O = 20 ft., and W = 300 cwt., while the pole itself weighs 2 cwt., find the tension of the rope.

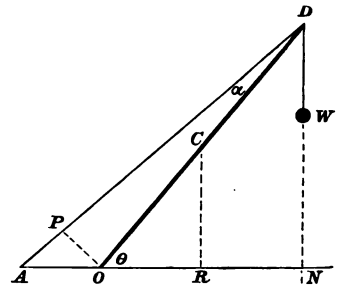

Fig. 142.

Let O P be perpendicular on A D, C R the vertical from the centre of gravity.

Tension $\times$ O P = pole $\times$ R O + W $\times$ O N.

Since $\frac{\text{A D} \times \text{O P}}{2} = \Delta = \sqrt{\frac{50+40+20}{2}(55-50)(55-40)(55-20)}$

$= \sqrt{55 \cdot 5 \cdot 15 \cdot 35} = 380$ nearly.

$\therefore \frac{50 \times \text{O P}}{2} = 380$, O P $= 15{\cdot}2$

$\frac{20 \times \text{D N}}{2} = 380$, D N $= 38$

O N $= \sqrt{40^2 - 38^2} = 12{\cdot}5$

O R $= \frac{1}{2}$ of O N $= 6{\cdot}25$.

$$\text{Tension} = \frac{30 \times 12{\cdot}5 + 2 \times 6{\cdot}2}{15{\cdot}2}.$$

Or, generally, if W = weight of pole.

Tension of rope $\times$ O P = weight of pole $\times$ O R + weight $\times$ O N

Let O D $= a$, $\therefore$ O P $= a \sin \alpha$; O N $= a \cos \theta$; O R $= \frac{1}{2}$ O N $= \frac{a}{2} \cos \theta$.

$$\therefore \text{Tension} = \frac{w \times \frac{a}{2} \cos\theta + \text{W} \cdot a \cos\theta}{a \sin \alpha} = \frac{\cos\theta}{\sin\alpha}(\text{W} + \tfrac{1}{2} w).$$

CHAPTER XII.

MOLECULAR FORCES.

164. **Action of molecular forces.**—It has been demonstrated in a former chapter, that the space included within the external surface of the body is not all occupied by the matter composing that body. We have seen that bodies, however dense or ponderous, are capable of being diminished in their volume by compression, or by diminution of temperature. It follows, therefore, that the component particles forming the mass of a body of uniform density are uniformly distributed throughout its volume, each particle being separated from those around it by a space of greater or less extent unoccupied by matter of the same kind.

These interstitial spaces are the regions which form the theatre of action of those important physical phenomena which are causally referred to what are called molecular forces. A multitude of phenomena, familiar to all observers, show that between the particles which compose the mass of a body, there are exerted attractive or repulsive forces, the sphere of whose action is in general limited to distances imperceptible to the senses, while their existence only admits of being proved by indirect means.

165. **Cohesion.**—The qualities of solidity and hardness, and, in general, those properties by which a body resists fracture or flexure, or any other derangement of its form, arise from the energy with which its component particles attract each other and resist any force which tends to separate them.

Molecular attraction manifested in this manner is called the *attraction of cohesion.*

166. **Adhesion.**—If the surfaces of two bodies be brought into very close contact, it is found that they cannot be separated without the exertion of some force of greater or less intensity, according to the circumstances of the contact.

Molecular force manifested in this manner is called the *attraction of adhesion.*

167. **Capillary attraction.**—Certain bodies being placed in contact with a fluid, the fluid will enter their dimensions and occupy their pores; as, for example, when a sponge or a lump of *sugar is* brought in contact with water. The fluid in these cases

rises in opposition to its gravity, and fills all the interstices of the sponge or the sugar. Molecular force manifested in this manner is called *capillary attraction.*[1] The effects of this force are explained in detail in the volume on HYDROSTATICS in this series.

168. **Chemical affinity.**—When two bodies of different kinds are brought together, their constituent particles will in certain cases unite, and form by their combination the constituent particles of a compound, differing in its physical and chemical qualities from either of the components. For example, if two gases called oxygen and hydrogen be mixed together in a certain proportion, and a light be applied to the mixture, an explosion will take place ; the atoms of the two gases will unite one with another, and the entire mass will be converted into water, the weight of the water being exactly equal to the sum of the weights of the two gases. In this case, each atom of the oxygen is attracted by two atoms of hydrogen, and their combination forms a molecule of water.

Molecular attraction manifested in this manner is called *chemical attraction* or *chemical affinity.*

169. **The atomic attraction and repulsion.**—It has been shown that all bodies submitted to the action of mechanical forces of sufficient energy are capable of being compressed and diminished in their volume. By such means, therefore, their component particles are forced into closer proximity.

But all of these resist such compression with a certain force, and most of them have a tendency to recover the volume which they had before compression. This general fact indicates the existence of another force, contrary in its direction to the attraction of cohesion, the sphere of whose action is within that of the latter attraction. To explain this phenomenon we are compelled to suppose that each atom composing a body is surrounded with a sphere of repulsion within which adjacent atoms cannot enter unless urged by a certain force. But outside this sphere of repulsion there exists the sphere of attraction, by which such atoms attract all the surrounding atoms, which gives the character of solidity and hardness to the mass. These relative molecular movements are, however, at present referred to other principles, of which a brief acccount is given in the Introduction to the volume on HYDROSTATICS in this series.

170. **Examples of molecular action.**—The attraction of cohesion is manifested in solid bodies by the force which is necessary to derange their form by fracture or flexure, or by any

[1] So called from *capilla*, a hair ; the magnitude of the interstices being in this case estimated at about the thickness of a hair.

other change of figure. The same force is manifested in liquids in their tendency to form into spherical drops, a globe being the greatest volume which can be contained within a given surface, the spherical form being determined by the equality of cohesive attraction in all directions.

Thus particles of water falling in the atmosphere attract each other, and collect in spherules forming rain. If such spherules after their formation be exposed to cold, they harden and form round hailstones. If a little mercury be let fall on a sheet of paper, it will collect in small silvery globules, notwithstanding the tendency of the gravity of its particles to make it spread over the paper in fine dust. Innumerable examples present themselves of this class of phenomena. The tear as it falls from the eye collects in a spherule upon the cheek ; the dew forms a translucent globule on the leaves of plants.

The manufacture of shot presents one of the most striking examples of this phenomenon in the arts. The lead, in a state of fusion, is poured into a sieve, the meshes of which determine the magnitude of the shot, at the height of about two hundred feet from the ground. The shower of liquid metal, after passing through the sieve, forms, like rain in the atmosphere, spherules, which, before they reach the ground, are cooled and solidified.

These spherules form the common shot used in sporting, and the precision of their spherical form shows how regularly the liquid obeys the geometrical and physical laws which have been already stated as the causes of its formation.

This disposition of fluids to assume the spherical form may be further elucidated in considering that any other figure which a body could take would necessarily place different parts of its surface at different distances from its central point, a circumstance which would be incompatible with the combined qualities of attraction of cohesion and fluidity. By fluidity, all the particles forming the mass are free to move amongst each other, and by the attraction of cohesion they are drawn round their common centre with the same force. To suppose that they could rest at different distances from their common centre, would necessarily involve the supposition either that the attraction by which they are affected was unequal, or that the mass had not perfect fluidity.

This principle, which is so evident, may be converted, and we may assume that in all cases where natural bodies are found in the spherical form, even though they be solid, they must have *been, at* the epoch of their formation, in a fluid state.

Hence it is inferred that the earth and the other bodies of the solar system were once fluid, and that our globe existed formerly in a liquid state.

The mutual repulsion of the atoms of gases is manifested by their indefinite expansibility and compressibility. Air included in a cylinder under an air-tight piston will expand so as to fill the increased volume as the piston is drawn up, and to this expansion there is no practical limit. This has been explained by supposing that around each molecule of the air or gas there is a sphere of repulsion, so that each particle repels those around it. When the piston is raised to twice its former height, the air beneath it will expand into double its former volume.

In this case it must be concluded that the vacant spaces between the particles of air are twice as great as they were before the piston was raised. If the piston be again raised to double its present height, the same effect will take place. The air will again expand in virtue of the repulsive force prevailing among its particles, and the interstitial spaces separating the particles will be proportionally augmented.

There is no known limit to this expansive quality, and it consequently follows that the region through which the repulsive forces of gases act has a corresponding extent.

Methods are explained in the volume on HEAT in this series by which several of the gases have been reduced to the liquid state, and analogy justifies the conclusion that all gaseous bodies are capable of this change. In the liquid state, the attraction of cohesion is rendered manifest, as has been already shown. But we have a still further evidence that the attraction of cohesion is only latent amongst the particles of gases, inasmuch as some of them have been reduced to the solid state ; and by analogy we may conclude that all are capable of this change. It has been already shown that the solid state is only a consequence of the attraction of cohesion.

These and other phenomena lead to the conclusion that in this case of the gaseous bodies, there exists beyond the sphere of cohesion an inherent molecular motion producing apparently the phenomenon of repulsion. When the particles, either by the application of cold or compression, or both of these agencies, are brought into such close contact as to be within the sphere of cohesion, then they become a liquid or solid, as the case may be.

The mutual repulsion found to prevail among the constituent particles of bodies is generally attributed to the agency of heat, and it is certain that the energy of this repulsion is increased, or diminished, according as heat is imparted to or subtracted

from bodies. In a solid body, such as a mass of gold, in its ordinary state, the attraction of cohesion between its particles greatly predominates over the influence of the repulsion already mentioned; but if heat be applied to this mass, the energy of the repulsion is gradually increased, until at length it becomes so nearly equal to that of cohesion, that the gravity of the particles overcomes that part of the cohesion not balanced by the repulsion, and the constituent parts of the mass no longer holding together in the solid form, the metal is converted into a liquid. If heat be still applied to this liquid, the temperature will rise and the liquid will expand; but after a certain quantity of heat has been imparted to it, the repulsive force between the particles themselves is so great, that, in spite of their gravity and of the attraction of cohesion, they separate and disperse into a vapour which possesses the qualities of a gas, being capable of expanding without limit.

Thus it appears that the same body may exist in the solid, liquid, and gaseous forms, according to the conditions under which it is placed in reference to heat.

171. **Adhesion of solids.**—If the surfaces of two pieces of metal, being rendered perfectly smooth, are brought into close contact by a strong pressure, they will adhere together with considerable force. That this adhesion is not due to atmospheric pressure can be demonstrated by showing that the adhesion will continue in a vacuum. In this case the superficial molecules of the two bodies are brought into contact so close as to be within the sphere of each other's attraction.

Innumerable examples of the adhesion of solid bodies are familiar to daily experience. We may write with chalk, or with a pencil, or charcoal on a wall or on a ceiling, although the effect of gravity would be to cause the particles abraded from the chalk, the lead, or the charcoal to fall from the wall or the ceiling. Dust floating in the air sticks to the wall or ceiling, in spite of the tendency due to its gravity to fall from them.

The force of adhesion of solid surfaces one to another may be ascertained by placing the adhering surfaces in a horizontal position, the lower one being attached to a fixed point, and the upper one connected with the arm of a balance. The weight necessary to separate them is the measure of the adhesion. If we desire to ascertain the amount of adhesion per square inch of surface, it is only necessary to divide such weight by the magnitude of the adhering surface expressed in square inches.

It is on the adhesion between metallic surfaces when pressed *strongly together* that the efficacy of a locomotive engine de-

pends. The driving-wheels press with a great weight upon the rails, and are made to revolve round their own centres by the force of the engine. If there were no adhesion, or even insufficient adhesion between the tire of the wheel and the rail on which it is pressed, the wheel would turn without advancing; and this actually does happen in cases where the rails are greasy, and very frequently when they are covered with a hoar frost, the contact being then interrupted, and the matter between the wheel and the rail not offering the necessary adhesion.

On the other hand, when the force applied to break the adhesion is directed perpendicularly to the adhering surfaces, a fluid or unctuous matter smeared upon the surface often increases the adhesive force.

Thus two metallic plates will adhere with greater force together if they are smeared with oil than when they are clean. This may partly arise, however, from the fact, that the film of oil which covers them excludes air more effectually than could be accomplished in the case of surfaces so considerable by mere pressure.

The effect known amongst workers in metal as the *bite* is the adhesion of two metallic surfaces brought into extremely close contact. It may be doubted whether this adhesion would not be diminished if some fluid were introduced between the surfaces.

The adhesion of the surface of solids may be rendered more intense than even the cohesion of the particles of the solids themselves by interposing between them some substance in a liquefied form, which hardens by cold, and which when hard has a strength equal to or greater than that of the solids which it unites. Glues, cements, and solders supply remarkable examples of this. Two pieces of wood glued together will break anywhere rather than at their joint. The processes of gilding and plating also supply examples of the adhesion of metals to each other.

The process of silvering mirrors is an example of the adhesion of metal to glass; and that of mortar in building is an example of the adhesion of earthy matters to each other.

Two pieces of caoutchouc, if pressed together upon freshly cut surfaces, will be found to unite as completely as if they composed one independent piece.

BOOK THE THIRD.

THEORY OF MACHINERY.

CHAPTER XIII.

GENERAL PRINCIPLES.

172. **What constitutes a machine.**—A machine is an instrument or apparatus by which a force applied at a certain point, and having a certain definite intensity and direction, is made to exert a force at another point, more or less distant from the former, and generally different in intensity and direction. Thus, for example, a horse moving on a horizontal road in a circle is made to raise a weight vertically in the shaft of a mine, or water from the shaft of a well.

Men pulling at a rope in some direction more or less oblique are enabled to raise a mass of heavy matter from the hold of a ship, and to transfer it to an adjacent wharf.

173. **Definitions and general considerations.**—The force which is applied to and transmitted by a machine is technically called the power; the point at which it is applied is called the point of application; its direction is the line in which the force has a tendency to make the point of application move; and its intensity is usually expressed by a weight which, acting at the same point of application, would produce a like effect upon it.

The moving powers applied to and transmitted by machinery are infinitely various. In the capstan of a ship, the moving power is human force applied to it; in a common pump, the *same moving force* is used; a horse is the moving power applied

to vehicles of transport on common roads, and a steam engine on railways; the wind is the moving power applied to a sailing-vessel, and to a windmill; the momentum of water acting against the float-boards of a wheel, or its weight acting in the buckets, is the moving power of a water-wheel; the elastic force of steam acting on the piston in the cylinder is the moving power of the steam-engine.

That part of a machine which is immediately applied to the resistance to be overcome is called the *working point.*

The resistance, whatever be its nature, to which the working point is applied, is technically called the *weight* or *load.* In many cases weight is the actual resistance which machines are applied to overcome; as, for example, in raising water from a well or from mines; also in raising ore. In some cases the resistance to be overcome is friction, used for the purpose of fracturing and pulverising material substances, that is, of overcoming cohesion. This is the case in flour-mills.

In some cases the resistance to be overcome is the friction of surfaces, and the resistance of the air. This is the case when carriages are moved on level roads or railways.

Whatever be the nature of the resistance, a weight which would produce an equivalent force acting against the moving power may be assigned. Thus, for instance, if the traces of a carriage drawn by horses, or the chain connecting a locomotive with a railway train, be stretched by the resistance of the carriage or the train, a weight may be substituted, which, being suspended vertically from the traces or the chain, would produce the same tension. The resistance in such case is expressed by stating the amount of this weight.

In the exposition of the theory of machinery, it is expedient to omit, in the first instance, the consideration of many circumstances, of which, however, a strict account must be subsequently taken before any practically useful application of them can be made. A machine, such as we must for the present contemplate it, is a thing which can have no real or practical existence. Its various parts are considered to be free from friction. Thus, the surfaces composing it, which move in contact with each other, are assumed to be perfectly smooth and polished; the solid parts are all considered to be absolutely rigid and inflexible.

The weight and inertia of the matter composing the machine itself are wholly neglected, and we reason upon it as if it were divested of these qualities. Cords, ropes, and chains are supposed to have neither stiffness, thickness, nor weight; they are regarded as mathematical lines, infinitely flexible and infinitely strong.

Finally, the machine, when it moves, is assumed to encounter no resistance from the air, and to be in all respects circumstanced as if it were in vacuo.

These suppositions being all imaginary, it follows that none of the consequences immediately deduced from them can be true. Nevertheless, as it is the business of art to bring machines as near to this state of ideal perfection as possible, the conclusions which are thus obtained, though false in a strict sense, yet deviate from the truth in no considerable degree.

These conclusions may, in fact, be regarded as a first approximation to truth.

The various effects which have been previously neglected are afterwards taken into account. The roughness of surfaces, the imperfect rigidity of the solid parts, the imperfect flexibility of cords and chains, the resistance of the air and other fluids, and the effects of the weight and inertia of the machine itself, are afterwards severally examined, their properties explained, and the manner in which they modify the transmission of the power to the weight developed. These modifications and corrections being applied to the conclusions obtained, a second approximation to the truth is made, but still only an approximation ; for, in investigating the laws which govern the several effects last mentioned, we are compelled to proceed upon a new group of imaginary suppositions.

To determine the laws which regulate the friction of surfaces, it is necessary to assume that the surfaces are uniformly rough, and subject to uniform pressure ; that the solid parts which are imperfectly rigid, and the cords and chains which are imperfectly flexible, are constituted throughout their entire dimensions of a uniform material, so that the imperfections do not prevail more in one part than in another. Thus, all irregularity is left out of account, and a general average of the effects taken. It is obvious, therefore, that, even in this second system of reasoning, we have still failed in obtaining a result exactly conformable to the real state of things. But it is equally obvious that we have obtained one much more conformable to that state than had been previously accomplished ; and, in fine, it is found that the conclusions thus obtained are sufficiently near the truth for practical purposes.

The imperfections in our process of investigation, manifested in this laborious system of successive approximation to the truth, is not peculiar to Physics. It pervades all departments of natural science. In Astronomy, the motions of the celestial bodies, and their various changes and appearances as developed *by theory,* assisted by observation and experience, consist of a

like series of approximations to the real motions and appearances which take place in nature.

It is the same in Art. The first labours of the artist produce from the rude block of marble a rough and rude resemblance of the human form. The next attempts remove the greater inequalities and protuberances, and reduce the form to a closer resemblance to the original. It is not, however, until after a long succession of operations, in which smaller and smaller portions of the stone are detached, that the last labour of the chisel of the master completes the resemblance.

174. **Machines provisionally regarded independent of weight and resistances to motion.**—We shall, therefore, for the present consider the machine, by which the effect of the power is transmitted to the working point, as divested of weight and inertia; we shall consider all the pivots, axles, and surfaces which move in contact absolutely devoid of friction; we shall consider all cords, ropes, and chains to be absolutely and perfectly flexible, and to be moved in contact with the grooves and wheels without friction; and, in fine, we shall consider the machine itself, as well as the agent exerting the power, and the matter composing the weight, to move without resistance from the air or any other fluid.

The exposition of the effects of machinery is often invested with the appearance of paradox. Astonishment is excited at what seems incompatible with the results of common experience, rather than admiration of the genius and skill by which simple and obvious principles are so applied as to produce unexpected results.

Thus it is stated that, by means of a machine, a power of comparatively insignificant amount is capable of supporting or raising a vast weight; as, for example, it is affirmed that the fingers of an infant pulling a thread of fine silk, which a pound weight could snap asunder, are capable by this or that machine of supporting or raising several hundredweight.

Statements like these, if literally understood, are fallacious; if rightly explained, they involve nothing which is not consistent with our habitual experience.

In every machine there are some fixed points or props, and the arrangement of the parts is always such that all that portion of the weight not directly acting against the power is distributed among these props. If the weight, for example, amount to 20 cwt., it is possible so to arrange it that any proportion of it, however great, may be thrown upon the fixed points or props of the machine: the remaining part only can properly be said to be

supported by the power, and this part so supported can never be greater than the power. Considering the effect of a machine in this manner, it appears that the power supports just so much of the weight, and no more, as is equal to itself, and that all the remaining part of the weight is sustained by the machine.

The truth of this observation will become more and more apparent when the conditions are explained, under which a power and weight can maintain each other in equilibrium, through the intervention of a machine, whether simple or complex.

But if the power, instead of merely supporting the weight at rest, be employed to raise this weight a given height, it may be asked how it can be explained that a power indefinitely small can lift a weight indefinitely great. The paradoxical character of this statement arises, as is the case generally in such propositions, from the omission of an important condition. It is quite true that a feeble power is capable of raising a great weight, but it is necessary to add that in doing this, the feeble power must act and move through a space just so much greater than that through which the weight is raised, as the weight itself is greater than the power. Thus, if a weight of 1000 lbs. be raised one foot by a power whose magnitude is only equal to 1 lb., then such power in raising the weight must move through 1000 feet.

Now when this condition is stated, the proposition is stripped altogether of its paradoxical character. There is nothing at all astonishing in the fact, that one thousand successive exertions of a force of one pound, each exertion being made through the space of one foot, should raise a 1000 lbs. weight through the height of one foot. There is nothing more surprising in such a fact than if the 1000 lbs. weight, being divided into 1000 equal parts, were raised by a thousand successive efforts of the power without the intervention of any machinery.

175. **Different relations of power and weight.**—It will be necessary to consider the effect produced by means of a machine under three distinct relations between the power and weight, viz.,

I. When the power equilibrates with the weight.

II. When the power is greater than that which equilibrates with the weight.

III. When the power is less than that which equilibrates with the weight.

The power and weight are said to be in equilibrium when they are so related to each other that when placed at rest they will remain so. It is a great, but very common error, to suppose *that equi*librium, as applied to a machine, necessarily im-

plies rest or the absence of motion. It is easy to show that if the power and weight, being in equilibrium, are put in uniform motion, they will continue that uniform motion exactly as a mass of matter would do in virtue of its inertia, if moving independently of any machine; for if we were to suppose that such motion would cease either suddenly or gradually, we must necessarily also suppose a definite force applied to the machine to stop its motion. Since the power and weight are in equilibrium, they cannot of themselves stop or retard the motion. It is true that the motion will in practical applications be gradually retarded, but that will be the effect of friction and atmospheric resistance, both of which are at present excluded from consideration.

We cannot, on the other hand, suppose the uniform motion imparted to the power and weight to be accelerated, without supposing the application of some adequate force to produce such acceleration, the power and weight being excluded by the very condition of their equilibrium.

Let us suppose the power and weight to be connected with the machine by cords, by which they are suspended from their respective points of application, both being, as usual, represented by equivalent weights. Now the cords by which they are suspended will be stretched with the same force whether the power and weight be at rest or in uniform rectilinear motion; and consequently the relation between them in both cases must be the same.

176. **Equilibrium involves either rest or uniform motion.**—The most common state of machines which are under the operation of equilibrating forces, is that of uniform motion, and not that of rest, as commonly stated. If a wind or water-mill be in regular operation, its driving-wheel moving with a uniform speed, then the power of the wind or water will be in equilibrium with the resistance, whatever that may be. If a steam engine be in regular operation, its piston will move at a uniform rate, and the force of the steam upon it will be in equilibrium with the resistance which it is applied to overcome. If a locomotive engine draw a railway train at a uniform speed, then the power exerted by the engine will be in equilibrium with the resistance opposed to the motion of the train.

177. **Investigation of different general cases.**—Let us now consider the case in which the power is greater than that which equilibrates with the weight of resistance. In this case the motion imparted to the object moved will be accelerated; for so much of the power as would equilibrate with the weight

or resistance would impart, as has been already shown, a uniform motion to the object moved. The surplus power above this amount, therefore, must be employed in accelerating the motion.

For example, if a locomotive engine exert a greater power than is equivalent to the resistance opposed by the train which it moves, then such excess of power can only act upon the mass of the train, and will impart to it an equivalent amount of motion. So long as this surplus power, therefore, acts, the mass of the train will receive from it a corresponding augmentation of its momentum, and consequently will receive a proportionate increase of speed. If the resistance, however, opposed by the train to the moving power augments with the speed, then it may at length become equal to the amount of the moving power ; and when it does, their equilibrium is established, and the train is moved by the power at a uniform speed.

These conditions are by no means imaginary. They are realized in every case in which a train is started from a state of rest, and in general when any machine whatever is first put in motion.

The power in commencing its action must necessarily be greater than the resistance opposed by the load ; for if it were not, it would only equilibrate with the resistance, and no motion would ensue. The surplus power is absorbed by the momentum acquired by the moving mass ; and as the velocity augments, more and more momentum is imparted. The velocity will at length become uniform, either because the energy of the power will be diminished, so as to become equal to the resistance, or because the resistance will be augmented, so as to become equal to the power ; or, in fine, as most generally happens in practice, both of these effects are combined, the resistance increasing and the power diminishing. This is always the case, therefore, when a machine is impelled by a surplus power, and when, on the other hand, there is a less than ordinary resistance on the side of the machinery and of the load. When first starting, the velocity being inconsiderable, the resistance of the air and other agencies depending upon speed is less. As the velocity increases, these resistances augment. This augmentation of resistance, however, as the speed increases, is generally much less than the diminution of the moving power ; in short, a considerable surplus power is generally necessary, at starting, to impart to the load and to the moving parts of the machinery the necessary momentum. But after this momentum has once been imparted, then nothing remains for the power but to balance the resistance *of the load, properly* so called.

This excess above the equilibrating power and the accelerated motion are reciprocal consequences. Such excess necessarily infers the accelerated motion of the load, and the accelerated motion of the load indicates such excess.

Effects directly the reverse of these are developed when the power applied is inferior to that which would equilibrate with the weight. Let us suppose, in this case, the machine to have been in uniform motion, and therefore the power and weight to have been in equilibrium. Let the power then be diminished by any amount, however small: the moment this diminution takes place equilibrium is destroyed, the power becomes inferior to the resistance, and there is an action in a direction contrary to that of the power, and therefore contrary to that of the motion which the load had already acquired, equivalent in amount to the difference between the resistance and the power. This force will act against the energy of the load, and will continually diminish it, until, at length, it brings the load to rest. From the moment, therefore, that the power becomes less than the resistance, the motion of the load will be gradually retarded. The inferiority of the power to the resistance, and a gradually retarded motion, are, therefore reciprocal consequences of each other.

It may be useful to illustrate still further these effects, which are of considerable importance in practice. Let us suppose the resistance which a machine is employed to overcome to be represented by the weight A, *fig.* 143, and let the power which acts against such resistance through the intervention of the cord A B C be represented by the force of an animal H. When the animal is at rest before starting, the cord A B C H is stretched with a force exactly equal to that of the weight. When the power begins to move, a momentum is imparted to the weight through the intervention of the cord. The cord is therefore stretched with an additional force proportional to this momentum. The speed of the power gradually increases from the moment its motion commences until it attains that speed which is continued uniform. During this increase of the speed of the power H, a corresponding and continual increase of momentum is imparted to the weight A, and consequently, during this interval, the tension

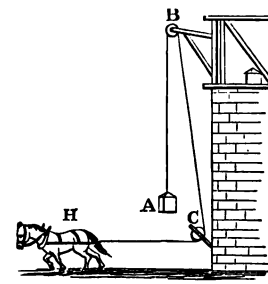

Fig. 143.

of the cord is constantly greater than the weight. When, however, the speed of the power H becomes uniform, then no further momentum will be imparted to the weight, and the force exerted by the power will diminish so as to become exactly equal to the weight. During this uniform motion the tension of the cord will be the same as it would be if the power and weight were at rest.

When the weight approaches its point of destination, and is about to be brought to rest, the power slackens its exertion, and, at the moment that it becomes less than the weight, a moving force takes effect equal in intensity to the difference between the power and weight directed from B to A. But against this there is the momentum of the weight directed from A to B in virtue of the uniform velocity with which it had been moving. The moving force, therefore, from B to A, represented by the difference between the power H and the weight A, will act against this momentum, and will gradually diminish it.

Although the upward motion of the weight, therefore, will continue after the diminution of the power, it will be gradually retarded, and after a certain interval will be altogether exhausted, and the weight will come to rest.

These effects take place in all machines whatever when they are started and stopped, and the circumstances and mechanical laws which govern them are precisely the same as in the illustration here given.

178. **Proper functions of a machine.**—The use of a machine is to adapt the power to the resistance. If the intensity, direction, and velocity of the power were identical with the intensity and direction of the resistance, and the velocity required to be imparted to it, then there would be no need of a machine; the power might be applied immediately to the resistance. But if a power of feeble intensity is required to act against a great resistance, then a machine must be interposed which will augment the intensity of the power. Or if a power moving in one direction be required to impart motion to a resistance in another direction, then a machine must be interposed which will transmit the effect of the power to a new direction. Or if a power having a certain velocity be required to impart a greater or less velocity to the resistance, then a machine must be interposed which will modify the velocity in the required proportion.

But even these, though the principal, are only a few of the infinite varieties of change and modification which machines are required to effect in the transmission of the power to the resistance. Independently of the directions, intensities, and veloci-

ties of the moving power and resistance, the character of the respective motions may differ in an infinite variety of ways: thus the moving power may be one which acts with a reciprocating motion between two points; as, for example, that of the piston of a steam-engine; and this moving power, again, may be required to produce a continuous motion in a straight line, like the motion of a train along the railway.

The machine which connects such a power with such a weight must therefore be so constructed as to convert the reciprocating motion between two points into a continuous motion in a straight line.

The moving power may act in a straight line, while the resistance requires a circular motion. Thus, the wind which acts upon the arms of a windmill is a continuous rectilinear force. The millstones to which that force is transmitted, revolve by a continual circular motion around vertical axes. The machinery of the windmill must therefore be adapted to convert the rectilinear force of the wind into the circular motion of the stones.

In every class of machines, and in every individual machine of each class, the relation between the velocities and directions of the power and weight, and the change produced in the character of the motion of the power when transmitted to the weight, depend solely on the structure of the machinery.

No variation in the magnitude of the power and weight can alter this relation. Thus the ratio of the velocities of the power and weight on a *lever* or an *inclined plane*, so long as their form and proportions remain the same, will be unaltered, and, whatever power or weight be applied to them, they will have that particular ratio.

179. **No machine can really add to the mechanical energy of the power.**—A machine being composed of inert matter cannot generate force, and consequently the working point cannot do more work than is transmitted to it from the point of application of the power.

It will, in fact, exert less, because friction and other sources of resistance must intercept a portion of the action of the power in its transmission from its point of application to its working point; but as, for the present, the consideration of this species of resistance is neglected, and machines are considered as exempt from them, we shall assume that the influence of the power is transmitted undiminished to the working point. But it is important on the other hand to remember, that *no more* moving force can be so transmitted.

180. **Method of measuring work.**—The work done by the power is determined by multiplying the weight which is equivalent to it, by the space through which it is moved ; and, on the other hand, the work done by the resistance is also estimated by multiplying the weight which is equivalent to this resistance by the space through which it is moved.

The moving force, as work has until recently been often called in practice, of the power is determined in the same manner as the moving force of a weight equivalent to it, and moving with the same velocity, would be determined. Thus, if we multiply the power, or its equivalent weight, by the space through which it moves in a given time, that is to say, by its velocity, we shall obtain a product which expresses its moving force or mechanical effect.

This latter product is also sometimes called the *moment of the power*. Thus if P express the power and p the space through which it moves in one second, then $P \times p$ will be its *moment*. In like manner, the moving force imparted to the resistance at the working point will be expressed by multiplying the resistance, or the weight equivalent to it, by the space through which it is moved in a given time. Thus if W be the weight, and w the space through which it is moved in one second, then $W \times w$ will be the moving force of the weight, and this product is called the *moment of the weight*.

The relation between the moments of the power and weight determines their mechanical state.

Three cases are here presented :—

I. When the moment of the power is equal to the moment of the weight ; that is, when

$$P \times p = W \times w.$$

II. When the moment of the power is greater than the moment of the weight ; that is, when

$P \times p$ is greater than $W \times w$.

III. When the moment of the power is less than the moment of the weight ; that is, when

$P \times p$ is less than $W \times w$.

In the first case, it is manifest that the power and weight [illegible] equilibrium, and that they will be either at rest or in [illegible]notion. For, since the moment of the power is the [illegible] of its moving force, and since this moving force is [illegible]d without increase or diminution by the machinery to [illegible]t, *and since*, by the supposition we have made, it is equal

to the moving force of the weight, these two forces must balance each other, and therefore be in equilibrium.

The condition therefore of equilibrium is, that the moment of the power is equal to the moment of the weight, or

$$\text{P} \times p = \text{W} \times w.$$

If the moment of the power be greater than the moment of the weight, then the moving force of the power, exceeding that of the weight, and being transmitted to the working point undiminished, will prevail over it, and the power and weight must either have an accelerated motion in the direction of the power, or a retarded motion in the direction of the weight.

If the moment of the power be less than the moment of the weight, then the moving force of the power being transmitted to the weight and being less than the moving force of the latter, the latter will prevail, and therefore the power and weight must have either a retarded motion in the direction of the power, or an accelerated motion in the direction of the weight.

If the moments of the power and weight be equal we may infer that the power will bear to the weight the same ratio as the velocity of the weight bears to the velocity of the power, or

$$\text{P} : \text{W} :: w : p;$$

or, as it is sometimes expressed, the power and weight will be to each other inversely as their velocities. This is another mode, then, of expressing the conditions under which the power and weight will be in equilibrium.

The considerations contained in the preceding articles will be still further extended, in accordance with more modern conceptions, in the chapter on work and energy.

181. **Power always gained at the expense of time.**—It is this inverse proportion which is intended to be expressed, when it is said that power is never gained save at the expense of time; the meaning of which is, that if a small power work against a great resistance, the rate at which it moves the resistance will be just so much slower than that at which the power itself moves, as the resistance is greater than the power.

This condition of equilibrium, when rightly understood, removes all paradox from the statement of the effects of machinery. A small power working through a large space, raising a great weight through a small space, is merely an expedient by which a feeble power is enabled to accomplish its task, by a long succession of efforts, without dividing the weight. To raise the weight of a ton by a single effort one foot, would require a force equivalent to the weight of a ton. But if, by the intervention

of a machine, a power is enabled to accomplish this object by 2240 distinct efforts, each effort working through one foot, then such power need not be more than one pound, or 2240 efforts made through the space of one foot, each effort exerting the force of one pound, will be mechanically equivalent to 2240 lbs., or one ton raised through one foot, and the effect produced will be the same as if the weight were actually divided into 2240 equal parts, and the power applied successively to raise each of these parts one foot.

A very inadequate estimate would, however, be formed of the objects and the utility of machinery, if we were to suppose them only directed to this particular class of problems. Cases innumerable occur, on the contrary, where small resistances are moved by great powers. For example, in a locomotive engine, while the piston in the cylinder moves once backwards and forwards, the train, which is the resistance overcome, is moved through a space equal to the circumference of the driving-wheel. Now, if we suppose the length of the cylinder, as frequently happens, to be one foot, and the circumference of the driving-wheel to be fifteen feet, then the velocity of the piston, or the power, will be to the velocity of the train, or the resistance, as 2 to 15; and consequently, the power which acts upon the piston must be greater than the resistance of the train, which is moved in the proportion of 15 to 2, omitting, as usual, the consideration of friction, &c. In like manner, in a watch or clock, the resistance of the object moved is merely that which is opposed to the motion of the hands on the dial-plate, while the moving power is the energy of the main-spring, or of a descending weight. In both these cases it is obvious that the power is vastly greater than the weight.

Machinery, therefore, may be stated generally as being the means by which the force and motion of the power are modified, so as to adapt them to the force or motion which is required to be imparted to the object moved.

In all that precedes, it has been assumed that the point of application of the power moves in the direction in which the power acts, and that the motion imparted to the working point is in a direction immediately opposed to the action of the weight or resistance. This, in fact, is what generally takes place in the practical construction and operation of machinery; for it is evident that if the point of application of the power were not free to move in the direction in which the power acts, a part of the power would necessarily be lost; and that if the working *point did not* move in a direction immediately opposed to the

weight or resistance, a part of the force transmitted to the working point would be inefficient. But as, in certain cases, these conditions would not be fulfilled, it will be useful to state how the principles which have been established in the present chapter must be modified in such cases.

If by the construction of the machinery the point of application of the power moves in a line different from that in which the power acts, then the effective part of the power will be found by the parallelogram of forces.

Let A (*fig.* 144) be the point of application of the power, and let B be the working point. Let A P represent the direction of the power, and B W the direction of the resistance or weight. Let A *p* be the direction in which the point of application is free to move, and let the working point B be free to move in the direction opposite to B *w*. Let right-angled parallelograms be

Fig. 144.

formed, having for their diagonals A P and B W, representing the power and resistance, and having their sides in the direction A *p* and B *w*, in which the point of application and the working point are respectively free to move.

The power will then be equivalent to two forces represented by A *m* and A *p*; the latter, being in the direction in which alone the point of application can move, is alone effective; that part of the power represented by A *m* will necessarily be expended in pressure and strain upon the fixed points of the machine. In like manner the weight or resistance represented by B W is equivalent to two forces, B *w* and B *n*; the force B *w*, being in the direction against which alone the working point can act, is that portion of the weight or resistance which the working point will act against: the remainder of the weight will produce strain or pressure on the fixed point.

In the application of the principles determining the relation of the power and weight in cases of equilibrium which have been established in the present chapter, the effective portion only of the power and weight must be taken into account. Thus, the power is to be considered as represented by A *p*, and

the weight by B w. These principles will be rendered more clearly intelligible when they have been illustrated in their application to the simple machines.

CHAPTER XIV.

SIMPLE MACHINES.

182. **Machines, simple and complex.**—Machines which are composed of two or more parts acting one upon another are called complex machines. Machines which consist only of one part are called *simple machines*. The several parts composing a complex machine are themselves simple machines. In a *complex machine* the effect of the power is transmitted successively through each of the parts composing it until it reaches the working point. The effect of complex machines is determined by combining together the separate effects of the simple machines of which they are composed.

To estimate the effects of machinery, therefore, it will be necessary, in the first instance, to explain the principles of simple machines.

183. **Classification of simple machines.**—Simple machines have been differently enumerated by different writers. If the object be to group in the smallest possible number of distinct classes those machines whose efficacy depends on the same principle, the simple machines may be comprised under the following three denominations :—

I. A solid body turning on an axis.
II. A flexible cord.
III. A hard and smooth inclined surface.

Notwithstanding the infinite variety of machinery, and of the parts composing it, it will be found that those parts may invariably be brought under one or other of the above classes.

184. **Conditions of equilibrium in a machine with a fixed axis.**—In a machine composed of a solid body turning on an axis, all the parts are carried round such axis as a common centre, and describe circles round it in the same time. It is evident that the magnitude of these circles, and consequently *the velocities* of the different parts, will be proportional to their

respective distances from the axis in which their common centre lies.

But since it has been already shown that when the power and weight are in equilibrium they must be inversely as the velocities of the points to which they are applied, it follows that any power and weight applied to such a machine will be in the inverse proportion of their distances from the axis when they are in equilibrium.

It must be understood, in the application of this important principle, that the power and weight are supposed to act in the direction of the motion of the parts to which they are respectively applied. If they do not act in this direction, then they must be resolved by the principle of the composition of force into two forces, one acting in the direction of the motion of the point of application, and the other in a direction passing through a point upon the axis. This has been already explained in art. 181.

185. **Machines with flexible cords.**—The second class of simple machines includes all those in which a force is transmitted by means of flexible threads, ropes, or chains. The principle by which the effects of these machines is estimated is, that the tension throughout the whole length of the same cord, provided it be flexible and free from the effects of friction, must be the same. Thus, if a force acting at one end be balanced by a force acting at the other end, however the cord may be bent, or whatever course it may be compelled to take, by any cause which may affect it between its ends, these forces must be equal, provided the cord be free to move over any obstacles which may deflect it. This class includes all the various forms of pulleys.

186. **Weights upon hard inclined surfaces.**—The third class includes all those cases in which the weight or resistance is supported or moved upon a hard surface inclined to the direction in which the weight or resistance itself acts. The effects of such machines may be estimated by the principles already explained. The force of the weight or resistance being resolved into two other forces by the principle of the composition of force, one of these two forces will be perpendicular to the surface, and thus supported by its reaction; the other will be parallel to it, and will act against the power.

187. **Mechanical powers.**—The first class of simple machines above mentioned, consisting of a solid body revolving on an axis, is usually subdivided into two.

1st. The lever, which consists of a solid bar, straight or bent, resting upon a prop, pivot, or axis.

2nd. A cylinder connected with a wheel of much greater diameter moving round a centre or axis. This combination is called the wheel and axle.

The second class includes the pulley. The third class includes the simple machines, commonly known as the inclined plane, the wedge, and the screw; the last being, as will appear hereafter, nothing more than an inclined plane rolled round a cylinder.

The classes, therefore, of the simple machines, as they are generally received, and which are known as the mechanical powers, are the six following:—

I. The lever.
II. The wheel and axle.
III. The pulley.
IV. The inclined plane.
V. The wedge.
VI. The screw.

We shall accordingly explain these, and show the most important varieties and combinations of which they are susceptible.

THE LEVER.

188. **Three kinds of lever.**—A straight and solid bar turning on an axis is called a *lever*. The *arms* of the lever are those parts of the bar extending on each side of the axis. The axis is called the *fulcrum* or *prop*.

Levers are commonly divided into three kinds, according to the position which the fulcrum has in relation to the power and weight.

If the fulcrum be between the power and weight, as in *fig.* 145, the lever is of the first kind.

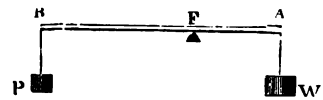

Fig. 145.

If the weight be between the fulcrum and power, as in *fig.* 146, the lever is of the second kind.

If the power be between the fulcrum and weight, as in *fig.* 147, the lever is of the third kind.

189. **Conditions of equilibrium.**—Of whatever kind the lever may be, the conditions of equilibrium of the power and *weight will be* such that they are inversely as their distances

from the fulcrum, this being the general condition of equilibrium for all machines which turn round a fixed axis (art. 184). It follows, therefore, that in *figs.* 145, 146, and 147 we shall have

$$P : W :: FA : FB;$$

or, if p express the distance of the power from the fulcrum, and w the distance of the weight from the fulcrum, we shall have

$$P : W :: w : p;$$

or, what is the same,

$$P \times p = W \times w.$$

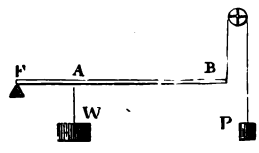

Fig. 146.

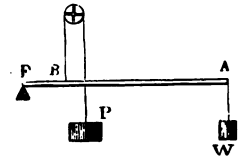

Fig. 147.

This statement, as will be perceived, is nothing more than a repetition of the general principle affecting machines which turn on an axis, in virtue of which forces upon them are in equilibrium when their moments round the axis are equal. The moment of the power is $P \times p$, and the moment of the weight is $W \times w$. The tendency of the power to turn the lever round its fulcrum in the direction of the power is expressed by the moment $P \times p$, and the tendency of the weight to turn the lever in the contrary direction is expressed by $W \times w$.

It follows, therefore, that the tendency of the power to turn the lever would be augmented either by increasing the amount of the power P, or by increasing its distance p from the fulcrum. In either case the effect will be increased in a corresponding proportion. Thus, if we remove the power to double its distance from the fulcrum, we shall double its effect; and if we remove it to half its distance, we shall diminish its effect one half. The distance of a force, whether power or weight, from the fulcrum, is called its *leverage*; and it is evident from what has been stated, that the effect of any force applied to a lever will be proportional to its leverage.

If the forces applied to a lever do not act perpendicular to it, their effect will be found by drawing from the fulcrum a perpendicular on their directions. This perpendicular will be their leverage. Thus in *fig.* 148, if the power act in the direction B P, draw F N perpendicular to the direction P B N; the power will have the same effect in turning the lever, as it

acted at N upon the lever N F. The moment of the power, therefore, in this case, will be found by multiplying it by F N,

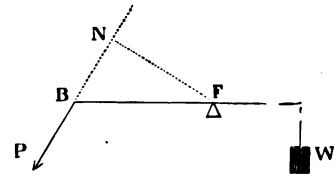

Fig. 148.

the perpendicular distance of its direction from the fulcrum. In general, therefore, the leverage of any force applied to such a machine is estimated by the perpendicular distance of the direction of such force from the fulcrum.

190. **Relation of power and weight in the three kinds.**—In a lever of the first kind, the power and weight may be equal, and will be so when their leverages are equal. The weight may be less than the power, and it will be so when it is at a greater distance from the fulcrum than the power.

In a lever of the second kind, the weight, being between the fulcrum and the power, must be at a less distance from the fulcrum than the power, and must consequently be always greater than the power.

In a lever of the third kind, the power being between the fulcrum and the weight, will be at a less distance from the fulcrum than the weight, and consequently in this case the power must always be greater than the weight.

191. **The balance.**—Numerous examples of levers of the first kind may be given. A balance is a lever of this kind with equal arms, in which the power and weight are necessarily equal. The dishes, or scale-pans, are suspended by chains or cords from points at precisely equal distances from the fulcrum, and being themselves adjusted so as to have precisely equal weights, the balance will rest in equilibrium when the dishes are empty. To maintain this equilibrium, it is evident that equal weights must be put into the two dishes; the slightest inequality would give a preponderance to one or the other dish.

To obtain all the necessary precision, the centre of the beam in well-constructed balances (*fig.* 149) has angular-shaped steel prisms attached to each side of it, with their edges presented downwards. These, which are called *knife edges*, rest upon a *hard surface, such as* steel or agate. The hooks to which the

chains supporting the dishes are attached are similarly supported. The instrument is so constructed that the centre of gravity of the beam and its appendages is a little below the knife edge on which it is suspended when the beam is horizontal, the index attached to the centre of the beam then pointing vertically upwards. If the beam be inclined downwards on either side, the centre of gravity will deviate to the other side, and, by its tendency to take the lowest position, it will bring the beam back to the horizontal position when the force which disturbed it is removed.

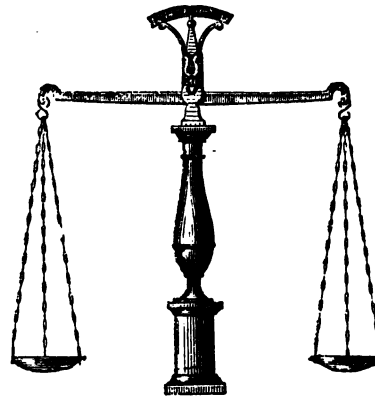

Fig. 149.

By the conditions of equilibrium of the lever, as explained above, the beam can only rest in the horizontal position, with the index vertical, when the weights in the dishes are equal. The article to be weighed being placed in one dish, known weights are placed in the other, until the index points to the zero on the graduated scale behind it, which is so arranged that the index is then vertical. The weight of the article is then equal to the known weights in the other dish.

To verify the exactitude of a balance, let it be first observed whether the index is vertical, and the beam horizontal, when the dishes are empty. The article to be weighed being then placed in one dish, and balanced by weights in the other, let the article and the weights be transposed, the former being placed in the dish containing the latter, and *vice versâ*. If the index is still vertical, and the beam horizontal, the balance is exact; if not, the arms are unequal, and the indications erroneous. Such a balance should be rejected.

192. **Sensibility of a balance.**—This term is used to express the facility with which the index is turned from its position by a preponderating weight placed in either dish, and it is greater or less, according as the distance of the centre of gravity below the point of support is less or greater. Let A B (*fig.* 150) be the beam, C the point of support, and G the centre of gravity. If the beam be slightly turned from the horizontal position to the position A′ B′, the centre of gravity will take the position G′, and would be balanced by a weight upon A′, which would be less than that of the beam in the proportion of the distance of G′ from G C to that of A′ from G C. The nearer G is to C, the nearer will also G′ be to G C, and therefore the less will be the weight which would suffice to turn the beam in a given degree from the horizontal position, and it is evident that the less this weight is, the more sensitive will be the balance.

Fig. 150.

In very sensitive balances the beam does not immediately come to rest when weights, whether equal or unequal, are put in the dishes, but oscillates for some time slowly from one side to the other. If the zero of the graduated arc on which the index plays be exactly in the middle of the arc of oscillation, the weights in this case will be equal ; but if the middle of the arc of oscillation is seen to be on one side of the zero, the preponderating weight will be on the same side.

However inaccurately a balance may be constructed, the exact weight of a body may be ascertained by it. For this purpose let the body to be weighed be placed in one dish, and let it be equipoised by sand placed in the other. Leaving the sand undisturbed, let the body be removed, and replaced by weights of known value which will equipoise the sand. The amount of these weights will then be the weight of the body.

193. **The Steelyard.**—A steelyard is a lever with unequal arms ; the power, being represented by a sliding weight, is adjusted so that its leverage may be changed at pleasure. These and similar instruments are used for the purpose of weighing in many commercial transactions.

This instrument, sometimes called the Roman balance, is

constructed in various forms, one of the most common of which is represented in *fig.* 151. The weight Q slides upon a graduated arm, and in some ascertainable position balances the article P which is to be weighed. The weight of P is just as many times that of Q as D C is greater than A C.

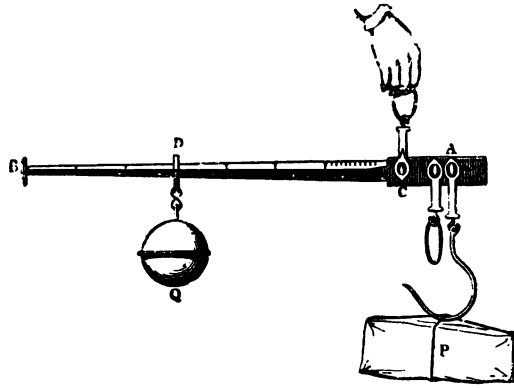

Fig. 151.

It may happen that even when Q is moved to the last division of the arm B C, the article P will still preponderate. In that case the steelyard is held by the ring nearer to A, which hangs down in the figure. By this means the distance of P from the fulcrum being less, the weight Q, at a given distance from the fulcrum, will balance a proportionally greater weight.

194. **Letter balance.**—One of the most simple forms of these useful instruments, which gives exact indications, is shown in *fig.* 152. It is a bent lever of the first kind. The dish E is suspended from a short arm A C, and is balanced by the arm C B, having a heavy bob at G, and a point at B, which moves upon a graduated arc. When A is depressed, C B rises, and its centre of gravity G moves to a greater distance from the vertical line through the point of support C, and takes a position in which it balances the weight placed in the dish E. The indications expressed on the arc show the weights in E, which correspond to the various positions of the index.

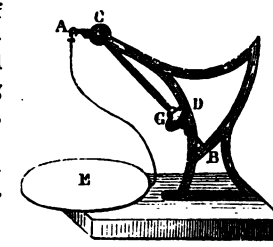

Fig. 152.

195. **Spring balances.**—Besides the instruments constructed upon the principle of the lever, various forms of weighing instruments are made, which depend on the elastic tension of springs. One of these is shown in *fig.* 153 and another in 154, which will be easily understood upon inspection, without further explanation.

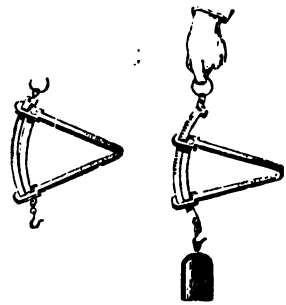

Fig. 153.

Fig. 154.

196. **Other examples of the lever.**—A crowbar is a lever of the first kind. In this instrument, when used, for example, to raise a block of stone, the fulcrum, *fig.* 155, is another stone F, placed near that which is to be raised, and the power of the hand H is placed at the other end of the bar. A poker applied to raise fuel is a lever of the first kind, the fulcrum being the bar of the grate.

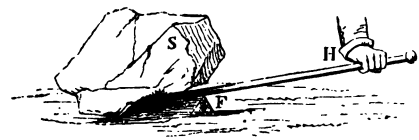

Fig. 155.

The force exerted by the hand at B (*fig.* 156) will in this case overcome a resistance greater than itself in the proportion of B C to A C. The instrument may be rendered still more efficacious by removing the point of support C to the line $r\ q$, or on the other hand a greater range of motion with less power may be obtained by removing it to the line $m\ n$.

Scissors, shears, nippers, pincers, and other similar instruments are composed of two levers of the first kind, the fulcrum

being the joint or pivot, and the weight the resistance of the substance to be cut or seized, the power being the fingers applied at the other end of the levers.

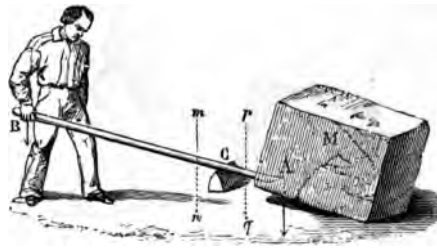

Fig. 156.

The brake of a pump is a lever of the first kind, the pump-rods and piston being the weight to be raised.

Examples of levers of the second kind, though not so frequent, are not uncommon.

An oar is a lever of the second kind. The reaction of the water against the blade is the fulcrum. The boat is the weight, and the hand of the boatman the power.

The rudder of a ship or boat is an example of this kind of lever, and explained in a similar way.

The chipping-knife (*fig.* 157) is a lever of the second kind. The end F attached to the bench is the fulcrum, and the weight the resistance of the substance R to be cut.

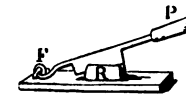

Fig. 157.

A door moved upon its hinges is another example.

Nutcrackers are two levers of the second kind, the hinge which unites them being the fulcrum, the resistance of the shell placed between them being the weight, and the hand applied to the extremities being the power.

A wheelbarrow is a lever of the second kind, the fulcrum being the point at which the wheel presses on the ground, and the weight being that of the barrow and its load collected at their centre of gravity M (*fig.* 158).

The same observation may be applied to all two-wheeled carriages which are partly sustained by the animal which draws them.

Levers of the third kind, acting, as has been explained, to mechanical disadvantage, the power being less than the weight,

are of less frequent use. They are adopted only where rapidity and dispatch are required more than power.

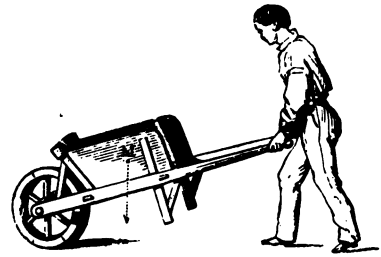

Fig. 158.

The most striking examples of levers of the third kind are found in the animal economy. The limbs of animals are generally levers of this description. The socket of the bone is the fulcrum, a strong muscle attached to the bone near the socket is the power, and the weight of the limb, together with whatever resistance is opposed to its motion, is the weight. A slight contraction of the muscle in this case gives a considerable motion to the limb: this effect is particularly conspicuous in the motion of the arms and legs in the human body; a very inconsiderable contraction of the muscles at the shoulders and hips gives the sweep to the limbs, from which the body derives so much activity.

The treadle of the turning-lathe is a lever of the third kind. The hinge which attaches it to the floor is the fulcrum; the foot applied to it near the hinge is the power; and the crank upon the axis of the fly-wheel, with which its extremity is connected, is the weight.

Tongs are levers of this kind, as also the shears used in shearing sheep. In these cases the power is the hand, placed immediately below the fulcrum or point where the two levers are connected.

197. **Pressure on fulcrum.**—The pressure on the fulcrum of a lever, when the power and weight are in equilibrium, is determined by the principle of the composition of forces. In a lever of the first kind, the resultant of the power and weight is a single force passing through the fulcrum equal to their sum; consequently, the pressure on such point will be equal to the sum of the power and weight.

In a lever of the second and third kind, the power and *weight, acting* in contrary directions, will have a resultant equal

to their difference passing through the fulcrum. This resultant will therefore express the pressure on the fulcrum.

198. **Rectangular lever.**—In the rectangular lever, the arms are perpendicular to each other, and the fulcrum, F (*fig.* 159), is at the right angle. The moment of the power in this case is P multiplied by A F, and that of the weight W multiplied by B F. When the instrument is in equilibrium, these moments must be equal.

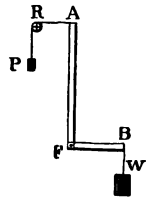

Fig. 159.

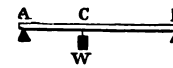

Fig. 160.

When the hammer is used for drawing a nail, it is a lever of this kind ; the claw of the hammer is the shorter arm, the resistance of the nail is the weight, and the hand applied to the handle is the power.

199. **Beam resting on two props.**—When a beam rests on two props, A B (*fig.* 160), and supports at some intermediate place, C, a weight W, this weight is distributed between the props in a manner which may be determined by the principles already explained.

If the pressure on the prop B be considered as a power sustaining the weight W by means of the lever of the second kind B A, then this power multiplied by B A must be equal to the weight multiplied by C A. Hence the pressure on B will be the same fraction of the weight as the part A C is of A B. In the same manner it may be proved that the pressure on A is the same fraction of the weight as B C is of B A. Thus, if A C be one third, and therefore B C two thirds of B A, the pressure on B will be one third of the weight, and the pressure on A two thirds of the weight.

It follows from this reasoning, that if the weight be in the middle, equally distant from B and A, each prop will sustain half the weight. The effect of the weight of the beam itself may be determined by considering it to be collected at its centre of gravity. If this point, therefore, be equally distant from the props, the weight of the beam will be equally distributed between them.

According to these principles, the manner in which a load borne on poles is distributed between the bearers may be ascertained. As the efforts of the bearers and the direction of the weight are always parallel, the position of the poles relatively to the horizon makes no difference in the distribution of the weights between them. Whether they ascend or descend, or move on a level plane, the weight will be similarly shared between them.

If the beam extend beyond the prop, as in *fig.* 161, and the weight be suspended at a point not placed between them, the props must be applied at different sides of the beam. The pressure which they sustain may be calculated in the same manner as in the former case.

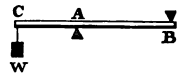

Fig. 161.

The pressure of the prop B may be considered as a power sustaining the weight W by means of the lever B C. Hence, the pressure of B multiplied by B A must be equal to the weight W multiplied by A C. Therefore, the pressure on B bears the same proportion to the weight as A C does to A B. In the same manner, considering B as a fulcrum, and the pressure of the prop A as the power, it may be proved that the pressure of A bears the same proportion to the weight as the line B C does to A B. It therefore appears that the pressure on the prop A is greater than the weight.

200. **Compound lever.**—A combination consisting of several levers acting one upon another, as represented in *fig.* 162, is called a compound lever.

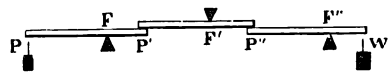

Fig. 162.

The manner in which the effect of the power is transmitted to the weight may be investigated by considering the effect of each lever successively. The power at P produces an upward force at P′, which bears to P the same proportion as P F to P′ F, Therefore, the effect at P′ is as many times the power as the line P F is of P′ F. Thus, if P F be ten times P′ F, the upward force at P′ is ten times the power. The arm P′ F′ of the second lever is pressed upwards by a force equal to ten times the power at P. In the same manner this may be shown to produce an effect at P″ as many times greater than that at P′ as P′ F′ is greater than P″ F′.

Thus, if P′ F′ be twelve times P″ F′, the effect at P″ will be

twelve times that at P′. But this last was ten times the power, and therefore the effect at P″ will be one hundred and twenty times the power. In the same manner it may be shown that the weight W is as many times greater than the effect at P″, as P″ F″ is greater than W F″. If P″ F″ be five times W F″, the weight will be five times the effect at P″. But this effect is one hundred and twenty times the power, and therefore the weight would be six hundred times the power.

In the same manner, the effect of any compound system of levers may be ascertained by taking the proportion of the weight to the power in each lever separately, and multiplying these numbers together.

In the example given, these proportions are 10, 12, and 5, which, multiplied together, give 600. In *fig.* 162 the levers composing the system are of the first kind; but the principle of the calculation will not be altered if they be of the second or third kinds, or some of one kind and some of another.

201. **Weighing-machines.**—These are usually compound levers, and are constructed in very various forms, but all depending nearly on the same mechanical principles; one of these varieties is shown in *figs.* 163 and 164.

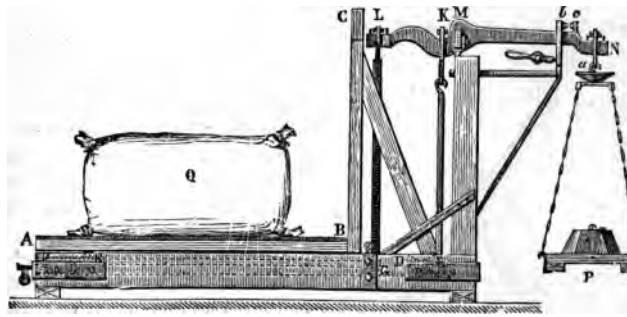

Fig. 163.

The platform, A B, upon which the object to be weighed is placed, is supported by the short arm M L, of a lever L N, through the intervention of a compound lever, K H D C B E F G L, the longer arm of which, M N, supports a dish in which the counterpoising weight is placed.

The platform, A B, is supported at two points, viz., 1st, by the lever E G L at E, and 2nd, by the arm L M at K, through the intervention of the rod K H, the diagonal piece D C, and the

upright piece C B. The proportions of the levers are so regulated that the pressure at L is less than at K, in the same proportion as K M is less than L M, and these two pressures together are balanced by the weight P.

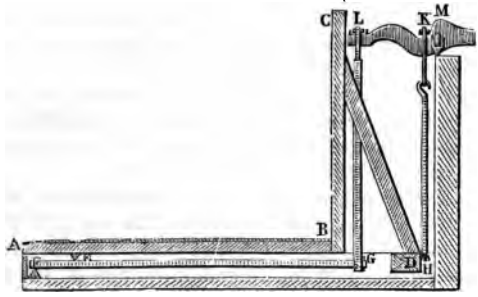

Fig. 164.

Before commencing the operation the lever L M N is rendered horizontal by small weights or sand put into the dish *a*, and its horizontal position is ascertained by the coincidence of the two points *b* and *c*, the former of which is fixed, and the latter attached to the arm M N of the lever.

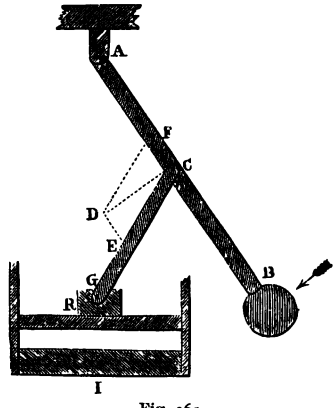

Fig. 165.

It is easy to understand that the proportion of the arms of the lever may be so arranged that each hundredweight of the article Q may be balanced by a pound or any lesser denomination of weight placed in P.

202. **The knee lever.**— A form of compound lever, known as the knee lever, is much used in the arts. This combination consists of a metal rod A B, *fig.* 165, having a fixed point of support A, on which it works. Another bar G C is jointed to it at C, a point intermediate between A and B. This bar C G is jointed at G to a plate, such as R, or any other object to which it is desired to transmit an intense force acting through a very *limited space,* as, for example, in the case of the printing press,

where the paper is pressed upon the type by a plate which is driven upon it by a sudden and severe force. The handle B of the lever being pressed in the direction of the arrow, exerts a corresponding pressure on the point C, which is driven in the direction C D, perpendicular to A B. This motion C D is resolved into two by the parallelogram of forces, one in the direction C E, and the other in the direction C F; the latter exerts pressure on the fixed point A, and the other acts upon the plate R, by means of the joint G forcing it downwards. As the joint C advances, the angle A C G becomes more and more obtuse, and the component C E of the force acting at B bears a rapidly increasing proportion to the force itself, so that when the levers A C and C G come nearly into a right line, the pressure exerted at B is augmented at G in an almost infinite proportion.

203. **Key of Erard's pianoforte.**—In this instrument, the object is to convey from the point where the finger acts upon the key, to that at which the hammer acts upon the string, all the delicacy of action of the finger, so that the piano may participate, to a certain extent, in that sensibility of touch which is observable in the harp, and which is the consequence of the finger acting immediately on the string in that instrument, without the intervention of any other mechanism.

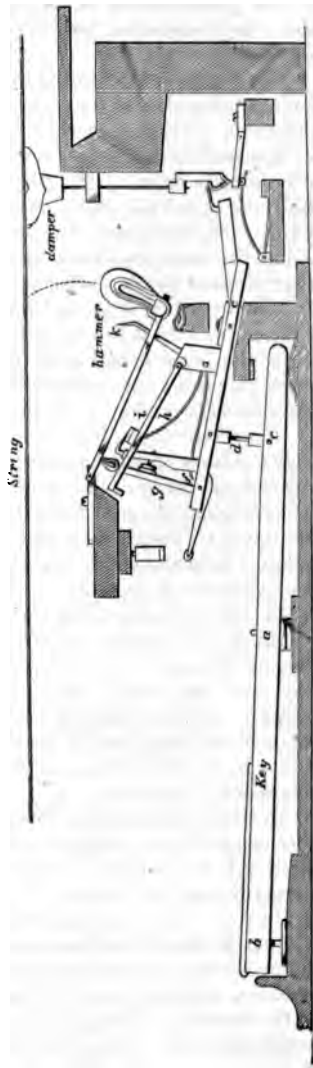

Fig. 166.

The combination of levers, by which the action of the finger is transmitted to the string, in Erard's pianoforte, is represented in *fig.* 166.

The key is represented at *b a c*, the fulcrum or pivot on which it plays being *a*, and the ivory table upon which the finger acts being at *b*. The point to which its motion is communicated is at *c*.

The motion is transmitted by a double-jointed piece *d* to an intermediate lever *e f*, the pivot of which is at *e*. At the joint *f* is a rod *g* called the *sticker*, which carries up the hammer to the string. The hammer is supported by the head of the sticker *g*, and at the same time rests upon the oblique lever *i*, which latter is acted upon by the spring *h*.

When the key is pressed down by the finger at *b*, the piece *d* is raised, and by it the lever *e f* and the sticker *g*. This lever raises the lever *i*, and acts upon the rod of the hammer at a point near the joint, making the hammer rise along the dotted curve so as to strike the string.

The proportions of this combination of levers are such, that, after the blow of the hammer on the string, the check *k* comes forward and receives the hammer in its fall, at about one third of its original distance from the string; so that while the finger continues to keep down the key, the hammer remains at a distance from the string, equal to one third of its distance when the key is not depressed.

In the meantime, the spring *h* has given way under the weight of the hammer, and, under these circumstances, the key *b* being allowed to rise by the finger through one third of its play, and then again being depressed, another stroke of the hammer on the string will be produced; for in this case the hammer will be brought back to the level of the head of the sticker *g*, by which means it will be driven upwards upon the depression of the key.

In the combination of levers used in other pianofortes, the note cannot be repeated without allowing the key to rise to the position it has before it is depressed; consequently in this case, a repetition of the note is produced with one third of the motion of the finger which is necessary in other pianofortes.

204. **Power of a machine.**—That number which expresses the proportion of the weight to the equilibrating power in any machine, is often called the *mechanical advantage*, or the *power of the machine*. Thus if, in a lever, a power of 1 lb. support a weight of 10 lbs., the power of the machine is 10. If a power of 2 lbs. support a weight of 11 lbs., the power of the machine is $5\frac{1}{2}$.

205. **Equivalent lever.**—As the distances of the power and weight from the fulcrum of a lever may be varied at pleasure, and any assigned proportion given to them, a lever may always be conceived having a power equal to that of any given machine. Such a lever may be called, in relation to that machine, the *equivalent lever.*

As every complex machine consists of a number of simple machines acting one upon another, and as each simple machine may be represented by an equivalent lever, the complex machine will be represented by a compound system of equivalent levers. From what has been proved, it therefore follows that the power of a complex machine may be calculated by multiplying together the powers of the several simple machines of which it is composed.

WHEEL-WORK.

206. **The wheel and axle.**—The form of simple machine denominated the wheel and axle, consists of a cylinder which rests in pivots at its extremities, or is suported in gudgeons, and is capable of revolving between those pivots, or in those gudgeons. Attached to this cylinder, and supported on the same pivots or gudgeons, a wheel is fixed, so that the two revolve together with a common motion. The wheel is often supplied with a rope or chain, which winds round the axle, and the power with another rope or chain which winds round the wheel.

Such an arrangement is represented in *fig.* 167, where W is the weight, A and B the pivots or gudgeons, C the wheel, and P the power.

The condition of equilibrium is, according to what has been already proved, the inverse proportion of the power and weight to the diameters of the wheel and axle, that is to say, the power is to the weight as the diameter of the axle is to the diameter of the wheel. The weight is generally applied, as represented in the figure, by means of a rope coiled upon the axle.

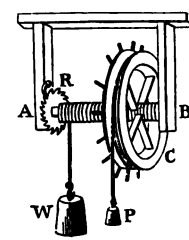

Fig. 167.

The manner of applying the power is very various. Sometimes the circumference of the wheel is furnished with projecting points, as represented in *fig.* 167, to which the hand is applied when human force is the power. Examples of this are numerous ; a familiar one is presented in the steering-wheel of a ship.

207. **The Windlass.**—In the common *windlass* the power is applied by means of a winch D C, as represented in *fig.* 168.

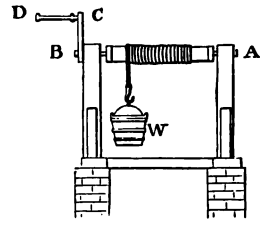

Fig. 168.

The arm B C of the winch represents the radius of the wheel, and the power is applied to D C at right angles to B C. In some cases no wheel is attached to the axle, but it is pierced with holes, directed towards its centre, in which long levers are incessantly inserted, and a continuous action produced by several men working at the same time, so that whilst some are transferring the levers from hole to hole, others are working the other levers, *fig.* 169.

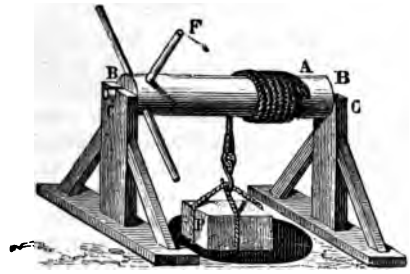

Fig. 169.

The axle is sometimes placed in a vertical position, the wheel or levers being moved horizontally. The *capstan* is an

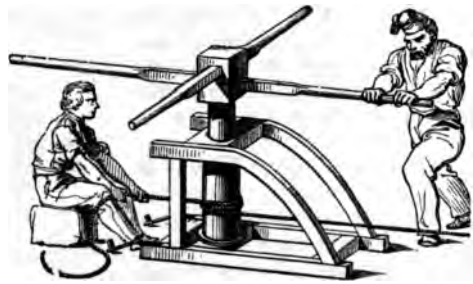

Fig. 170.

example of this. A vertical axis is fixed in the deck of the

ship, the circumference being pierced with holes presented towards its centre. These holes receive long levers, as represented in *fig.* 170. The men who work the capstan walk continually round the axle, pressing forward the levers near their extremities.

More generally the head of the capstan is circular, and pierced with many holes, in each of which a lever can be inserted, so that many men can work together when great power is required, as in weighing anchor, *fig.* 171.

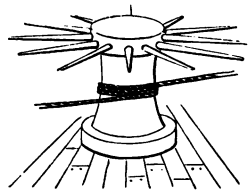

Fig. 171.

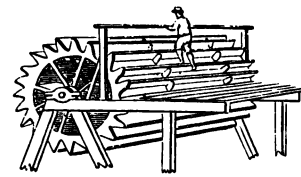

Fig. 172.

208. **The tread-mill, &c.**—In some cases the wheel is turned by the weight of men or animals placed at its circumference, who move forward as fast as the wheel descends, so as to maintain their position continually at the extremity of the horizontal diameter. The tread-mill, *fig.* 172, and certain cranes, such as *fig.* 173, are examples of this.

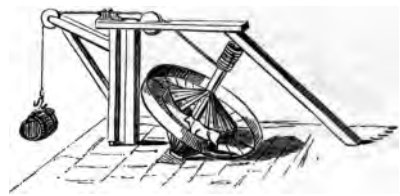

Fig. 173.

209. **French quarry wheels.**—In the neighbourhood of Paris, and other parts of France, stone for building is obtained in subterranean quarries, from which it is raised through deep vertical shafts, by means of large wheels, worked by the weight of men, on the same principle as that of the tread-mill. One of these quarry wheels, and the manner of working it, is shown in *fig.* 174.

From what has been already explained, it is evident that the effect of the power upon the weight would be augmented by

diminishing the thickness of the axle, and diminished by increasing that thickness.

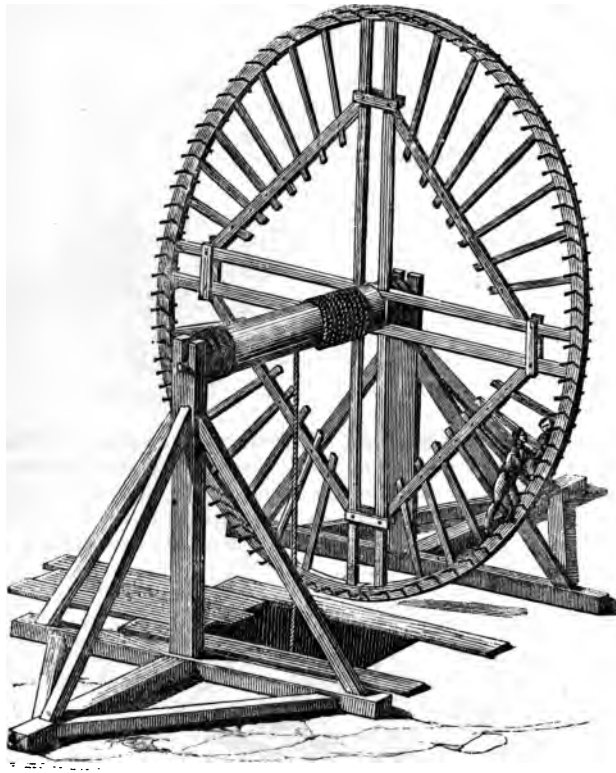

Fig. 174.

210. **Differential wheel and axle.**—It sometimes happens that an invariable power has to act against a variable resistance, or a variable power against a constant resistance. In such a case, the effect of the wheel and axle as just described would vary: an augmentation of the power or diminution of the resistance would throw the power and weight out of equilibrium.

If, however, the axle were made to increase in thickness in the same proportion as the ratio of the power to the weight is *augmented, then* the change of such ratio would be compen-

sated by a corresponding change in the leverage, and an equilibrium would be maintained between the power and weight, notwithstanding their variation. Numerous instances of this are presented in the arts, some of which will be noticed hereafter.

When a weight or resistance of comparatively great amount is to be raised by a very small power by means of the simple wheel and axle, either of two inconveniences would ensue; either the diameter of the axle would become too small to support the weight, or the diameter of the wheel would become so great as to be unwieldy in its operation. This has been remedied, without having recourse to a complex machine, by a simple expedient represented in *fig.* 175. The axle of the windlass

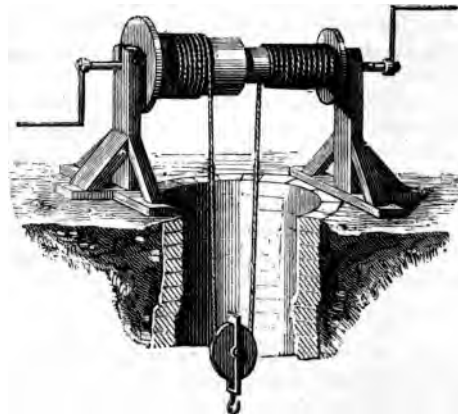

Fig. 175.

here consists of two parts, one thicker than the other, and the rope by which the weight is raised rolls on the thicker while it rolls off the thinner. In each revolution, therefore, the part which is rolled on exceeds that which is rolled off by the difference between the circumference of the two parts of the axle. The effect, accordingly, is the same as if an axle had been used, whose diameter is equal to the difference between the diameters of the thicker and thinner part. Since, then, without diminishing the thickness of the axle, we may diminish without limit the difference between the thicker and thinner parts, the ratio of the weight to the power may be augmented indefinitely without diminishing the strength of the axle.

211. **Combinations.**—When great power is required, wheels and axles may be combined in a manner analogous to the compound lever already explained. The power being supposed to act on the circumference of the first wheel, its effect is transmitted to the circumference of the first axle ; this circumference acts on the circumference of the second wheel, and transmits motion thereby to the circumference of the second axle, which, in its turn, acts on the circumference of the second wheel, transmitting motion to the circumference of the third axle, and so on.

There is nothing different in the mechanical effect of such a combination from that of a system of compound levers, except that it admits more conveniently of a continuous action, and produces continued and regular motion. The relation between the power and the weight of resistance, when in equilibrium, is determined in exactly the same manner as in the case of the compound lever.

If the diameters of all the wheels be multiplied together, and the diameters of all the axles be also multiplied together, then the power will be to the weight as the product of the diameters of all the axles to the product of the diameters of all the wheels. Thus, if the diameters of all the axles be expressed by the numbers 2, 3, 4, and the diameters of all the wheels be expressed by the numbers 20, 25, and 30, then the ratio of the power to the weight will be as

$$2 \times 3 \times 4 = 24 \text{ to } 20 \times 25 \times 30 = 15{,}000.$$

The manner in which the wheels and axles act one upon another is very various. Sometimes a strap or cord is placed in a groove in the circumference of the axle, and carried round

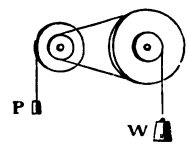

Fig. 176.

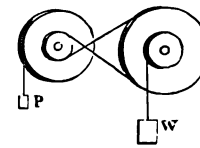

Fig. 177.

a similar groove in the circumference of the wheel. This, which is called an endless band, is represented in *figs.* 176 and 177.

In the case represented in *fig.* 176 the wheels are driven in the same direction ; in that represented in *fig.* 177, they are *driven in opposite* directions. Examples of this method of

transmitting the motion from wheel to wheel are presented in every department of the arts and manufactures. In the turning-lathe and the grinding-wheel a catgut cord carried round the treadle wheel imparts motion to the mandril or the grindstone. In the great factories shafts, as shown in *fig.* 178, are carried along the ceilings of the room, round which, at certain points, endless straps are carried, which are conducted round the wheels, thus giving motion to the lathes or other machines. One of the chief advantages of this method of transmitting motion by wheels and axles is, that the bands by which the

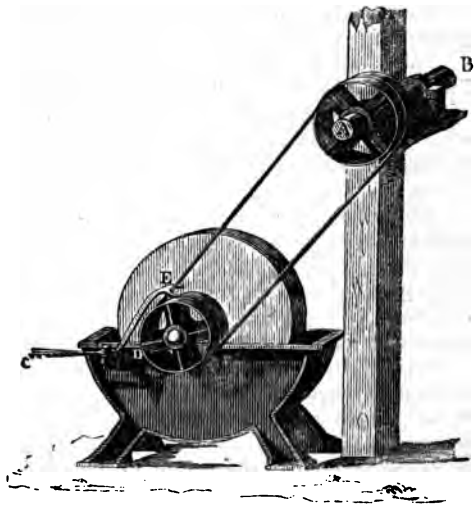

Fig. 178.

motion is conveyed may be placed at any distance from each other, and even in any position with respect to each other, and may, by a slight adjustment, receive motion in either one direction or the other.

When the circumference of the axle acts immediately on that of the wheel, which it moves without the intervention of a strap or cord, means must be adopted to prevent them from moving in contact (*fig.* 179) without transmitting motion, which they would do if both surfaces were perfectly smooth and free from friction.

This is accomplished by different expedients. In cases where

great power is not required, motion is communicated through a series of wheels and axles by rendering their surfaces rough, either with facing them with rough leather, or making them of wood cut across the grain. This method is used in spinning machinery, where a large buffed wheel, placed in a horizontal position, is surrounded by a series of small buffed rollers pressed close against it, each roller communicating motion to a spindle. As the wheel revolves, revolution is imparted to the rollers, the velocity of which exceeds that of the wheel in the same proportion as the diameter of the wheel exceeds that of the roller. This method is very convenient in cases where the motion of the rollers requires to be occasionally suspended, each roller being provided with a means by which it can be thrown out of contact with the wheel, and thus stopped.

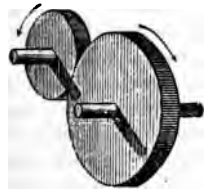

Fig. 179.

212. **Toothed wheels** —The most frequent method of transmitting motion through a train of wheel-work is by the construction of teeth upon their circumference, so that the teeth of each falling between those of the other, the one wheel necessarily pushes forward the other. When teeth are used, the axles are usually called *pinions*, and the teeth raised upon them are called *leaves*.

In the formation of the teeth of wheels and pinions, expedients are adopted to prevent them from rubbing one upon another, when they move in contact with each other. A particular form is adopted for the teeth, in virtue of which the surfaces are applied one to the other with a rolling motion like that of a carriage wheel upon the road. By this expedient the rapid wear of the teeth, which would be produced by constant friction accompanied by pressure, is prevented.

In computing the mechanical effects of toothed wheels and pinions, the number of teeth may be substituted for their circumferences and diameters.

The condition of equilibrium will therefore be obtained by multiplying together the number of teeth in all the wheels, and the number of teeth in all the pinions, the power being to the weight, when in equilibrium, as the latter product to the former.

213. **Spur wheels.**—Teeth are formed upon the edges of wheels and pinions in different positions. When they are formed in the plane of the wheel so as to diverge from the axis as a *centre*, the wheels are called *spur wheels*; and a train of such

wheels and pinions, such as *fig.* 180, is called spur gearing. By such gearing the several axes round which motion is produced are parallel to one another, as appears by the figure.

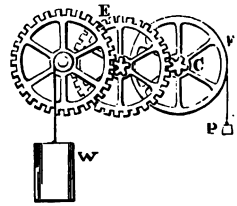

Fig. 180.

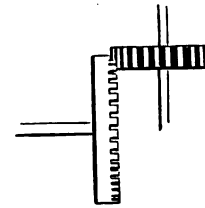

Fig. 181.

It is evident in this case that the wheel and pinion, which are upon the same axis, need not be in immediate juxtaposition, but may be separated one from the other by any length of the shaft. In this way, if the shaft be carried along the factory, a pinion at one end of a room may give motion to or receive it from a wheel upon the same shaft at the other end.

214. **Crown wheels.**—When the teeth are formed in the surface of a hoop or cylinder, so as to be directed parallel to the axis, as shown in *fig.* 181, the wheel is called, from its form, a *crown wheel.* If the teeth of such a wheel be engaged in those of a spur wheel, as in the figure, the axis of the latter must obviously be at right angles to that of the former.

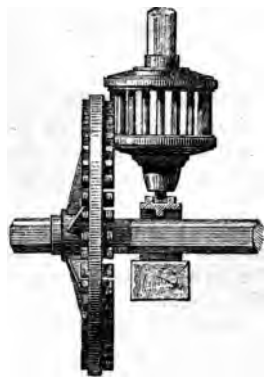

Fig. 182.

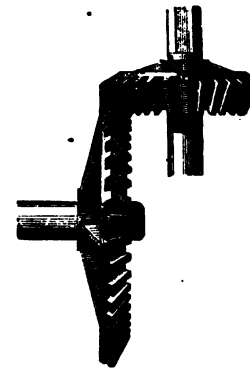

Fig. 183.

Sometimes the crown wheel works in a sort of cylindrical basket on a shaft at right angles to its axis, as shown in *fig.* 182.

215. **Bevelled wheels** are such as have their teeth inclined to the shaft, as shown in *fig.* 183. By this expedient motion may be imparted from one shaft to another inclined to it at any angle.

216. **Rack and pinion.**—When it is desired to impart a rectilinear motion of limited range by means of the rotation of an axle, the object is attained by the combination of a spur wheel or pinion with a straight bar having teeth of corresponding form and magnitude formed upon it. Such a toothed bar is called a *rack*. The mode of action of such a combination is shown in *fig.* 184.

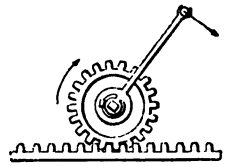

Fig. 184.

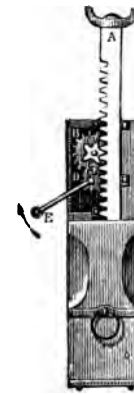

Fig. 185.

The efficacy of the power is generally increased by the combination of two or more wheels and pinions, as shown in *fig.* 185, where the handle drives the pinion D which works in the wheel B, on the axle of which is the pinion which drives the rack A.

217. **Rachet wheel.**—When it is desired to have an axle or shaft to revolve in one direction, but not in the other, the object is accomplished by fixing upon it a wheel having teeth, *n* (*fig.* 186), inclined in the direction contrary to that in which it is intended the shaft should be free to move. A catch, *o m*, falls between these teeth, and stops all motion of the wheel directed against it; but when the wheel turns in the other direction, the catch falls from tooth *to tooth, producing* a clicking noise.

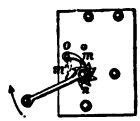

Fig. 186.

PULLEYS.

218. **Ropes neither perfectly flexible nor smooth.** — Although the term pulley implies the combination of a rope and a wheel on which it runs, the practical effects of the simple machine so denominated depend altogether on the rope ; the wheel being introduced for the mere purpose of diminishing the effects of friction and imperfect flexibility, the consideration of both of which are omitted in theory. If a rope were perfectly flexible, and were capable of being bent over a sharp edge, and of moving upon it without friction, we should be enabled by its means to make a force in any one direction overcome a resistance or communicate a motion in any other direction.

Thus, if a perfectly flexible rope, F S (*fig.* 187), pass over a sharp edge P, and be connected with a weight R vertically, a force acting obliquely in the direction P F will raise the weight vertically in the direction R Q ; but as no material of which ropes can be made can render them perfectly flexible, and as in proportion to the strength by which they are enabled to transmit force their rigidity increases, it is necessary in practice to adopt means to remedy or mitigate those effects which attend the absence of perfect flexibility, and which would otherwise render cords practically inapplicable as machines.

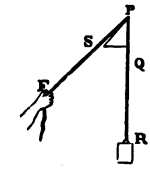

Fig. 187.

But, besides the want of perfect flexibility, the surface of the rope is always rough, and often considerably so. This surface in passing over an edge, would produce a degree of friction which would altogether stop its movement. If a rope were applied in some direction to a resistance acting in another, some force would be necessary to bend it over the angle P, which the two directions form one with the other ; and, if the angle were sharp the effect of such a force might be the rupture of the rope.

219. **Use of sheaves.**—But if, instead of bending the rope at one point over a single acute angle, the change of direction were produced by successively deflecting it over several angles, each of which would be less sharp, the force necessary for the deflection and the liability of breaking the cord would be diminished. But such object will be still more effectually attained if the cord be deflected over the surface of a curve.

If the rope were applied merely to sustain a weight without moving it, a curved surface would therefore be sufficient to remove the inconvenience arising from imperfect flexibility ; but

when motion is required, the rope, in passing over such a surface, would be subject to great friction and rapid wear. This inconvenience is removed by causing the surface on which the rope runs to move with it, so that no more friction is produced than would arise from the curved surface itself rolling upon the rope. These objects are attained by the common pulley, which consists of a wheel called a sheave, fixed in a running block turning on a pivot. A groove is formed in the edge of the wheel, in which the rope runs, the wheel revolving with it.

This apparatus is represented in *fig.* 188. Notwithstanding, however, that this expedient removes the friction and rigidity to so great a degree as to render the use of the cord practically available, it must not be supposed that these effects are altogether overcome; they still produce some impediment to the full efficiency of the power, as will be explained more fully hereafter. For the present, however, we shall consider the rope as rendered by this expedient flexible and free from friction.

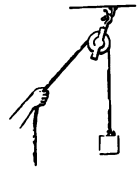

Fig. 188.

By means of a single sheave, a power acting in any one direction may be made to transmit its effects to a resistance acting at some distance in any other direction, provided the two lines of direction be in the same plane, and not parallel. Thus, let A B (*fig.* 189) be the direction in which the power acts, and let C D be the direction in which the weight or resistance acts.

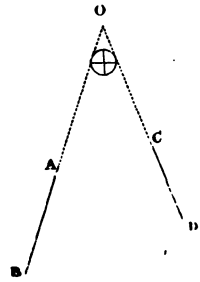

Fig. 189.

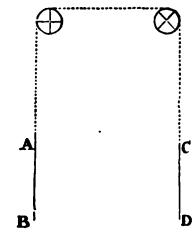

Fig. 190.

To transmit in this case the power to the weight, let the two directions B A and D C be prolonged until they meet, which they will do at O. In the angle O, formed by the two directions, let a sheave be placed, and let the power be connected with a rope *in the direction* B A. This rope, being carried from A over the

sheave at O, must be brought down in the direction C D, and connected with the resistance.

220. **Power and resistance parallel.**—But if the direction of the power and resistance be parallel, as in *fig.* 190, then the effect of the power might be transmitted to the weight by means of a cord and two fixed pulleys, one placed over the direction of the force, and the other over the direction of the weight, as represented in the figure.

In fine, if the direction of the power and the weight be not placed in the same plane, then the effect of the power may still be transmitted to the weight by means of two pulleys. Let us suppose, for example, that a power acting in a given horizontal line is required to be transmitted to a weight acting at some distance from it, in a certain vertical line, which is not in the same plane with the direction of the power.

Let two fixed pulleys be placed at two points in any convenient positions on the lines of direction of the power and weight, and let a line be supposed to join these points. Let the axis of one of the pulleys be placed at right angles to the plane formed by the line of direction of the power and the line joining the two pulleys, and let the other be placed with the axis at right angles to the plane passing through the line of direction of the weight and the line joining the two pulleys. By this arrangement, the cord being passed successively over both pulleys, the effect of the power will be transmitted to the weight.

221. **Single rope and fixed pulley.**—In all these cases, the same cord by which the weight is suspended being directly connected with the power, and its tension throughout its entire length being the same, the weight and the power must be equal when they are in equilibrium. This condition is also rendered manifest by the fact, that from the motion of the mechanism connecting it, the weight and power will move with the same velocity. It appears, therefore, that no mechanical advantage is gained by a single rope acting over one or more fixed pulleys; nevertheless, there is scarcely any engine, simple or complex, which is attended with more convenience.

In the application of power, whether of man or animals, or arising from other natural forces, there are always some directions in which it may be exerted to greater convenience and advantage than others, and in many cases the power is capable of acting in only one particular direction. Any expedient, therefore, which can give the most advantageous direction to the moving power, whatever be the direction of the resistance opposed to it, contributes as much practical convenience as one

which enables a small power to balance or overcome a great weight.

By means of the fixed pulley a man may raise himself to a considerable height, or descend to any proposed depth. If he be placed in a chair or a basket attached to one end of a rope which is carried over a fixed pulley, by laying hold of this rope on the other side, he may, at will, descend to a depth equal to half of the entire length of the rope, by continually yielding rope on the one side, and depressing the basket or chair by his weight on the other. Fire escapes have been constructed on this principle, the fixed pulley being attached to some part of the building.

222. **Single movable pulley.**—A single movable pulley is represented in *fig.* 191 ; a cord is carried from a fixed point F,

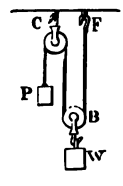

Fig. 191.

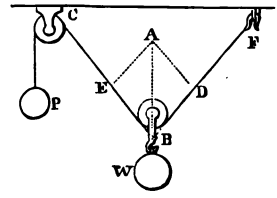

Fig. 192.

and passing through a block B attached to a weight W, passes over a fixed pulley C, the power being applied at P. We shall first suppose the parts of the cord on each side the wheel B to be parallel : in this case the whole weight W being sustained by the parts of the cords B C and B F, and these parts being equally stretched, each must sustain half the weight, which is therefore the tension of the cord. This tension is resisted by the power at P, which must therefore be equal to half the weight.

In this machine, therefore, the weight is twice the power.

If the parts of the cord B C and B F be not parallel, as in *fig.* 192, a greater power than half the weight will be necessary to sustain it. To determine the power necessary to support a given weight in this case, take the line B A in the vertical direction, consisting of as many inches as the weight consists of ounces ; from A draw A D parallel to B C, and A E parallel to B F : the force of the weight represented by A B will be equivalent to two forces represented by B D and B E. The number of inches in these lines respectively will represent the number of ounces which are equivalent to the tensions of the parts B F and B C of *the cord. But* as these tensions are equal, B D and B E must

be equal, and each will express the amount of the power P, which stretches the cord at P C.

It is evident that the four lines A E, E B, B D, and D A are equal, and as each of them represents the power, the weight which is represented by A B must be less than twice the power which is represented by A E and E B taken together. It follows, therefore, that as the parts of the rope which support the weight depart from parallelism, the machine becomes less and less efficacious; if the inclination of the strings is as much as 120°, the power is equal to the weight; if the angle becomes greater the equilibrating power would become greater than the weight.

223. **Combinations.**—If several sheaves be constructed in the same movable block, the mechanical advantage may be proportionally augmented. In *fig.* 193 a system is represented in which three sheaves are inserted in the movable block bearing the weight, the same number being inserted in the fixed block. The cord is carried from the power first over the fixed sheave A, then over the movable sheave B, then over the fixed sheave C, the movable sheave D, the fixed sheave E, and the movable sheave F, and finally attached to the block at G.

Fig. 193.

Now, since the cord throughout its whole length is stretched with the same force, and since it is evident that at the part where the power is applied, this force of tension must be equal to the power, it follows that the six parts of the cord which support the weight will each be stretched by a force equal to the power, and that consequently the weight, when in equilibrium, must be equal to six times the power.

This condition may also be inferred from the fact, that if the power move the cord, its velocity will be six times that of the weight; for if six feet of the rope be drawn over the fixed pulley, these six feet must be equally distributed between the six parts of the rope which sustain the weight, and consequently each part must be raised through one foot, which is therefore the height through which the weight would be raised for every six feet through which the power passes.

In general it may therefore be inferred, that, in a pulley which consists of a single movable block containing one or more sheaves, the weight, when in equilibrium, will be just as many times the power as is represented by the number of cords, or the number of parts of the cord, which sustain the weight.

In the form of movable block represented in *fig.* 194, the cord, after passing successively over the sheaves, is finally attached to the lower block. This, by increasing the parts of the cords supporting the lower block by one, augments the efficiency of the instrument without increasing the number of sheaves.

Two of the most powerful forms of pulley, consisting of a single movable block, are represented in *figs.* 195 and 196. The combination represented in *fig.* 195 is called Smeaton's pulley, having been invented by that celebrated engineer. The fixed and movable block contain each ten sheaves, and the order in which the rope is carried over them is represented in

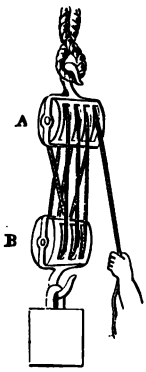

Fig. 194.

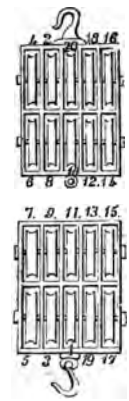

Fig. 195.

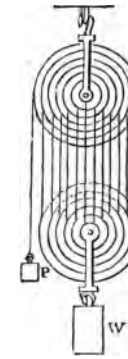

Fig. 196.

the figure by the numbers 1, 2, 3, 4, &c. The total number of parts of the cord supporting the lower block is in this case twenty, and consequently the power is to the weight as 1 to 20.

The form of pulley represented in *fig.* 196 is called White's pulley.

If, instead of one movable block and a single rope, two or more movable blocks with independent ropes be used, the power of the pulley may be augmented on the same principle as in the case of compound levers or compound wheel-work.

Different combinations of this kind are represented in *figs.* 197, 198, 199, 200.

The figures which are annexed to the ropes in each case *represent* the power they respectively exert.

The first rope in *fig.* 197 is stretched by the force of the power only. The first movable block being supported by two parts of this first rope will exert a force equal to double the power on the second rope; and the second movable block being supported by two parts of this rope will exert a force upon the third rope equal to four times the power; and in the same way it follows that the third block will exert a force in supporting the weight equal to eight times the power. In such a system, the addition of each movable block doubles the mechanical effect.

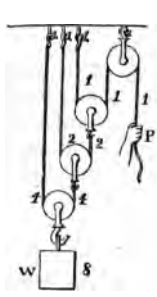

Fig. 197.

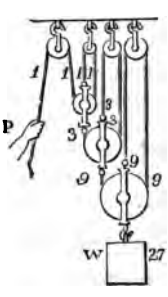

Fig. 198.

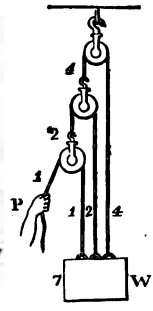

Fig. 199.

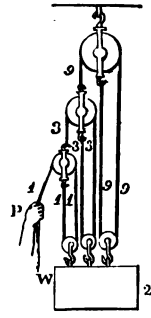

Fig. 200.

But without augmenting the number of movable blocks, by only adding fixed blocks the effect may be augmented in a three-fold instead of a two-fold proportion, as represented in *fig.* 198, where each successive movable block is supported by three parts of the same cord. In *fig.* 199 the ends of the cords, instead of being attached to fixed points, are attached to the weight. In this case, the weight is supported by each of the several cords, these cords being stretched by different forces. The first is stretched with a force equal to the power, the second with a force equal to double the power, and the third with a force equal to four times the power, and so on. In such a system, the mechanical effect for the same number of blocks is greater than in that represented in *fig.* 197, where three blocks only support a weight four times the power; whereas in the system represented in *fig.* 199, three blocks support seven times the power. The effect of this system may be still further increased by attaching blocks to the weight, as represented in *fig.* 200, and carrying the ropes to the pulleys above. The effect produced is indicated by the numbers fixed to the cords and to the weights.

224. **Practical effect very different from theoretical.**—From its portable form, cheapness of construction, and the facility with which it may be applied in almost every situation, the pulley is one of the most useful of the simple machines. The mechanical advantage, however, which it appears to possess in theory, is considerably diminished in practice, owing to the stiffness of the cordage and the friction of the wheels and blocks. By these means it is computed that in most cases so great a proportion as two-thirds of the power is lost. The pulley is much used in building when weights are to be elevated to great heights; but its most extensive application is found in the rigging of ships, where almost every motion is accomplished by its means.

In all these examples of pulleys, we have supposed the parts of the rope sustaining the weight, and each of the movable pulleys, to be parallel to each other. If they be subject to considerable inclinations, the relative tensions of the different ropes must be estimated according to the principle explained in the case of a single oblique cord.

INCLINED PLANE.—WEDGE AND SCREW.

225. **Effect of an inclined surface.**—A hard surface pressed against a weight or resistance in a direction at right angles to it would support it, and the whole amount of such weight or resistance would in this case press upon the surface. But if, instead of being at right angles to it, it were placed in an oblique direction, then the weight or resistance would be resolved in accordance with the parallelogram of forces into two, one of which would act perpendicularly to the plane and produce pressure upon it, and the other would be parallel to the plane, and be free to produce motion.

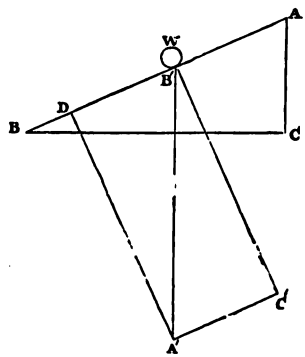

Fig. 201.

Let A B, *fig.* 201, be such a surface, and let W be a body producing some resistance, or having a tendency to move in the direction B′ A′ oblique to B A. Let the whole force with which W would move if not supported by the plane be expressed by B′ A′. *Then, taking* B′ A′ as the diagonal of a parallelogram one

of whose sides is B′ C′ at right angles to B A, and the other B′D in the direction of the plane, the force B′ A′ will be equivalent to two forces, one represented by B′ C′, and the other by B′ D. The former, being perpendicular to the plane, will be resisted by its reaction, and the other only will take effect. To support the weight, therefore, in this case, would require a force acting parallel to the plane, and opposite to the force represented by B′ D. If we take on the plane the length B A equal to B′ A′, and draw A C parallel to B′ A′, and B C perpendicular to it, then the triangle A B C will be in all respects equal and similar to A′ B′ C′; in fact, it may be considered as the same triangle, but in a different position. Since, therefore, A′ B′, B′ C′, and A′ C′ represent respectively the whole force acting on the body W, its pressure on the plane, and its tendency to move in the direction of the plane, these three forces will be exactly represented in their effects by the lines A B, B C, and A C.

We have here taken the general case, and supposed the body W to exercise a force in any direction whatever; but if we apply the principle to the case of a heavy body resting upon a plane inclined to the vertical direction, then the arrangement becomes what is commonly called the inclined plane. A B is called the *length* of the plane, A C its *height*, and B C its *base*.

From what has been just proved, then, it follows, that if a weight be placed upon an inclined plane, the weight consisting of as many pounds as there are inches in the length of the plane, the pressure on the plane will consist of as many pounds as there are inches in the base, and the tendency to move down the plane would be balanced by as many pounds as there are inches in the height.

The apparatus represented in *fig.* 202 is intended to prove this experimentally. The weight placed upon the plane is a roller so formed as to move freely upon it. A string is attached to it, which being carried parallel to the plane is conducted over a fixed pulley, and supports a dish bearing a weight. On comparing this weight, including the weight of the dish, with the weight of the roller upon the plane, and by varying the angle of elevation of the plane, we find that in every case the weight necessary to produce equilibrium will be expressed by the height of the plane, the entire weight of the roller being expressed by its length.

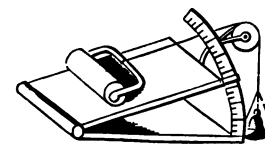

Fig. 202.

It is evident from what has been just explained that the less

the elevation of the plane is, the less will be the power requisite to sustain a given weight upon it, and the greater will be the pressure upon it; for the less the elevation of the plane is, the less will be its height and the greater will be its base.

226. **Inclined roads.**—Roads which are not level may be considered as inclined planes, and loads drawn upon them in carriages, regarded in reference to the powers which impel them, are subject to all the conditions which have been established for inclined planes.

The inclination of the road is estimated by the height corresponding to some proposed length: thus we say, a road rises one foot in twenty-five, or one foot in thirty; meaning that if twenty-five or thirty feet of the road be taken as the length of an inclined plane, the corresponding height of such plane would be one foot; and if twenty-five or thirty feet be measured upon the road, the difference of the level of the two extremities will be one foot. According to this method of estimating the inclination of roads, the power required to sustain a load upon them, friction apart, is always proportional to this rate of elevation. If a road rise one foot in twenty, then a power of one ton will be sufficient to sustain twenty tons, and so on.

227. **Inclined planes on railways.**—When a power is employed in moving a load upon a road thus inclined, the action of the power may also be regarded in another point of view. Let us suppose a railway train weighing 200 tons moving up an inclined plane, which rises at the rate of one in two hundred; what mechanical effect does the moving power produce in moving up 200 feet of such a plane? First, it acts against the friction, the atmospheric and other resistances to which it would be exposed if the plane had been level. Secondly, it is employed in raising the entire weight of the train through the elevation which corresponds to 200 feet in length, that is, through one perpendicular foot.

The mechanical effect, therefore, is precisely the same as if the load of 200 tons had been first moved along a level plane 200 feet long, and then elevated up a step one foot high; but instead of being called upon to make this great exertion of raising 200 tons directly through one perpendicular foot, the moving power is enabled gradually to accomplish the same object by a longer continuance of a more feeble exertion of force, such exertion being spread over 200 feet instead of being condensed into a single foot.

In all that precedes, we have assumed that the power acts [illegible]*allel to the* plane; in some cases, however, it acts obliquely

Let W P, *fig.* 203, be the direction of the power. Taking that as the diagonal of a parallelogram, it will be equivalent to W D and W E, the former perpendicular, and the latter parallel to the plane. W D will have the effect of diminishing the pressure on the plane, and W E will be efficient in drawing the weight up the plane. In some cases the direction of the power is below the plane, as in *fig.* 204. In this case, as before, the

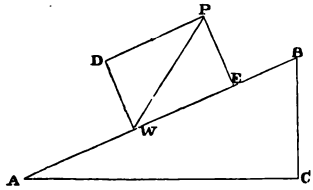

Fig. 203.

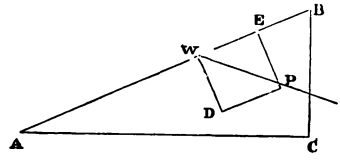

Fig. 204.

power W P is resolved into two forces, W E parallel to the plane, and W D perpendicular to it. The latter augments the pressure of the weight on the plane, and the former is efficient in drawing it up the plane.

The method of drawing up or letting down barrels from or into a cellar, represented in *fig.* 205, and that of drawing them

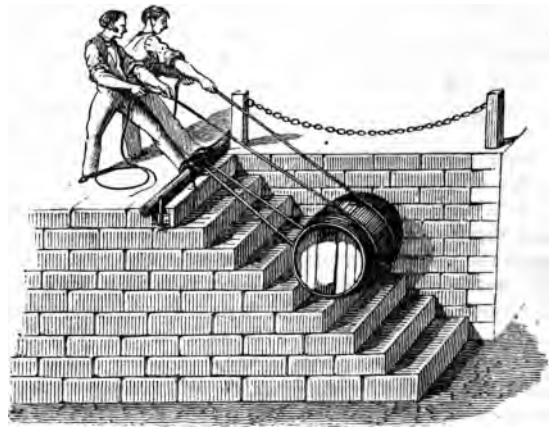

Fig. 205.

upon a dray, shown in *fig.* 206, are cases of the inclined plane which will be easily understood.

228. **Double inclined plane.**—It sometimes happens that a weight upon one inclined plane is raised or supported by another weight upon another inclined plane. Thus, if A B and A B′, *fig.* 207, be two inclined planes, forming an angle at A, and W W′

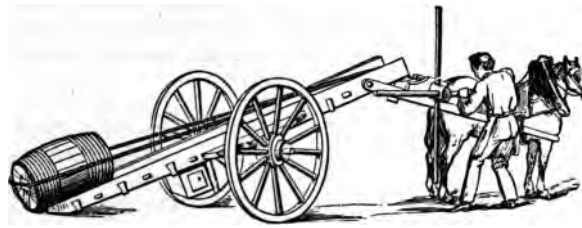

Fig. 206.

be two weights placed upon these planes, and connected by a cord passing over a pulley at A, the one weight will either sustain the other, or one will descend, drawing the other up. To determine the circumstances under which these effects will ensue, draw the lines W D and W′ D′ in the vertical direction, and take upon them as many inches as there are ounces in the weights respectively. W D and W′ D′ being the lengths thus taken, and therefore representing the weights, the lines W E and W′ E′ will represent the effects of these weights respectively down the planes. If W E and W′ E′ be equal, the weights will sustain each other without motion; but if W E be greater than W′ E′, the weight W will descend, drawing the weight W′ up; and if W′ E′ be greater than W E, the weight W′ will descend, drawing the weight W up. In every case W F and W′ F′ will represent the pressures upon the planes respectively.

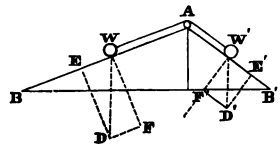

Fig. 207.

229. **Self-acting planes.**—It is not necessary, for the effect just described, that the inclined planes should, as represented in the figure, form an angle with each other. They may be parallel, or in any other position, the rope being carried over a sufficient number of wheels, placed so as to give it the necessary deflection. This method of moving loads is frequently applied in great public works where railroads are used. Loaded waggons descend one inclined plane, while other waggons, either empty or loaded, so as to permit the descent of those with which *they are connected*, are drawn up the other.

230. **The wedge.**—When the weight is not moved upon the plane, but is stationary, the plane being itself moved under the weight, the machine is called a wedge.

Let D E, *fig.* 208, be a heavy beam, secured in a vertical position between guides, F G and H I, so that it is free to move upwards and downwards, but not laterally. Let A B C be an in-

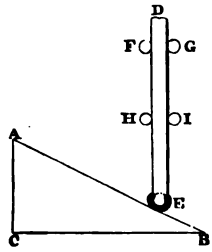

Fig. 208.

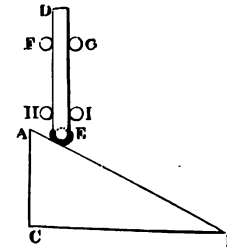

Fig. 209.

clined plane, the extremity of which is placed beneath the end of the beam. A force applied to the back of this plane A C, in the direction C B, will urge the plane under the beam so as to raise the beam to the position represented in *fig.* 209. Thus, while the inclined plane is moved through the distance C B, the beam is raised through the height C A.

It follows, therefore, that in the case of the wedge the velocity of the resistance is to the velocity of the weight as the base of the inclined plane, which forms the wedge, is to its height.

Wedges, however, are more generally formed of two inclined planes, connected base to base, as represented in *fig.* 210. In this case, the back of the wedge is the sum of the heights of the two inclined planes, and the length of the wedge is their common base. The force, therefore, which drives the wedge is to the resistance with which it equilibrates, as half the back of the wedge is to its length.

Fig. 210.

This theory of the wedge is not applicable in practice with any degree of accuracy. This is owing chiefly to the enormous disproportion which friction in these machines bears to the power; but independently of this there is another difficulty in the theory of this machine.

The power commonly used in the case of a wedge is not pressure, but percussion. The force of a blow is of a nature so different from continued force, such as the pressure of weights, that it admits of no numerical comparison with the resistance offered by cohesion, to overcome which it is generally applied. We cannot exactly state the proportion which a blow bears to a weight. The wedge is almost invariably urged by percussion, while the resistances which it has to overcome are as constantly forces of another kind. Although, however, no exact numerical computation can be made, yet it may be stated in general that the wedge is more and more powerful as the angle is more acute.

The cases in which wedges are most generally used in the arts and manufactures, are those in which an intense force is required to be exerted through a very small space. This instrument is therefore used for splitting masses of timber or stone, for raising vessels in docks, when they are about to be launched, by being driven under their keels, in presses where the juice of seeds, fruits, or other substances are required to be extracted, as, for example, in the oil mill, in which the seeds from which the oil is extracted are introduced into hair bags, which being placed between planes of hard wood are pressed by wedges. The pressure exerted by the wedges is so intense that the dry seeds are converted into solid masses as hard and compact as the most dense woods. Wedges have been used occasionally to restore to the perpendicular, edifices which have inclined owing to the sinking of their foundations.

231. **Practical examples.**—All cutting and piercing instruments, such as knives, razors, shears, scissors, chisels, nails, pins, needles, &c., are wedges. The angle of the wedge in all these cases is more or less acute, according to the purpose to which it is applied. Chisels intended to cut wood have their edge at an angle of about 30°; for cutting iron from 50° to 60°, and for brass about 80° to 90°. In general, tools which are urged by pressure admit of being sharper than those which are driven by percussion. The softer or more yielding the substance to be divided is, the more acute the wedge may be constructed.

232. **Utility of friction.**—In many cases the efficiency of the wedge depends on that which is entirely omitted in its theory, viz., the friction which arises between its surface and the substance which it divides. This is the case when pins, bolts, or nails are used for binding the parts of structures *together, in which* case, were it not for the friction, they would

recoil from their places and fail to produce the desired effect. Even when the wedge is used as a mechanical engine the presence of friction is absolutely indispensable to its practical utility.

The power, as has already been stated, generally acts by successive blows, and is therefore subject to constant intermission, and but for the friction the wedge would recoil between the intervals of the blows with as much force as it had been driven forward. Thus the object of the labour would be continually frustrated. The friction in this case is of the same use as a ratchet-wheel, but is much more necessary, as the power applied to the wedge is much more liable to intermission than in the cases where ratchet-wheels are generally used.

233. **The screw.**—In ascending a steep hill it has been the practice of road engineers, instead of making an inclined plane directly from the base to the summit, to carry the road round the hill, gradually rising as it proceeds. If we desire to ascend with ease to the top of a high column, we could do so if a path or ledge were formed on the outer surface, gradually winding round and round the column from the bottom to the top. Such a path would be, in fact, an inclined plane carried round the column. But it will be evident that such an arrangement would constitute *a screw*. This will be rendered still more apparent by the following contrivance :—

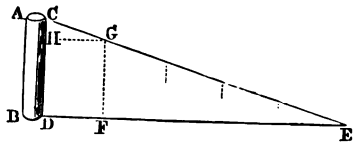

Fig. 211.

Fig. 212.

Let A B, *fig.* 211, be a cylindrical roller, and let C D E be an inclined plane cut in paper, the height of which C D is equal to the length of the roller. Let the edge C D be pasted on the roller, and then let the roller be turned so that the paper shall be wrapped round it. When it makes one revolution of the roller, the portion of the edge C G will have made one spiral coil ; the next revolution will make an equal spiral coil, and so on until all the paper has been rolled upon the roller, when the edge of the paper so coiled will show a regular spiral line round the roller, as represented in *fig.* 212. Taking C H G, *fig.* 211, as the portion of the inclined plane thus rolled in every turn round the roller, it is evident that C H is its height, and H G its

base. But C H is the distance between two successive coils of the spiral, and H G is the circumference of the roller. The coils of the spiral are called the *threads* of the screw, and the distance C H between the successive coils is called the *distance between the threads.*

In the application of the screw, the weight or resistance is not, as in the inclined plane and wedge, placed upon the surface of the plane or thread. The power is usually transmitted by causing the screw to move in a concave cylinder, on the interior surface of which a spiral cavity is cut, corresponding exactly to the thread of the screw, and in which the thread will move by turning round the screw continually in the same direction. This hollow cylinder is usually called the *nut* or concave screw. The screw surrounded by its spiral thread is represented in *fig.* 213.

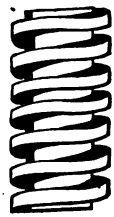

Fig. 213.

There are several ways in which the power is transmitted to the resistance by means of a screw; but by whatever means it may be so transmitted, it is evident that the screw will move the resistance in a single revolution through a space equal to the distance between two contiguous threads. The comparative velocities, therefore, of the power and weight will always be found in this class of simple machines by comparing the space described by the power, in imparting one revolution to the screw, with the distance between two contiguous threads.

The most common manner of urging the screw is by a lever attached to its head, as represented in *fig.* 214, at E F. Supposing the power to be applied at F, then in producing one revolution of the screw, and therefore in moving the resistance through a space equal to the distance between two contiguous threads, it will make one complete revolution in a circle whose radius is the length of the lever on which it acts. The velocity, therefore, of the power will be to the velocity of the weight as the circumference of the circle described by the power is to the distance between two contiguous threads; and consequently, the condition of equilibrium between the power and weight will be this, that the power is to

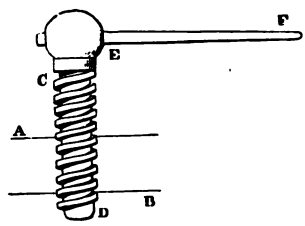

Fig. 214.

the weight as the distance between the contiguous threads is to the circumference described by the power.

The great mechanical force exerted by the screw will hence be easily understood. There is no limit to the smallness of the distance between the threads, except the strength which is necessary to be given to them, and there is no limit to the magnitude of the circumference to be described by the power, except the necessary facility of moving in it. We can conceive the power acting by a lever, and therefore moving through a great circumference while the screw moves through a comparatively minute space ; and consequently, in such case, the power will be so much less than the resistance, as the distance between the threads is less than the circumference described by the power.

The manner of acting upon the resistance by means of the screw is very various. Sometimes the nut is fixed and the screw movable ; sometimes the screw is fixed and the nut movable ; sometimes the nut, though incapable of revolving, can be moved progressively ; and sometimes the screw is incapable of revolving, but is moved progressively. These conditions admit of various combinations which are severally adopted in practice. In *fig.* 214, the nut A B being supposed to be fixed, if the lever F be turned, the end D of the screw will descend or ascend, according to the direction in which E F is turned, and will act upon the resistance accordingly. If the screw be fixed, the nut may be moved upon it, either by turning the nut or the screw. In either case the nut will ascend or descend, according to the direction of the motion. In each revolution it will move through a space equal to the distance between two contiguous threads.

If we suppose the nut A B, *fig.* 214, to be incapable of ascending or descending, but to be capable of revolving, then, by turning it round, the screw which plays in it will ascend or descend through a space equal to the distance between two contiguous threads for every revolution made by the nut.

On the other hand, the apparatus may be so arranged that the screw, though capable of revolving, is incapable of a progressive motion, and the nut, though capable of a progressive motion, is incapable of revolving. In this case, when the screw is made to revolve, the nut in which it plays will be moved upwards or downwards, through a space equal to the distance between two threads, by the revolution of the screw.

The screw is generally used in cases where severe pressure is

to be exercised through small spaces; it is, therefore, the agent in most presses.

In *fig.* 215 the nut is fixed, and by turning the lever which passes through the head of the screw a pressure is exercised upon any substance placed upon the plate immediately under the end of the screw. In *fig.* 216 the screw is incapable of revolving, but is capable of advancing in the direction of its

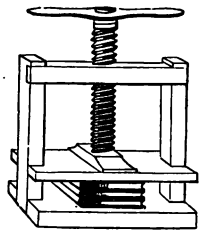

Fig. 215.

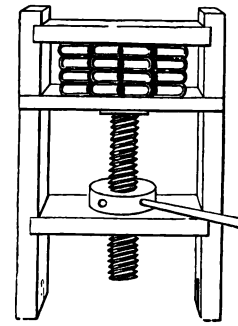

Fig. 216.

length. On the other hand, the nut is capable of revolving, but does not advance in the direction of the screw. When the nut is turned by means of the lever inserted in it, the screw advances in the direction of its length, and urges the board which is attached to it upwards, so as to press any substance placed between it and the fixed board above.

234. **Examples.**—In cases where liquids or juices are to be expressed from solid bodies, the screw is the agent generally employed. It is also used in coining, where the impression of a die is to be made upon a piece of metal, and in the same way in producing the impression of a seal upon wax or other substance adapted to receive it. When soft and light materials, such as cotton, are to be reduced to a convenient bulk for transportation, the screw is used to compress them, and they are thus reduced into hard dense masses. In printing, the paper is sometimes urged by a severe and sudden pressure upon the types by means of a screw.

A screw may be cut upon a cylinder by placing the cylinder in a turning-lathe, and giving it a rotatory motion upon its axis. The cutting point is then presented to the cylinder and moved in the direction of its length at such a rate as to be carried through the distance between the intended threads while the

cylinder revolves once. The relative motions of the cutting point and the cylinder being preserved with perfect uniformity, the thread will be cut from one end to the other. The shape of the threads may be either square, as in *fig.* 213, or triangular, as in *fig.* 218.

If the lever by which the power acts on the screw were capable of indefinite increase, or the thread of indefinite fineness, there would be no limit to the mechanical effect of the instrument; but to both of them there are practical limits. The lever cannot be increased so as to render the operation of the power unwieldy and impracticable, and the thread cannot be diminished beyond that limit which will give it sufficient strength; and the cases in which the greatest mechanical efficacy is needed, are precisely those in which the thread requires to be strongest.

235. **Hunter's screw.**—To obtain an indefinite augmentation of the power of the screw, without diminishing the strength of the thread, Mr. Hunter proposed an arrangement which is known by his name, as Hunter's screw. This (represented in *fig.* 217) consists in the use of two screws, the threads of which may have any strength and magnitude, but which have a very small difference of breadth.

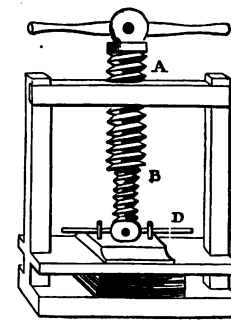

Fig. 217.

While the working point is urged forward by that which has the greater thread, it is drawn back by that which has the less; so that during each revolution, the screw, instead of being advanced through a space equal to the magnitude of either of the threads, moves through a space equal to their difference. The mechanical power of such a machine will be the same as that of a single screw having a thread whose magnitude is equal to the difference of the magnitudes of the two threads just mentioned.

Thus, without inconveniently increasing the sweep of the power on the one hand, or on the other diminishing the thread until the necessary strength is lost, the machine will acquire an efficacy limited by nothing but the smallness of the difference between the two threads.

236. **Micrometer screw.**—The slow motion which may be imparted to the end of a fine screw by a considerable motion of

the power, renders it an instrument peculiarly well adapted to the measurement of very minute motions and spaces, the magnitude of which could scarcely be ascertained by any other means. To explain the manner in which it is applied: suppose a screw to be so cut as to have fifty threads in an inch, each revolution of the screw will advance its point through the fiftieth part of an inch. Now, suppose the head of the screw to be a circle whose diameter is an inch, the circumference of the head will be something more than three inches: this may be easily divided into a hundred equal parts, distinctly visible. If a fixed index be presented to this graduated circumference, the hundredth part of a revolution of the screw may be observed by noting the passage of one division of the head under the index. Since one entire revolution of the head moves the point through the fiftieth of an inch, one division will correspond to the five-thousandth of an inch. In order to observe the motion of the point of the screw in this case, a fine wire is attached to it, which is carried across the field of view of a powerful microscope, by which the motion is so magnified as to be distinctly perceptible.

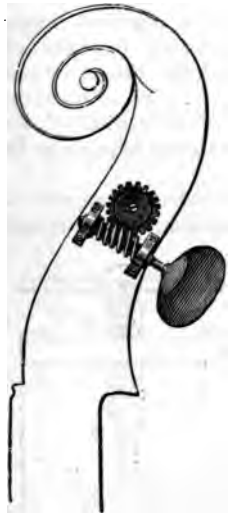

Fig. 218.

237. **Endless screw.**—When the thread of a screw acts in the teeth of a wheel, the screw being fixed so that it can revolve without advancing, it is called an endless screw. It has the effect of giving motion to the wheel in the same manner as does a pinion, one tooth of the wheel being driven forward by each revolution of the screw. Such an arrangement is applied, for example, to tighten the strings of double basses, *fig.* 218.

CHAPTER XV.

THE PENDULUM, THE BALANCE WHEEL, AND THE FLY WHEEL.

Of the class of mechanical contrivances whose purpose is to render motion uniform, the principal are the pendulum, the balance wheel, and the fly.

238. **Simple pendulum.**—The pendulum consists of a heavy mass attached to a rod, the upper extremity of which rests upon a point of support in such a manner as to have as little friction as possible. Such an instrument will remain at rest when its centre of gravity is in the vertical line immediately under the point of suspension or support. But if the centre of gravity be drawn from this position on either side, and then disengaged, the instrument will swing from the one side to the other of the position in which it would remain at rest, the centre of gravity describing alternately a circular arc on the one side or the other of its position of rest. If there were neither friction nor atmospheric resistance, this motion of vibration or oscillation on either side of the position of equilibrium would continue for ever; but, in consequence of the combined effects of these resistances, the distances to which the pendulum swings on the one side and on the other are continually diminished, until, after the lapse of an interval more or less protracted, it comes to rest.

Suppose a small ball of lead suspended by a fine silken string, the length of which is incomparably greater than the diameter of the leaden ball. Such an arrangement is called the *simple pendulum*.

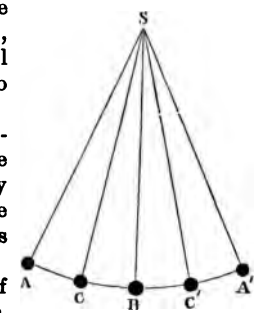

Fig. 219.

Let S, *fig.* 219, be the point of suspension; let S B be the fine silken thread by which the ball B is suspended, and the weight of which, in the present case, is neglected. Let B be the position of the ball when in the vertical under the point of suspension S. In that position the ball would remain at rest; but if we suppose the ball drawn aside to the position A, it will, if disengaged, fall down the arc A B,

of which the centre is S, and the radius the length of the string. Arriving at B, it will have acquired a certain velocity, which, in virtue of its inertia, it will have a tendency to retain, and with this velocity it will commence to move through the arc B A′. Supposing neither the resistance of the atmosphere nor friction to act, the ball will rise through an arc B A′ equal to B A; but it will lose the velocity which it had acquired at B, for it is evident that it will take the same space and the same time to destroy the velocity which has been acquired, as to produce it. Thus, the velocity at B, being acquired in falling through the arc A B, will be destroyed in rising through the equal arc B A′.

Having arrived at A′, the ball, being brought to rest, will again fall from A′ to B, and at B will have again acquired the same velocity which it had obtained in falling from A to B, but in the contrary direction; and in the same manner it may be explained that this velocity will carry it from B to A. Having arrived at A, the ball, being again brought to rest, will fall once more from A to B, and so the motion will be continued alternately between A and A′.

The motion of the pendulum from A to A′, or from A′ to A, is called an *oscillation*, and its motion between either of those points and B is called a semi-oscillation, the motion from B to A or from B to A′ being called the ascending semi-oscillation, and the motion from A or A′ to B, the descending semi-oscillation.

The time which elapses during the motion of the ball between A and A′ is called the *time of one oscillation*.

It is evident, from what has been stated, that the time of moving from either of the extremities A A′ of the arc of oscillation to the point B, is half the time of an oscillation. If, instead of falling from the point A, the ball had fallen from the point C, intermediate between A and B, it would have then oscillated between C and C′, two points equally distant from B; and the arc of oscillation would have been C C′, more limited than A A′. But in commencing its motion from C, the declivity of the arc down which it falls towards B would be evidently less than the declivity at A; consequently, the force which would accelerate it, commencing its motion at C, would be less than that which would accelerate it, commencing its motion at A. The ball, therefore, commencing its motion at A, would be more rapidly accelerated than when it commences its motion at C.

The result of this is, that, although the arc A B may be twice *as long as the* arc C B, the *time* which the ball takes to fall from

A to B will not be sensibly different from the time it takes to fall from C to B, provided that the arc of oscillation A B A′ is not considerable.

It was at first supposed, as we have just stated, that, whether the oscillations were longer or shorter, the times would be absolutely the same. Accurately speaking, however, this is not the case; but if the total extent of the oscillation A A′ do not exceed 5° or 6°, the time of oscillation in it may be considered, practically, the same as in the lesser arcs.

This important principle may be easily experimentally verified. Let two small leaden balls be suspended from the same point of support, but one being in advance of the other, so that in oscillating the two balls shall not strike each other. This being done, let one of the balls be drawn from its point of rest through an angle less than 3°, and let it be disengaged. It will oscillate as described above. Let the other ball be now drawn from its point of rest through a much less angle, and let it be so disengaged that it shall commence its oscillation at the same moment with the commencement of one of the oscillations of the other ball.

Let it, in short, be so managed, that when the one ball is at A, the other shall be at C; and that both shall commence their descending motion towards B at the same moment. It will be then found that their oscillations will be synchronous for a considerable length of time, that is to say, the balls will arrive at A′ and C′, respectively, at the same instant; and returning, will simultaneously arrive at A and C respectively.

If, in this case, the oscillation of the ball A were made through an arc, even as great as 10°, that is to say, 5° on each side of the vertical, the oscillation of the ball C being made through an arc of 2°, it would be found that 10001 oscillations of the latter would be equal to 10000 oscillations of the former, so that the actual difference between their times of oscillation would not exceed the ten thousandth part of such time.

It might be expected that the time of oscillation of different pendulums would depend, more or less, upon the weight of the matter composing them, and that a heavy body would oscillate more rapidly than a lighter one. Both theory and experience, however, prove the result to be otherwise. The force of gravity which causes the pendulum to oscillate acts separately on all the particles composing its mass; and if the mass be doubled, the effect of this force upon it is also doubled; and, in short, in whatever proportion the mass of the pendulum be increased or diminished, the action of the force of gravity upon it will be

increased or diminished in exactly the same proportion, and consequently the velocity imparted by gravity to the pendulous mass at each instant will be the same.

It is easy to verify this by experiment. Let different balls of small magnitude, of metal, ivory, and other materials, be suspended by light silken strings of the same length, and made to oscillate: their oscillations will be found to be equal.

If pendulums of different lengths have similar arcs of oscillation, the times of oscillation of those which are shorter will be less than the times of oscillation of those which are longer. Let a, b, c, d, and e (*fig.* 220) be five small leaden balls, suspended by light silken strings to the point of suspension S, and

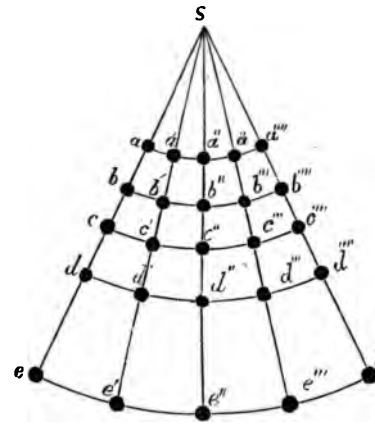

Fig. 220.

let all of those be supposed to form pendulums, having the same angle of oscillation. The arc of oscillation of the ball a will be $a\ a''''$, and that of b will be $b\ b''''$, that of c, $c\ c''''$, and so on. In commencing to fall from the points a, b, c, d, e, towards the vertical line, these five balls are accelerated equally, inasmuch as the circular arcs down which they fall are all equally inclined at this point to the vertical line. The same will be true if we take them at any corresponding points, such as a', b', c', e'. It may, therefore, be concluded that throughout the entire range of oscillation of each of these five pendulums, they will be impelled by equal accelerating forces.

Now it has been shown that when bodies are impelled by the same or equal accelerating forces, the spaces through which

they move are proportional to the squares of the times of their motion; therefore it follows that the lengths of these arcs of oscillation are proportional to the squares of the times. But the lengths of these arcs are evidently in the same proportion as the lengths of the pendulums; that is to say, the arc $a\ a''''$ is to $b\ b''''$ as s a is to s b, and the arc $b\ b''''$ is to $c\ c''''$ as s b is to s c, and so on.

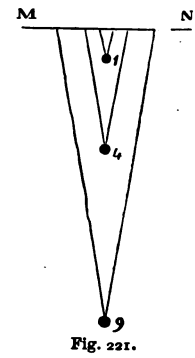

Fig. 221.

It follows, therefore, that the squares of the times of oscillation of pendulums are as their lengths, or, what is the same, the times of oscillation are as the square roots of their lengths. This principle is easily verified experimentally.

239. **Experimental illustration.**—Let three small leaden balls be suspended vertically under each other by means of loops of silken threads, as represented in *fig.* 221, and in such a manner that they can all oscillate in the same plane at right angles to the plane of the diagram, the suspending loops not interfering with each other.

Let the loops be so adjusted that the distance of the ball 1 below the line M N shall be 1 foot, the distance of the ball 4, 4 feet, and the distance of the ball 9, 9 feet.

Let the ball 9 be put in a state of oscillation through small arcs, and let the ball 4 be then drawn from its vertical position, and disengaged so as to commence one of its oscillations with an oscillation of the ball 9; and in the same manner let the ball 1 be started simultaneously with one of the oscillations of the ball 9.

It will be found that two oscillations of the one-foot pendulum are made in exactly the same time as a single oscillation of the four-foot pendulum; consequently, the time of each oscillation of the latter will be double that of the former, while its length is fourfold that of the former.

In the same manner, while the one-foot pendulum makes three oscillations, the nine-foot pendulum will make one, and, consequently, the time of oscillation of the latter will be three times that of the former, while its length is nine times that of the former.

By this principle, the length of a pendulum which would oscillate in any proposed time, or the time of oscillation of a pendulum of any proposed length, can be ascertained, provided we know the length of a pendulum which oscillates in any given

time. Thus, suppose L to be the length of a pendulum which oscillates in the time T. Let it be required to determine the length of a pendulum L′, which would oscillate in any other time T′. We shall have the following proportion :

$$L : L' :: T^2 : T'^2.$$

From this proportion, if L and T be both given, we can find the time T′ of oscillation of the other pendulum if L′ be given ; or we can find the length L′ if T′ be given.

In the first case we have

$$T'^2 = T^2 \times \frac{L'}{L};$$

in the second we have

$$L' = L \times \frac{T'^2}{T^2}.$$

Since the force which produces the oscillation of a pendulum is the accelerating force of gravity urging the pendulous body alternately from the extremities of the arc of oscillation to the middle point of that arc, it is evident that if this force were increased in its intensity, the velocity with which the pendulous body would be precipitated to its lowest position would be increased, and consequently the time of oscillation diminished ; and if, on the other hand, the impelling force of gravity were diminished, the force urging the pendulous body being enfeebled, it would be moved with a diminished velocity, and consequently the time of oscillation would be increased. It follows, therefore, that the same pendulum will oscillate more slowly or more rapidly, according as the force of gravity which acts upon it is diminished or increased.

But it is not enough to state that a variation in the force of gravity will change the time of oscillation of the pendulum. It is required to ascertain in what proportion it will produce this change ; that is to say, if the force of gravity acting on the pendulum be augmented in any given ratio, in what corresponding ratio will the time of oscillation of such pendulum be diminished. It is proved in the theory of accelerating forces, that, under such circumstances, the squares of the times of oscillation will vary in the inverse proportion of the force ; that is to say, in whatever ratio the force of gravity be augmented, the squares of the times of oscillation of the pendulums will be diminished.

240. **Pendulum indicates variation of gravity in different latitudes.**—But as the squares of the times of oscillation

are proportional to the lengths of the pendulums, it follows from this that the lengths of the pendulums which oscillate in the same time under the influence of different accelerating forces will be proportional to these forces; and that, consequently, if in any two places it be found that the pendulums which oscillate in the same time have different lengths, it must be inferred that the forces of gravity in these two places are in the exact proportion of these lengths.

It is in virtue of this principle that the pendulum supplies means of determining the variation of the forces of gravity upon different parts of the earth's surface.

241. **Compound pendulum.**—We have hitherto supposed that the pendulous body is a heavy mass of indefinitely small magnitude, suspended by a wire or string having no weight. These are conditions which cannot be fulfilled in practice. Every real pendulous body has a definite magnitude, its component parts being at different distances from the point of suspension; the rod which sustains it has considerable weight, and all the points of this rod, as well as those of the pendulous mass itself, are at different distances from the point of suspension. In estimating, therefore, the effect of pendulums, it is necessary to take into account this circumstance.

Let us suppose a, b, c, d, e, f, g (*fig.* 222) to be as many small heavy balls connected by independent strings, the weight of which may be neglected, with a point of suspension S, and let these seven balls be supposed to vibrate between the positions S M and S M′. Now, if these balls were totally independent of each other, and connected with the point of suspension by independent strings, they would all vibrate in different times, those which are nearer the point S vibrating more rapidly than those which are more distant from it. If, therefore, they be all disengaged at the same moment from the line S M, those which are nearest to S will get the start of those which are more distant, and at any intermediate position between the extremes of their vibration they will assume the positions a', b', c', d', e', f', g'. That which is nearest to the point S, and which is the shortest pendulum, will be foremost, since it has the most rapid vibration. The next in length, b, will follow it, and so on; the most remote from S, being the longest pendulum, will be the last in order.

Now if, instead of supposing these seven balls to be suspended by independent strings, we imagine them to be fixed upon the same wire, so as to be rendered incapable of having any independent motion, and compelled to keep in the same

straight line, then it is evident that while the whole series vibrates with a common motion, those which are nearest to the point of suspension will have a tendency to accelerate the motion of those which are more distant, while those which are more distant will have a tendency to retard the motion of those which are nearer.

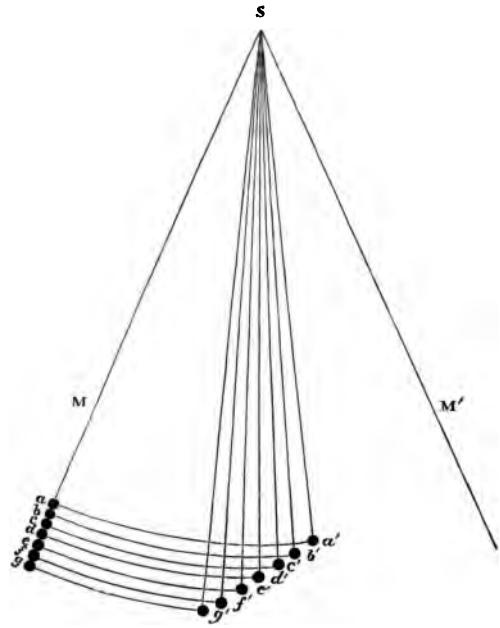

Fig. 222.

These effects will produce a mutual compensation; *b* and *c* will vibrate slower than they would if they were moving freely, while *e* and *f* will evidently move more rapidly than if they were moving freely. Among the series there will be found a certain point which will separate those which are moving slower than their natural rate from those which are moving faster than their natural rate; and a ball placed at this point would vibrate exactly as it would do if no other balls were placed either above or below it. Such a ball would, as it were, be the centre which would divide those which are accelerated from those which are retarded.

242. **Centre of oscillation.**—Such a point has been denominated the *centre of oscillation*. It is evident that a pendulous mass, of magnitude more or less considerable, will vibrate in the same time as it would do if the entire mass were concentrated at its centre of oscillation, and formed there a material point of insensible magnitude. By the length of a pendulum, no matter what be its form, is always to be understood the distance of its centre of oscillation from its point of suspension.

The centre of oscillation has the following remarkable and important quality, which is established in higher mathematics, and verified by experiment:—If a pendulum be inverted and suspended by its centre of oscillation, its former point of suspension will become its new centre of oscillation, and the time of vibration will remain the same as before. This property is usually expressed by stating that the 'centres of suspension and oscillation are convertible.'

This property can be verified by experiment. If the centre of oscillation of any pendulous body, ascertained by mathematical calculation, be taken as a point of suspension, it will be found that the time of oscillation of the pendulum will be the same as it was with the first point of suspension. Since the length of a pendulum and, therefore, the time of oscillation in a given place and subject to a given intensity of the force of gravity, depends upon the distance between its point of suspension and its centre of oscillation, it is evident that any variation which may take place in this distance will cause a corresponding change in the time of oscillation; and if the pendulum be applied to an instrument for measuring time, it will cause a variation in the rate of that instrument, and a corresponding error in its indications.

Now, it is found in practice that all pendulums are subject to a change, more or less, in their form and magnitude, in consequence of the change of temperature of the atmosphere to which they are exposed. With this change they expand and contract, and with every expansion and contraction the distance between their centres of oscillation and suspension will be varied, unless expedients be adopted to counteract such an effect. Although the variations produced by these causes are not sufficiently great to render it necessary to provide a correction for them in common timepieces, yet, in cases where extreme accuracy is required, expedients have been adopted to prevent the consequent error.

These expedients are called compensation pendulums.

The principle upon which all these depend is the combination of two substances in the structure of the pendulum, which expand in unequal degrees for the same change of temperature; and they are so arranged that while the expansion of the one increases the distance of the centre of oscillation from the point of suspension, the expansion of the other has a contrary effect, and the dimensions of these two substances are so adjusted that the increase of distance produced by the one shall be exactly equal to the diminution of distance produced by the other; so that the result is that the centre of oscillation remains at the same distance from the point of suspension, and therefore the time of oscillation of the pendulum remains unaltered.

The first, the most important, and the most universal use of the pendulum, is as a measure of time. The uniformity of the rate of its vibration is the property which renders it so eminently qualified for this purpose. A pendulum vibrating alone, independently of any mechanism, would measure the time which elapses during its motion. It would be only necessary for an observer to sit by it and count the number of its oscillations. If the time of one oscillation were previously known, then the number of oscillations performed in any interval would at once give the length of such interval. But, in order to supersede the necessity for attention and vigilance on the part of such an observer, a train of wheel-work is placed in connection with the pendulum, the movement of which it regulates; and in connection with this train of wheel-work are fixed the dial-plate and the hands of the clock, by which the number of oscillations of the pendulum which take place in a day, or in any part of a day, are indicated and registered.

When the same pendulum is transported to different parts of the earth's surface, it is found that the rate of its vibration varies, and this variation is proved to take place even after precautions have been taken to keep the centres of oscillation and suspension at the same distance from each other. Now this change in the rate of vibration under such circumstances can only be explained by a change in the intensity of the force of gravity by which the pendulum is moved. It is found that when the pendulum is carried towards the terrestrial equator, the time of its vibrations is longer; and that when it is carried towards the pole, the time of its vibrations is shorter: the inference deduced from which is, that the force of gravity diminishes as we approach the equator, and increases as we approach the pole.

If the earth had the form of an exact sphere, did not revolve

on its axis, and was of uniform density, the force of gravity of all parts of its surface would be the same, and no such variation in the rate of a pendulum could take place when transported from one point of the surface of the earth to another. But if the earth be an exact sphere, revolving upon its polar axis in 23 h. 56 min., then the effect of such motion of rotation would be to produce a certain small diminution of the intensity of the force of gravity in approaching the equator, and an increase in such intensity in approaching the pole. The amount of such diminution or increase produced by such rotation is capable of calculation, and, being computed and compared with such change of intensity of the force of gravity indicated by the variations of a pendulum, is found not to correspond exactly with it. This absence of complete correspondence indicates another cause affecting the force of gravity besides the rotation of the earth.

If the earth be not an exact sphere, but have a form of which a turnip and an orange are exaggerated representations, called in geometry an oblate spheroid, such a form, combined with the rotation of the earth, would produce a further effect in varying the force of gravity in proceeding towards the equator or towards the pole. Now it is found, by calculation, that a certain degree of this form, combined with the diurnal rotation of the earth, would produce exactly that variation in the force of gravity going towards the equator and going towards the pole which is indicated by the variation in the time of vibration of the same pendulum.

Hence it appears that the pendulum becomes an instrument by which not only the doctrine of the diurnal rotation of the earth is verified and corroborated, but by which the departure of the figure of the earth from an exact globular form is also established. From what has been stated, it will appear that the length of a pendulum which vibrates seconds in different parts of the earth will be different; the force of gravity in lower latitudes being less than in higher, the length of the pendulum which vibrates seconds will be proportionally less.

243. **Pendulum measures the velocity of falling bodies.** —As the pendulum thus supplies a measure of the intensity of the force of gravity, it necessarily also affords the means of calculating the height from which a body falling freely would descend in a second if it moved in vacuo. The method of determining this by the pendulum is susceptible of much greater accuracy than that which has been already indicated by Atwood's machine. To find the space through which a body will fall at any

place in a second of time, let the length of a pendulum which vibrates seconds in that place, expressed in inches, be multiplied by 4·9348, and the product will express in inches the height through which a body would fall in a second in that place, independently of the resistance of the air.

244. **Property of a swing.**—It is well known to all persons who have amused themselves with exercising in this species of pendulum, that by humouring the motion by certain changes of attitude, the oscillation to and fro, which, if they remain stationary, would be gradually diminished and would ultimately cease, by reason of the resistance of the air and friction, can be not only sustained, but increased in its range. This is explicable by the well established principles which govern the motion of pendulous bodies.

Let A (*fig.* 223) be a small weight suspended upon a fine thread from the centre B, and let it be supposed to fall from the position A to the vertical line B C; when it arrives at C, it will have exactly the same velocity in the horizontal direction as a body would have after having fallen vertically from F to C. Now let us suppose that at the moment of arriving at C, the body A is suddenly raised to some point such as D, nearer to B, retaining nevertheless the horizontal velocity it had acquired. With this velocity it will begin to move up the arc D N, and it will continue to rise in that arc, until it arrives at a point H, whose height D K, above D, is equal to C F.

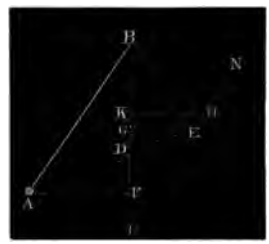

Fig. 223.

Now it is evident that, in this case, the angle H B D, through which it will swing from the vertical B D, will be much greater than the angle A B C. This will be still more evident if it be considered that if B E be drawn, making the angle E B D equal to A B C, the height D G will be less than C E, and therefore less than D K in the same proportion as D B is less than C B; and since D G is less than D K, the angle D B E, and therefore the angle A B C, is less than the angle D B H.

It follows, therefore, that if by any expedient the centre of gravity of a pendulous body be lowered in each descending semi-oscillation, and raised in each ascending one, the range of the oscillation will be constantly augmented; and this, as will

be apparent, is precisely what the swinger does when, by humouring his attitude with the changes of oscillation, he in-

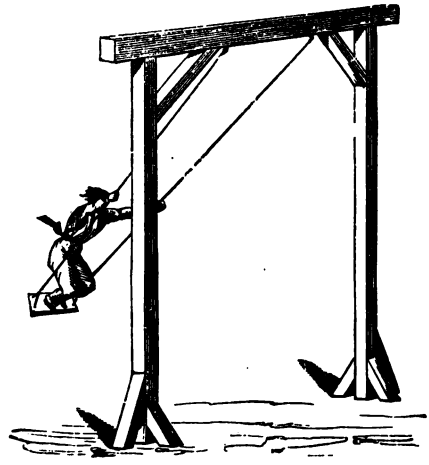

Fig. 224.

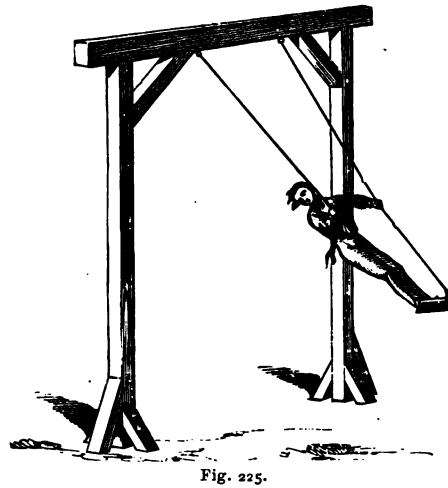

Fig. 225.

creases the range of the swing. If he stand upon the swing board he lowers his body, as shown in fig. 224, in each descend-

ing motion of the swing, and raises it, as in *fig.* 225, in each ascending motion; in the one case he lowers the centre of gravity, and in the other he raises it.

If he sit upon the swing board, he lies back with his body at right angles to the cord and his legs downwards in descending, and with his body upwards and his legs at right angles to the cord in ascending. By these changes of attitude, the centre of gravity, as before, is lower in descending than in ascending.

In both cases the same effect is produced as we have described above, in supposing the body A (*fig.* 223) to be lower upon the string while it descends than while it ascends.

245. **The actual length of a pendulum which vibrates seconds** is found by accurate experiments to be 39·139 inches at London, 39·021 inches at the Equator, and 39·215 at Spitzbergen, lat. 79° 49′ 58″ N.

246. **The balance wheel** applied to regulate watchwork is a light steel wheel A B C (*fig.* 226) nicely balanced upon its axis. Under this wheel a highly elastic spiral spring, made of very fine steel wire, is placed, the inner end of which is attached to the centre of the wheel, and the outer end *e* is attached to the fixed plate of the watch under the wheel. The wheel naturally assumes a certain position of equilibrium, from which if it be disturbed in either direction, by turning it so as to coil or uncoil the spring, it will, when disengaged, return to its position of equilibrium; but at the moment of arriving at that position it will have acquired such a velocity as will cause it to go on in virtue of the inertia of the wheel, and it will thus swing to the other side of the position of equilibrium. In this manner the wheel will swing alternately on the one side and the other of its position of equilibrium, with a motion altogether analogous to the motion of a pendulum; and this analogy is rendered still more close by the fact that the spiral spring has a property which renders the oscillations of the wheel, whatever be their range, great or small, isochronous; that is to say, they will be performed in exactly equal times. It is therefore by this property that the balance wheel enjoys the same regulating power as the pendulum.

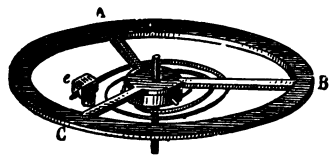

Fig. 226.

247. **The fly** is a regulator, consisting of a wheel having four arms, upon each of which a thin oblong vane of metal is

so fixed that it can be turned with its plane at any desired inclination to the axis ; the wheel is accurately balanced so as to be in equilibrium upon its axis, whatever be the position of the vane. When a motion of revolution is imparted to such a wheel, the four vanes acting against the atmosphere will be resisted by it with a force which will be greater as the velocity of revolution is augmented, and it is demonstrable that the rate of increase of this resistance is proportional to the square of the

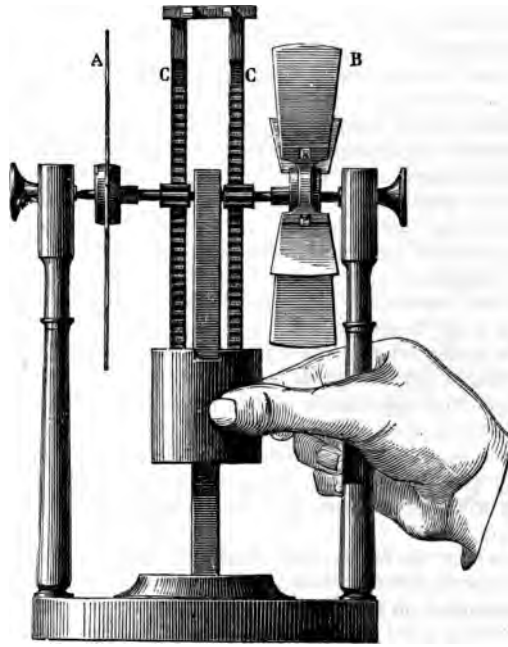

Fig. 227.

velocity of the wheel :—thus, with a double velocity, there will be a fourfold resistance ; with a triple velocity, a ninefold resistance, and so on. An apparatus for illustrating experimentally the effect of such a wheel is represented in *fig.* 227, where two such wheels, A and B, are mounted on horizontal axles so as to be capable of turning with very little friction. Upon each of these axles is fixed a small pinion, engaged in the teeth of a rack C ; to the lower ends of these racks is attached a heavy

weight. When this weight is pushed up to the pinions, the lowest teeth of the racks will be engaged in those of the latter, and if the weight be then let go, it will fall, drawing the racks with it, and imparting revolution to the two pinions, so that when the weight shall have dropped upon the bottom of the stand, the two pinions, and consequently the two flies, will have received precisely the same velocity of rotation; but the racks having terminated just before the arrival of the weight upon the stage, the pinions are no longer engaged in their teeth, and will therefore be free to revolve, as well as the flies, with the velocity acquired. As represented in the figure, the vanes of the fly B are presented with their surfaces at right angles to the direction of the motion, while the vanes of A are presented with their edges to the motion; it is evident, therefore, that one must encounter more resistance than the other from the air, and this is proved by the fact that while B is very soon brought to rest, A will continue to move for a considerable time. The experiment may be varied by inclining the vanes at various angles, when it will be found that the nearer their position is to that represented at B, the sooner the wheel will be brought to rest by the resistance; and the nearer it is to the position shown in A, the longer will be the continuance of the motion. The velocity imparted to the wheels may also be varied by varying the weight attached to the racks.

The practical application of the fly as a regulator depends upon the two properties here explained: first, that with a given position of the vanes, the resistance of the air will increase indefinitely with the velocity; and, secondly, that this resistance will vary with the inclination of the vanes to the direction of the motion.

Let us suppose, for example, that a fly be applied to equalise the accelerating force produced by a descending weight; the fly is then mounted on an axis which receives motion, in common with the other parts of the mechanism to which it is applied, from the weight. At the commencement, the motion will be accelerated because the weight will be greater than the resistance, and consequently a continually increasing velocity will be imparted to the fly; according to what has been explained, the fly will therefore encounter a resistance which, increasing as the square of the velocity, will soon become equal to the force of the descending weight, and then all acceleration must cease, and the motion must become absolutely uniform, since any, even the least, increase of velocity would produce a resistance greater than the moving power; the uniform velocity thus at-

tained will continue so long as the moving power continues to act with the same force upon the mechanism.

If it be desired to increase the uniform velocity which a given moving force will thus impart, it is only necessary to turn the vanes of the fly so as to present them more obliquely to the direction of the motion.

The fly possesses a property as a regulator which renders it available in many cases in which neither the pendulum nor the balance wheel would be applicable: these two regulators, as we shall more fully explain hereafter, produce, not an uniform, but an intermitting motion, as will be readily comprehended by observing the motion of the seconds-hand of a time-piece, whether it be regulated by a pendulum or a balance wheel; such a hand, as everyone knows, moves not continuously, but by starts; and the same is true of the minute and hour hands, only that in these the intermissions are made through spaces so minute that they are not observable. Now, in many cases, such an intermitting motion is altogether inadmissible, as, for example, in the apparatus for illustrating the motion of falling bodies represented in *fig.* 55, where the fly placed at the top regulates the rotation of the cylinder, which must be continuous and uniform.

Musical boxes, self-acting pianofortes, barrel organs, played by mechanism, are all examples where the regulator must be a fly, and where neither pendulum nor balance-wheel would be admissible.

CHAPTER XVI.

REGULATION AND MODIFICATION OF FORCE AND MOTION.

248. **Regularity of motion.**—Regularity and uniformity are two of the conditions most universally indispensable in the operation of machinery; sudden changes of speed are always a source of loss of power, often injurious and sometimes destructive to the apparatus, and never fail to produce imperfection in the articles fabricated.

Much attention, therefore, has been directed to, and much mechanical ingenuity expended on, contrivances for insuring these conditions of regularity and uniformity in the movement

of machinery, by removing those causes of inequality which can be avoided, and by compensating those which cannot.

Irregularity in the motion of machinery will result in any one of the following causes:—

1°. When a varying power is opposed to a uniform resistance.

2°. When a uniform power is opposed to a varying resistance.

3°. When the power and resistance both vary, but not proportionally to each other.

4°. When the power is not transmitted with uniform effect to the working point in the successive positions assumed by the machine.

249. **Varying power opposed to uniform resistance.**—The force of the prime mover is seldom regular. The force of water varies with the copiousness of the stream; the force of wind is proverbially capricious; the power of steam varies with the intensity of combustion in the furnace; and the force of animal power, depending on the temper and health, is difficult of control, human labour being the most unmanageable of all. No machine works so irregularly as one that is depending on manipulation.

In some cases the prime mover is subject, by the very conditions of its existence, to constant variation; as, for example, where it is a main-spring, which gradually loses its energy as it is relaxed. In some cases, the prime mover is liable to intermission, and is totally suspended during certain intervals. An example of this is presented in the single-acting steam-engine, where the force of the steam acts only during the descent of the piston, but is suspended during its ascent.

250. **Uniform power opposed to varying resistance.**—In almost all the applications of machinery the load or resistance is subject to continual fluctuation. In mills, a multiplicity of parts are liable to be occasionally and irregularly disengaged, and to have their operations suspended. In large factories for spinning, printing, dyeing, &c., a great number of separate spinning-machines, looms, presses, and other engines, are usually driven by a common power, such as a water-wheel or a steam-engine. In such cases, the number of machines worked at the same time must necessarily vary according to the employment supplied to the factory, and to the fluctuating demand for the articles produced. Under such circumstances, the velocity with which the machinery is moved would suffer corresponding changes, increasing with each diminution, and being retarded *with each increase* of the resistance.

In many cases, the variation of the power and the variation of the resistance are both from their conditions inevitable, and yet a uniform effect is indispensable. It is evident that this can only be insured by a class of contrivances which have for their object to proportion the power to the resistance, by either causing a diminution or increase of resistance to diminish or augment the supply of power; or, on the other hand, by causing the variation of the power to act in a corresponding manner upon the resistance or load. In a word, uniformity of action in machinery can only be insured by providing means by which the power and the resistance, no matter what be their respective variations, shall always be proportional to each other.

Whenever the power is less than that which is in equilibrium with the resistance, the motion will be retarded, and if this condition continue, it will ultimately stop; and whenever the power is greater than that determined by the condition of equilibrium, the motion will be accelerated; and if this condition should continue, the acceleration would continue until the machine would be destroyed by its own momentum.

There is scarcely any machine in which the energy of the power is transmitted uniformly to the resistance in all the phases of the mechanism. In all machines the moving parts assume in succession a variety of positions, in each of which their effect in transmitting the power to the resistance is different; and thus the effective energy of the machine in acting against the resistance is subject to continual fluctuation. It is not easy to convey, without numerous examples, a general idea of those causes of inequality to those who are not familiar with machinery. It will, however, be more clearly understood when we come to explain the methods of equalising the action of the power and the resistance.

251. **Regulators.**—The class of contrivances which have for their object to render the power and resistance proportionate to each other are called *regulators*. They generally act upon that point of the machine which commands the supply of the power by means of some mechanical contrivances, which check the quantity of the moving principle conveyed to the machine whenever the motion becomes accelerated, and increase the supply whenever it becomes retarded. In a water-wheel, for example, this is accomplished by acting upon the shuttle, in a windmill by the adjustment of the sails, and in a steam-engine by acting on a valve called the throttle-valve placed in the main pipe, through which steam flows from the boiler to the cylinder.

252. **The governor.**—One of the most interesting and in-

structive examples of this class of contrivances is called the *governor*. This expedient, which was long used to regulate mill work and other machinery, owes its beautiful adaptation to the steam-engine to the ingenuity of Watt. It consists of two

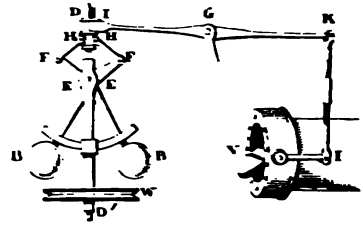

Fig. 228.

heavy balls, B B, *fig.* 228, attached to the extremity of rods B F jointed at E, and passing through a mortice in the vertical stem D D'.

When the balls B are driven from the axis by the central force arising from their rotation, their upper arms E F are caused to increase their divergence in the same manner as the blades of scissors are opened by separating the handles. These acting upon the ring H, by means of the short rods F H, draw it down the vertical axis from D towards E. A contrary effect is produced when the balls B are brought closer to the axis, and the divergence of the rods B E diminished. A horizontal wheel W is attached to the vertical axis D D', having a groove to receive a rope or strap upon its rim. This strap passes round the wheel or axis by which motion is transmitted to the machinery to be regulated, so that the spindle or shaft D D' will be always made to revolve with a speed proportionate to that of the machinery.

As the shaft D D' revolves, the balls B are carried round it with a circular motion, and consequently acquire a central force, which causes them to recede from the axle, and therefore to depress the ring H. On the edge or rim of this ring is formed a groove, which is embraced by the prongs of a fork I at the extremity of one arm or a lever whose fulcrum is at G. The extremity K of the other arm is connected by some means with the part of the machine which supplies the power. In the present instance we shall suppose it a steam-engine, in which case the rod K I communicates with a flat circular valve V placed in the principal steam-pipe, and so arranged that when K is elevated as far as by their divergence the balls B have power over

it, the passage of the pipe will be closed by the valve v, and the passage of steam entirely stopped; and, on the other hand, when the balls subside to their lowest position, the valve will be presented with its edge in the direction of the tube, so as to interrupt no part of the steam.

The property which renders this instrument so well adapted to its purpose is, that there is but one velocity at which the balls can remain in equilibrium.

253. **Fusee.**—The effect of a power of variable energy may be rendered uniform by transmitting it to the working point through the agency of a leverage of corresponding variation so regulated that, in the same proportion as the power diminishes in energy, the leverage shall increase, and *vice versâ*. A well-known example of this occurs in the construction of certain watches, where the moving power, being a main-spring inclosed in a barrel, has a gradually diminished energy as the spring is relaxed. The chain as it is discharged from the barrel is coiled

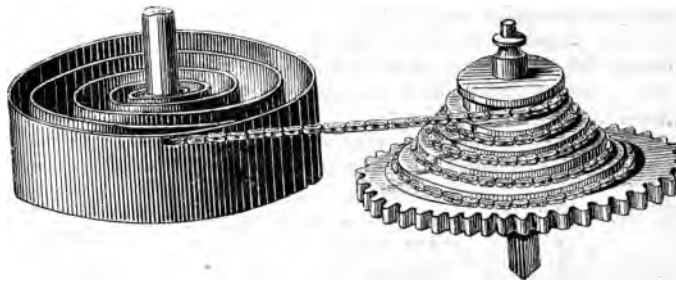

Fig. 229.

upon a conical spiral, called a fusee, represented in *fig.* 229. The leverage by which the force of the spring is transmitted being the semi-diameter of the fusee; and the motion commencing from the top, or narrowest end, it follows that when the energy of the spring is greatest the leverage is least; and as the chain coils upon the barrel containing the spring, and is discharged from the fusee, the radius of each part of the fusee which discharges the chain gradually increases. The form of the fusee is such that this increase of leverage is in the exact proportion of the diminished force of the spring.

The several parts of a machine have certain periods of motion, in which they pass through a variety of positions, returning constantly to similar positions after equal intervals.

In the different positions assumed by the moving parts during these periods, the effect of the power transmitted to the working point is different; and cases even occur in which for a moment this effect is altogether interrupted, and the machinery is then in a predicament in which the power loses all effect upon the resistance. The consequence of this would be, that, supposing the power and resistance to be both uniform, the action of the former upon the latter would be subject to periodical variation, being at one time more and at another time less than what would be necessary to keep the whole in equilibrium.

Under these circumstances, it is possible to suppose that the movement of the machine may continue, and even that its average rate may be uniform; but its motion would be subject to periodical variations, being alternately accelerated and retarded. This would be attended not only with an injurious effect upon the work produced by the machine, but would be also detrimental to the machine itself, whose moving parts would be subject to continual starts and strains arising from the alternate reception and destruction of momentum.

254. **Crank.**—To render these general observations more clearly intelligible, we shall take, as an example, the action of a common crank, used in steam-engines and many other machines.

A crank is nothing more than a double winch. It is represented complete with both its arms in *fig.* 230. Attached to the middle of C D, by a joint, is a rod, which is the means of imparting the effect of the power to the crank. This rod is driven by an alternate motion like the brake of a pump. The bar C D is carried with a circular motion round the axis A F.

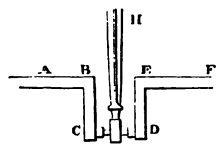

Fig. 230.

Let the machine viewed in the direction A B E F of the axis be conceived to be represented in *fig.* 231, where A represents the centre round which the motion is to be produced, and G the point where the connecting rod G H is attached to the arm of the crank. The circle through which G is to be urged by the rod is represented by the dotted line. In the position represented in *fig.* 231, the rod acting in the direction H G has its full power to turn the crank G A round the centre A. As the crank comes into the position represented in *fig.* 232 this power is diminished; and when the point G comes immediately below A, as in *fig.* 233, the force in the direction H G has no effect in turning the crank round A, but,

on the contrary, is entirely expended in pulling the crank in the direction G A, and therefore only acts on the pivots or gudgeons which support the axle.

At this crisis of the motion, therefore, the whole effective energy of the power is annihilated.

After the crank has passed to the position represented in *fig.* 234, the direction of the force which acts upon the connecting rod is changed, and now the crank is drawn upward in the direction G H. In this position the moving force has some efficacy to produce rotation round A, which efficacy continually increases

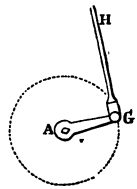

Fig. 231.

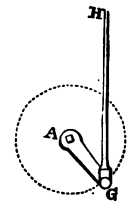

Fig. 232.

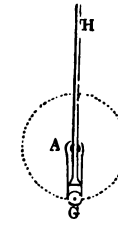

Fig. 233.

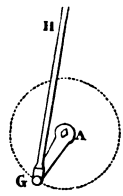

Fig. 234.

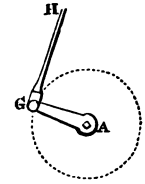

Fig. 235.

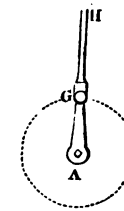

Fig. 236.

until the crank attains the position shown in *fig.* 235, when its power is greatest. Passing from this position, its efficacy is continually diminished until the point G comes immediately above the axis A (*fig.* 236). Here again the power loses all its efficacy to turn the axle A force in the direction G H or H G can obviously produce no other effect than a strain upon the pivots or gudgeons.

It will be evident from this that the action of the power transmitted to the working point G is very variable. At the dead points represented in *figs.* 233 and 236, the machine, if depending solely upon the moving power, must come to rest, for at both points the whole effect of the power would be exerted in producing pressure on the axle and gudgeons of the crank.

Through a small space at either side of these dead points, the effect transmitted to G, though not absolutely nothing, is almost evanescent, so that it may be considered that through a small arc at either side of each of the dead points the machine is quite inert.

It must, however, be considered that, in virtue of its inertia, the motion which the machinery had previously to its arrival at its dead points has a tendency to continue; and if the resistance of the load and the effects of friction be not too great, this disposition to preserve its state of motion will extricate the machinery from the mechanical dilemma in which it is involved

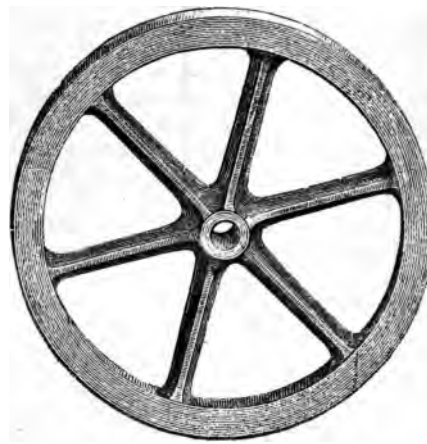

Fig. 237.

in these cases by the particular disposition of its parts. Although, however, the motion will not therefore be actually suspended, on the arrival of the crank at the dead points, it will be greatly retarded; and, on the other hand, when the power acquires its greatest activity, as it does in the position represented in *figs.* 231 and 235, it will be unduly accelerated.

255. **Fly wheel.**—These irregularities are equalised by fixing upon the axis of the crank, or at any other convenient part of the machine, a fly-wheel, which is a massive ring of metal (*fig.* 237), connected with a central box or nave by comparatively light spokes, and turning on an axis with but little friction. If any force be applied to it, with that force, making some slight deduction for friction, it will move and will continue to move until

some obstacle retard it, which obstacle will receive from it as much force as the fly-wheel loses.

The effect of such a wheel applied to the parts moved by the crank will equalise the inequality which has just been described. When the crank assumes the position represented in *figs.* 231 and 235, where the power has full play upon it, the effect of the power is partly transmitted to the machine, and partly received by the movable rim of the fly-wheel, to which it imparts increased momentum. There is here, it is true, an acceleration of the motion, but one which is comparatively small, inasmuch as the great mass of the fly-wheel receives the momentum without sensible increase of speed. When the crank gets into the predicament represented at the dead points (*figs.* 233 and 236), the momentum of the fly wheel, received when the crank acted with the most advantage, immediately conveys its force to the working-point G, extricates the machine, and carrying the crank out of the neighbourhood of the dead point, brings the power again to bear upon it.

It happens frequently in the practical application of machinery, that the moving power is much too intense, or much too feeble, for the resistance; and in the one case contrivances are required by which it may be greatly attenuated, and in the other by which it may be greatly augmented.

256. **Main-spring.**—In the case of watchwork, the resistance to be overcome is nothing more than that presented by the hands which move upon the dial-plate. In this case the moving power is the force of the fingers, by which once in twenty-four hours the main-spring is wound up. The main-spring itself must be regarded, in this case, as a mere depository for the power exerted in winding up the watch, and not as a prime mover. The force which is thus deposited once in twenty-four hours in the main-spring is delivered gradually and regularly, by such spring, to the fusee, and transmitted, through the system of wheels, to the hands.

In the case of clocks moved by main-springs instead of a weight, this attenuation of the moving power is still more extensive. These, being wound up, will frequently go for fifteen days and even much longer. In this case, therefore, the mechanical force exerted by the hand in winding them up, and which is developed in less than a minute, is spread over at least fifteen times twenty-four hours by the mechanism of the clock.

257. **Clockwork moved by a weight.** — In the case of clocks which move by a descending weight, the original moving force is also the application of the human hand in winding up

the weight. The weight, being lifted a height of three or four feet, descends slowly through the same height, imparting its descending force gradually and regularly to the clockwork. In this case, therefore, the descending force of a weight through a small height is so attenuated as to impart a motion to the hands which will continue sometimes for a month or longer.

It is frequently required, on the contrary, to impart to the resistance a force vastly greater in intensity than the moving power. Numerous examples of this have been already given in illustrating the simple machines. In all cases where the leverage of the power is greater than the leverage of the resistance, there will be an augmented intensity of mechanical action in the same proportion; and this intensity, by combining levers or other simple machines, may be augmented without any practical limit.

258. **Hammers, sledges, &c.**—But in some cases a force is required more intense than can be obtained even by these means. In such cases it becomes necessary to convert the continued agency of the moving power into one which acts instantly and by intermission. If, for example, it be required to cause a nail to penetrate a beam of wood, we should attempt in vain to accomplish this by producing any pressure, however great, on the head of the nail. A few blows of a hammer, nevertheless, easily effect this. In this case, the moving power is the hand, or other force which raises the hammer. The mass of the hammer, in falling on the head of the nail, instantly imparts to the nail the entire force which was exerted in lifting it, but with this difference, that such force, in raising the hammer, was developed in a certain definite time, whereas it is discharged upon the head of the nail in an instant.

The same observations apply to all cases in which percussion is used. In all these cases the force is developed in a definite time, but is discharged upon the resistance in an instant.

259. **Inertia supplies means of accumulating force.**—In some cases where a severe instantaneous action is required, the moving power is accumulated by means of the inertia of matter. A mass of matter retains, by virtue of its inertia, the whole amount of any force which may be given to it, except that part of which friction and the atmospheric resistance deprives it. To render this method of accumulating force intelligible, let us first imagine a polished level plane, on which a heavy globe of metal, also polished, is placed. It is evident that the globe will remain at rest on any part of the plane without a tendency to move. Suppose, then, a slight impulse be given to it, which will cause

it to move with any given velocity— for example, three feet per second. It will then continue to move with this velocity, for any length of time, except so far as it may be impeded by the resistance already mentioned.

Let us then imagine a second impulse given to it equal in force to the former : this will increase its velocity to six feet per second ; a third impulse will augment it to nine feet per second, and so on. Now, there is no limit to the number of impulses which may be successively given to the moving body, provided only space were given for its motion. Thus, ten thousand repetitions of the impulse would make the body move at the rate of 30,000 feet per second. If the body to which these impulses were transmitted were a cannon-ball, it might, by the constant application of a feebly impelling force, be made to move at length with as much force as if it were impelled from a powerful piece of ordnance. The force with which such a ball would strike a buttress might be sufficient to reduce it to ruins ; and yet such force may be nothing more than the accumulation of a number of feeble forces, not beyond the power of a child to exert, which are stored up and preserved in the moving mass, and then brought to bear at the same moment on the resistance against which the force is directed. It is the same for any number of actions exerted successively and during a long interval, brought into operation at one and the same moment.

But the case here supposed cannot actually occur, because we have not in general practical means of moving a body for a considerable time in the same direction, without much friction, and without encountering other obstacles which would impede its progress. If, however, a leaden ball be attached to the end of a string and whirled rapidly round, a great force would be given to it, and it will strike a board with such intensity as to produce a very destructive effect.

A weapon called a life-preserver consists of a piece of lead sometimes attached to the end of a piece of cane or whalebone, with which a blow may be given with great force. Innumerable examples of the application of this principle will present themselves to every mind. Flails used in threshing, clubs, canes, whips, and all instruments used for striking, axes, hatchets, cleavers, and all instruments which act by a blow, present examples of this principle.

Where very intense force is required, as for example, in certain presses, two heavy balls are attached to the ends of a horizontal lever A B, with equal arms, *fig.* 238. This lever works

a screw, at the lower end of which is the working point. A rapid motion is imparted to the balls by the hand, and the working point is driven against the resistance by the accumulated momentum acquired by the balls, augmented by the leverage of the arms to which they are attached, and the mechanical force of the screw.

The surprising effects produced by the accumulation of force are apt to lead to erroneous suppositions, that instruments thus acting by inertia have the effect of actually augmenting the amount of moving power. When the quality of inertia, however, is rightly understood, such an error cannot occur. The instruments by which force is thus accumulated, so far from augmenting the effect of the moving power, must to some extent diminish it; inasmuch as they are liable to friction and atmospheric resistance, by which more or less force is intercepted. An accumulator of force, whatever be its form, can never have more force than has been applied to put it in motion. Whether it be a falling weight, a revolving mass, a spring which is coiled up, or air which is condensed, it cannot develop a greater amount of force than that which is imparted in raising it if it be a weight, in putting it in motion if it be a moving mass, in winding it up if it be a spring, or in compressing it if it be air. The only difference between the power which is imparted to these agents, and the effects which they produce respectively upon the resistance, is to be found in the time during which the effects are developed. The power is in general imparted slowly, while the effects are produced almost instantaneously.

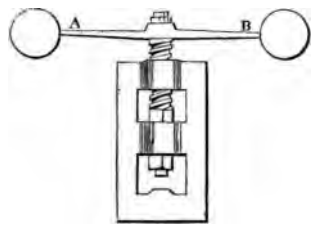

Fig. 238.

Mills for rolling metals, or for punching boiler-plates, supply striking examples of this. The water-wheel, or steam-engine, or whatever other power be used, is allowed for some time to act upon the fly-wheel alone, no load being placed upon the machine. When a sufficient momentum has been imparted to the mass of metal forming the fly-wheel, the metal to be rolled or pierced is submitted to the machine, and is immediately flattened or perforated by it, depriving at the same time the fly-wheel of a corresponding quantity of its momentum.

In the same manner, a force may be obtained by the arms of men acting on a fly for a few seconds, sufficient to produce an

impression on a piece of metal by an instantaneous stroke. The fly is therefore the principal agent in coining-presses.

Some presses used in coining have flies with arms four feet long, bearing a hundredweight at each of their extremities. If such a velocity be imparted to such an arm that it shall make one revolution per second, the die will be driven against the metal with the same force as that with which $3\frac{3}{4}$ tons would fall from the height of 16 feet, which is an enormous power, if the simplicity and compactness of the machine be considered.

260. **Position of the fly-wheel.**—The place to be assigned to a fly-wheel relatively to the other parts of the machinery is determined by the purpose for which it is used. If it be intended to equalise the action, it should be near the working point. Thus, in a steam-engine, it is placed near the crank which turns the axle by which the power of the engine is transmitted to the object it is finally designed to accomplish. On the contrary, in hand-mills, such as those commonly used for grinding coffee, &c., it is placed upon the axis of the winch by which the machine is worked.

The open work of fenders, fire-grates, and similar ornamental articles constructed in metal, is produced by the action of a fly in the manner already described.

The cutting tool, shaped according to the pattern to be executed, is attached to the end of the screw, and the metal being held in a proper position beneath it, the fly is made to urge the tool downwards with such force as to stamp out pieces of the required figure. When the pattern is complicated, and it is necessary to preserve with exactness the relative situation of its different parts, a number of punches are impelled together, so as to strike the entire piece of metal at the same instant, and in this manner the most elaborate open work is executed by a single stroke of the hand.

261. **Modification of motion.**—Although the simple machines, classed under the general denomination of *mechanical powers*, include in principle all the solid parts which can enter into the composition of any piece of mechanism, there are still many expedients which, on account of their simplicity of construction and extensive utility in practice, merit more particular notice.

In the arts and manufactures, the kind of motion produced by a given force is often of much greater importance than either the velocity imparted to the working-point, or the amount of the useful effect obtained from the moving power; the latter may affect the quantity of work done in a given time; but the former,

in particular cases, is essential to the performance of the work, in any quantity whatever.

In the practical application of machinery, the object aimed at is often to communicate to the working point some peculiar sort of motion adapted to the uses to which the machine is applied, and it rarely, indeed almost never, happens that the movable power is capable of this sort of motion. Expedients must, therefore, be discovered by means of which the motions which the moving power is capable of directly producing can be converted into those which are necessary for the purposes to which the machine is applied.

The varieties of motion which most commonly present themselves in the practical application of mechanics may be divided into *rectilinear* and *rotatory*. In rectilinear motion the several parts of the moving body proceed in parallel straight lines with the same speed. In rotatory motion the several points revolve round an axis, each performing a complete circle, or similar parts of a circle, in the same time.

Each of these may again be resolved into continued and reciprocating. In a continued motion, whether rectilinear or rotatory, the parts move constantly in the same direction, whether that be in parallel straight lines or in rotation on an axis. In reciprocating motion the several parts move alternately in opposite directions, tracing the same spaces from end to end continually. Thus, there are four principal species of motion which more frequently than any others act upon, or are required to be transmitted by, machines:—

1. *Continued rectilinear motion.*
2. *Reciprocating rectilinear motion.*
3. *Continued circular motion.*
4. *Reciprocating circular motion.*

These will be more clearly understood by examples of each kind.

Continued rectilinear motion is observed in the flowing of a river, in a fall of water, in the blowing of the wind, in the motion of an animal upon a straight road, in the perpendicular fall of a heavy body, in the motion of a body down an inclined plane.

Reciprocating rectilinear motion is seen in the piston of a common syringe, in the rod of a common pump, in the hammer of a pavier, the piston of a steam-engine, the stampers of a fulling-mill.

Continued circular motion is exhibited in all kinds of wheel-work, and is so common that to particularise it is needless.

Reciprocating circular motion is seen in the pendulum of a clock, and in the balance-wheel of a watch.

We shall now explain some of the contrivances by which a power having one of these motions may be made to communicate either the same species of motion changed in its velocity or direction, or any of the other three kinds of motion.

262. **Continued rectilinear motion.**—The most obvious expedient for producing this motion is by means of one or more pulleys. It has been already shown that, by a single fixed pulley, a moving power acting in the direction of any line may impart continued rectilinear motion to a resistance in any other line, provided only that the two directions can be connected by a fixed pulley. This, however, can only be done when lines drawn in the two directions intersect each other. In that case a fixed pulley, being placed in the angle formed by the two

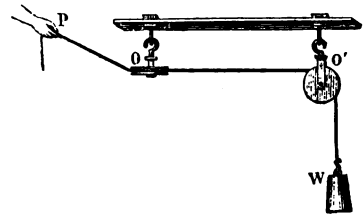

Fig. 239.

lines, with its axis perpendicular to them, will serve to guide a rope, one part of which will be directed to the resistance, and the other to the moving power.

If, however, the direction of the power do not intersect that of the resistance, they must either be parallel or in different planes.

If they be parallel, as, for example, when the power acts vertically downwards, while the weight is moved vertically upwards, the object will be attained by two fixed pulleys having their axes horizontal, one placed directly above the power, and the other directly above the resistance.

If the direction of the power and that of the weight be in different planes, the object may also be attained by two fixed pulleys, placed at any two points arbitrarily taken. The axis of the pulley O (*fig.* 239), which guides that part of the rope directed to the power P, must in that case be at right angles to the direction of the power and that of the line joining the pulleys; and the axis of the pulley O′, which guides that part

of the rope directed to the resistance w, must be at right angles to the direction of the line joining the pulleys and that of the resistance.

If it be necessary to change the velocity, any of the systems of pulleys described in Arts. 223 *et seq.* may be used in addition to the fixed pulleys.

By the wheel and axle any one continued rectilinear motion may be made to produce another in any other direction, and with any other velocity. It has been already explained that the proportion of the velocity of the power to that of the weight is as the diameter of the wheel to the diameter of the axle. The thickness of the axle being therefore regulated in relation to the size of the wheel, so that their diameter shall have that proportion which subsists between the proposed velocities, one condition of the problem will be fulfilled. The rope coiled upon the axle may be carried, by means of one or more fixed pulleys, into the direction of one of the proposed motions, while that which surrounds the wheel is carried into the direction of the other by similar means.

By the wheel and axle a continued rectilinear motion may be made to produce a continued rotatory motion, or *vice versâ*. If the power be applied by a rope coiled upon the wheel, the continued motion of the power in a straight line will cause the machine to have a rotatory motion. Again, if the weight be applied by a rope coiled upon the axle, a power having a rotatory motion applied to the wheel will cause the continued ascent of the weight in a straight line.

Continued rectilinear and rotatory motions may be made to produce each other, by causing a toothed wheel to work in a *rack*. Such an apparatus is represented in *fig.* 240. In some cases the teeth of the wheel work in the links of a chain. The wheel is then called a *rag-wheel* (*fig.* 241). Straps, bands, or ropes may communicate rotation to a wheel, by their friction in a groove upon its edge.

A continued rectilinear motion is produced by a continued circular motion in the case of a screw. The lever which turns the screw has a continued circular motion, while the screw itself advances with a continued rectilinear motion.

The continued rectilinear motion of a stream of water acting upon a wheel produces continued circular motion. In like manner the continued rectilinear motion of the wind produces a continued circular motion in the arms of a windmill.

Cranes for raising and lowering heavy weights convert a circular motion of the power into a continued rectilinear motion *of the weight.*

263. **Reciprocating rectilinear motion.**—Continued circular motion may produce reciprocating rectilinear motion, by a great variety of ingenious contrivances.

Reciprocating rectilinear motion is used when heavy stampers are to be raised to a certain height, and allowed to fall upon some object placed beneath them. This may be accomplished by a wheel bearing on its edge curved teeth, called *wipers*. The stamper is furnished with a projecting arm or peg, beneath which the wipers are successively brought by the revolution of the wheel. As the wheel revolves the wiper raises the stamper,

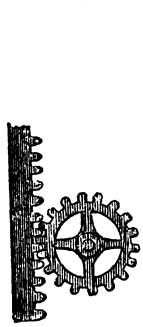

Fig. 240.

Fig. 241.

Fig. 242.

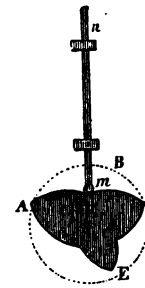

Fig. 243.

until its extremity passes the extremity of the projecting arm of the stamper, when the latter immediately falls by its own weight. It is then taken up by the next wiper, and so the process is continued. A similar effect is produced if the wheel be partially furnished with teeth, and the stamper carry a rack in which these teeth work. Such an apparatus is represented in *fig.* 242.

It is sometimes necessary that the reciprocating rectilinear motion shall be performed at a certain varying rate in both directions. This may be accomplished by the arrangement represented in *fig.* 243. A wheel turns uniformly in the direction of A B D E. A rod *m n* moves in guides, which only permit it to ascend and descend perpendicularly. Its extremity *m* rests upon a path or groove raised from the face of the wheel, and shaped into such a curve that as the wheel revolves the rod *m n* shall be moved alternately in opposite directions through the guides with the required velocity. The manner in which the velocity varies will depend on the form given to the groove or channel raised upon the face of the wheel, and this may be

shaped so as to give any variation to the motion of the rod *m n* which may be required for the purpose to which it is to be applied.

The *rose-engine* in the turning-lathe is constructed on this principle. It is also used in spinning machinery.

It is often necessary that the rod to which the reciprocating motion is communicated shall be urged by the same force in both directions. A wheel partially furnished with teeth, acting on two racks placed on different sides of it, and both connected with the bar or rod to which the reciprocating motion is to be communicated, will accomplish this. Such an apparatus is represented in *fig.* 244, and needs no further explanation.

Fig. 244.

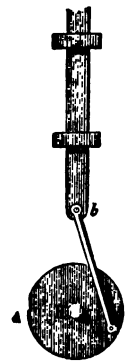

Fig. 245.

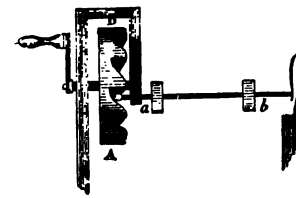

Fig. 246.

Another contrivance for the same purpose is shown in *fig.* 245, where A is a wheel turned by a winch H, and connected with a rod or beam moving in *guides*, by the joint *a b*. As the wheel A is turned by the winch H, the beam is moved between the guides alternately in opposite directions, the extent of its range being governed by the length of the diameter of the wheel. Such an apparatus is used for grinding and polishing plane surfaces, and also occurs in silk machinery.

An apparatus applied by M. Zureda in a machine for pricking holes in leather is represented in *fig.* 246. The wheel A B has its circumference formed into teeth, the shape of which may be varied according to the circumstances under which it is to be applied. One extremity of the rod *a b* rests upon the teeth of the wheel upon which it is pressed by a spring at the other extremity. When the wheel revolves it communicates to this rod a reciprocating rectilinear motion.

Leupold applied this mechanism to move the pistons of pumps. Upon the vertical axis of a horizontal hydraulic wheel is fixed another horizontal wheel, which is furnished with seven teeth in the manner of a crown wheel. These teeth are shaped like inclined planes, the intervals between them being equal to the length of the planes. Projecting arms attached to the piston rods rest upon the crown of this wheel; and, as it revolves, the inclined surfaces of the teeth, being forced under the arm, raise the rod, upon the principle of the wedge. To diminish the obstruction arising from friction, the projecting arms of the piston rods are provided with rollers, which run upon the teeth of the wheel. In one revolution of the wheel each piston makes as many ascents and descents as there are teeth.

264. **Continued circular motion.**—Wheelwork furnishes numerous examples of continued circular motion round one axis, producing continued circular motion round another. If the axles be in parallel directions, and not too distant, rotation may be transmitted from one to the other by two spur wheels (Art. 213); and the relative velocities may be determined by giving a corresponding proportion to the diameter of the wheels.

If a rotatory motion is to be communicated from one axis to another parallel to it, and at any considerable distance, it cannot in practice be accomplished by wheels alone, for their diameters would, in that case, be too large. In this case the

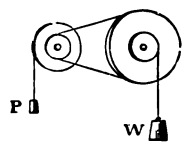

Fig. 247.

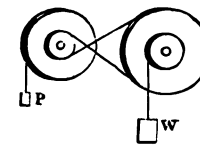

Fig. 248.

motion is communicated from wheel to wheel by endless bands. If the two wheels are desired to turn in the same direction, the bands are placed as in *fig.* 247. If in contrary directions, they are crossed, as in *fig.* 248.

In this case, as in toothed wheels, the relative velocities are determined by the proportion of the diameters of the wheels.

In some cases it happens that the strain upon the wheels is too great to allow of the motion being transmitted by a band. In such cases the motion may be transmitted from one shaft to the other by means of bevelled wheels. In fig. 249 an

arrangement is shown in which a revolving shaft imparts motion to a series of other shafts, perpendicular to it. Each of the latter might impart motion to other shafts, at any convenient distance, by like means.

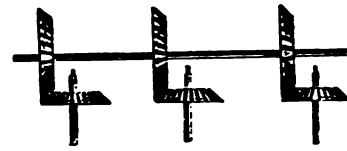

Fig. 249.

In *figs.* 250 and 251 another arrangement is shown, by which a shaft W imparts motion to another shaft W′, by the intervention of a third shaft *w w′* on which bevelled wheels are fixed. The velocities may in this case be varied in any desired proportion by establishing a corresponding relation between the magnitudes of the wheels.

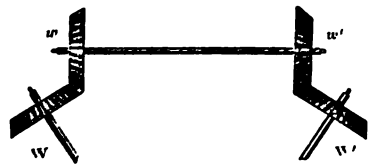

Fig. 250.

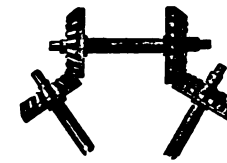

Fig. 251.

The methods of transmitting rotation from an axis or shaft to another at right angles to it, by spur and crown wheels, and by an endless screw and a spur wheel, have been already explained in Arts. 214 and 237.

265. **Universal joint.**—The axis to which rotation is to be given, or from which it is to be taken, is sometimes variable in its position. In such cases an ingenious contrivance, called a *universal joint*, invented by the celebrated Dr. Hook, may be used. The two shafts or axles A B, *fig.* 252, between which the motion is to be communicated, terminate in semicircles, the diameters of which, C D and E F, are fixed in the form of a cross, their extremities moving freely in bushes placed in the extremities of the semicircles. Thus, while the central cross remains unmoved, the shaft A and its semicircular end may revolve round C D as an axis; and the shaft B and its semicircular end may revolve round E F as an axis. If the shaft A be made to revolve without changing its direction, the

points C D will move in a circle whose centre is at the middle of the cross. The motion thus given to the cross will cause the points E F to move in another circle round the same centre, and hence the shaft B will be made to revolve.

This instrument will not transmit the motion if the angle under the directions of the shafts be less than 140°. In this case a double joint, as represented in *fig.* 253, will answer the purpose. This consists of four semicircles united by two crosses, and its principle and operation are the same as in the last case.

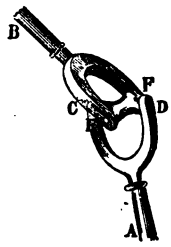

Fig. 252.

Fig. 253.

Universal joints are of great use in adjusting the position of large telescopes, where, while the observer continues to look through the tube, it is necessary to turn endless screws or wheels whose axes are not in an accessible position.

The cross is not indispensably necessary in the universal joint. A hoop, with four pins projecting from it at four points equally distant from each other, or dividing the circle of the hoop into four equal arches, will answer the purpose. These pins play in the bushes of the semicircles in the same manner as those of the cross.

The universal joint is much used in cotton-mills, where shafts are carried to a considerable distance from the prime mover, and great advantage is gained by dividing them into convenient lengths, connected by a joint of this kind.

266. **Alternate circular motion.**—In the practical application of machinery it is often necessary to connect a part having a continued circular motion with another which has a reciprocating or alternate motion, so that either may move the other. There are many contrivances by which this may be effected.

One of the most remarkable examples of it is presented in the escapements of watches and clocks.

A beam vibrating on an axis, and driven by the piston of a steam-engine, or any other power, may communicate rotatory motion to an axis by a connecter and a crank.

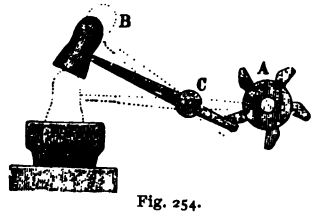

Fig. 254.

A wheel A (*fig.* 254) armed with wipers, acting upon a sledge-hammer B, fixed upon a centre or axle C, will, by a continued rotatory motion, give the hammer the reciprocating motion necessary for the purposes to which it is applied. The manner in which this acts must be evident on inspecting the figure.

The large shears used in factories for cutting plates of metal are worked upon a similar principle. The lower edge of the shears (*fig.* 255) is fixed, and the upper movable upon a joint. Under the extremity of the movable arm is placed a wheel, the

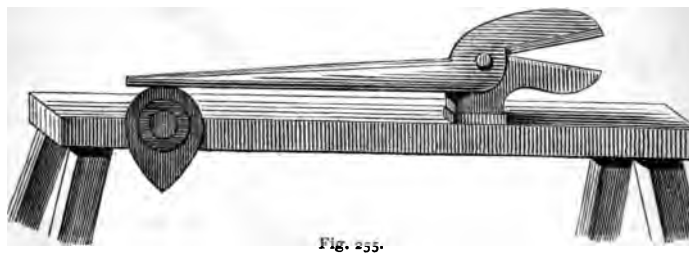
Fig. 255.

contour of which has the form shown in the figure. In the position there shown, the arm is in its lowest position, and therefore the blades of the shears diverge as much as possible. When the wheel turns, so as to raise the movable arm, the shears will be closed. In this way, by the continual revolution of the wheel, the shears will be alternately closed and opened.

The treadle of the lathe furnishes an obvious example of a vibrating circular motion producing a continued circular one. The treadle acts upon a crank, which gives motion to the principal wheel, in the same manner as already described in reference to the working beam and crank in the steam-engine.

By the following ingenious mechanism an alternate or

vibrating force may be made to communicate a circular motion continually in the same direction:—Let A B (*fig.* 256) be an axis receiving an alternate motion from some force applied to it, such as a swinging weight. Two ratchet wheels *m* and *n* are fixed on this axle, their teeth being inclined in opposite directions. Two toothed wheels, C and D, are likewise placed upon it, but so arranged that they turn upon the axle with a little friction. These wheels carry two catches *o p*, which fall into the teeth of the ratchet wheels *m n*, but fall on opposite sides conformably to the inclination of the teeth already mentioned. The effect of these catches is, that if the axis be made to revolve in one direction, one of the two toothed wheels is always compelled (by the catch *against* which the motion is directed) to revolve with it, while the other is permitted to remain stationary in obedience to any force sufficiently great to overcome its friction with the axle on which it is placed. The wheels C and D are both engaged by bevelled teeth with the wheel E.

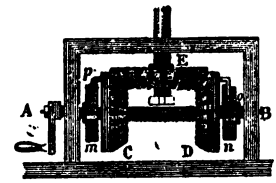

Fig. 256.

According to this arrangement, in whichever direction the axis A B is made to revolve, the wheel E will continually turn in the same direction; and therefore, if the axle A B be made to turn alternately in the one direction and the other, the wheel E will not change the direction of its motion. Let us suppose the axle A B is turned against the catch *p*. The wheel C will then be made to turn with the axle. This will drive the wheel E in the same direction. The teeth on the opposite side of the wheel E being engaged with those of the wheel D, the latter will be turned upon the axle; the friction, which alone resists its motion in that direction, being overcome. Let the motion of the axle A B be now reversed. Since the teeth of the ratchet wheel *n* are moved against the catch *o*, the wheel D will be compelled to revolve with the axle. The wheel E will be driven in the same direction as before, and the wheel C will be moved on the axle A B, and in a contrary direction to the motion of the axle, the friction being overcome by the force of the wheel E. Thus, while the axle A B is turned alternately in the one direction and the other, the wheel E is constantly moved in the same direction.

It is evident that the direction in which the wheel E moves

may be reversed by changing the position of the ratchet wheels and catches.

It is often necessary to communicate an alternate circular motion, like that of a pendulum, by means of an alternate motion in a straight line. A remarkable instance of this occurs in the steam-engine. The moving force in this machine is the pressure of steam, which impels a piston from end to end alternately in a cylinder. The force of this piston is communicated to the working beam by a strong rod, which passes through a collar in one end of the piston. Since it is necessary that the steam included in the cylinder should not escape between the piston rod and the collar through which it moves, and yet that it should move as freely and be subject to as little resistance as possible, the rod is turned so as to be truly cylindrical, and is well polished. It is evident that, under these circumstances, it must not be subject to any lateral or cross strain, which would bend it towards one side or the other of the cylinder. But the end of the beam to which it communicates motion, if connected immediately with the rod by a joint, would draw it alternately to the one side and the other, since it moves in the arc of a circle, the centre of which is at the centre of the beam. It is necessary, therefore, to contrive some method of connecting the rod and the end of the beam, so that while the one shall ascend and descend in a straight line, the other may move in the circular arc.

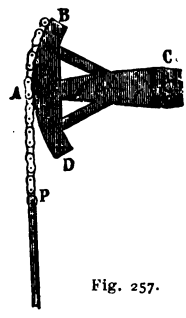

Fig. 257.

The method which first suggests itself to accomplish this is, to construct an arch-head upon the end of the beam, as in *fig.* 257. Let C be the centre on which the beam works, and let B D be an arch attached to the end of the beam, being a part of a circle having C for its centre. To the highest point B of the arch a chain is attached, which is carried upon the face of the arch B A, and the other end of which is attached to the piston-rod. Under these circumstances it is evident that when the force of the steam impels the piston downwards, the chain P A B will draw the end of the beam down, and will, therefore, elevate the other end.

When the steam-engine is used for certain purposes, such as pumping, this arrangement is sufficient. The piston in that case is not forced upwards by the pressure of steam. During its ascent it is not subject to the action of any force of steam,

and the other end of the beam falls by the weight of the pump-rods drawing the piston at the opposite end, A, to the top of the cylinder. Thus the machine is, in fact, passive during the ascent of the piston, and exerts its power only during the descent.

If the machine, however, be applied to purposes in which a constant action of the moving force is necessary, as is always the case in manufactures, the force of the piston must drive the beam in its ascent as well as in its descent. The arrangement just described cannot effect this; for although a chain is capable of transmitting any force by which its extremities are drawn in opposite directions, yet it is, from its flexibility, incapable of communicating a force which drives one extremity of it towards the other. In the one case the piston first *pulls* down the beam, and then the beam *pulls* up the piston. The chain, because it is inextensible, is perfectly capable of both these actions; and, being flexible, it applies itself to the arch-head of the beam, so as to maintain the direction of its force upon the piston continually in the same straight line. But when the piston acts upon the beam in both ways, in pulling it down and pushing it up, the chain becomes inefficient, being, from its flexibility, incapable of the latter action.

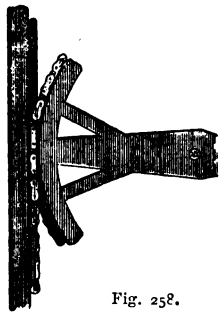

Fig. 258.

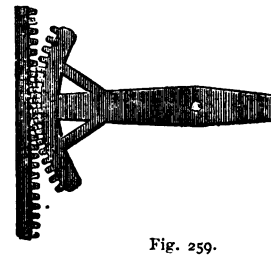

Fig. 259.

The problem might be solved by extending the length of the piston-rod, so that its extremity shall be above the beam, and using two chains; one connecting the highest point of the rod with the lowest point of the arch-head, and the other connecting the highest point of the arch-head with a point on the rod below the point which meets the arch-head when the piston is at the top of the cylinder, *fig.* 258.

The connection required may also be made by arming the arch-head with teeth, *fig.* 259, and causing the piston-rod to

terminate in a rack. In cases where, as in the steam-engine, smoothness of motion is essential, this method is objectionable; and under any circumstances such an apparatus is liable to rapid wear.

The method contrived by Watt, for connecting the motion of the piston with that of the beam, is one of the most ingenious and elegant solutions ever proposed for a mechanical problem. He conceived the notion of two straight rods, A B, C D, *fig.* 260, moving on centres or pivots A and C, so that the extremities B and D would move in the arcs of circles having their centres at A and C. The extremities B and D of these rods he conceived to be connected with a third rod B D united with them by pivots on which it could turn freely. To the system of rods thus connected let an alternate motion on the centres A and C be communicated; the points B and D will move upwards and downwards in the arcs expressed by the dotted lines, but the middle point P of the connecting rod B D will move upwards and downwards without any sensible deviation from a straight line.

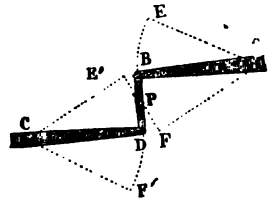

Fig. 260.

To prove this demonstratively would require some elaborate mathematical investigation. It may, however, be rendered in some degree apparent by reasoning of a brief and more obvious nature. As the point B is raised to E it is also drawn aside towards the right. At the same time the other extremity D of the rod B D is raised to E′, and is drawn aside towards the left. The ends of the rod B D being thus at the same time drawn equally towards opposite sides, its middle point P will suffer no lateral derangement, and will move directly upwards. On the other hand, if B be moved downwards to F, it will be drawn laterally to the right; while D being moved to F′ will be drawn to the left. Hence, as before, the middle point P sustains no lateral derangement, but merely descends. Thus, as the extremities B and D move upwards and downwards in circles, the middle point P moves upwards and downwards in a straight line.

An elegant contrivance, by which a continued motion of rotation can be made to produce a reciprocating rectilinear motion, is shown in *fig.* 261, where a wheel A B is surrounded on the inside with teeth, and a wheel C of half the diameter

revolves within it; the wheel A B being fixed. While the centre of C moves in a circle round the centre of A B, any point taken on the circumference of C will move alternately across A B, in the direction of one of its diameters. Thus, that point of C which, in the figure, is at the highest point of A B, will move along the vertical diameter of A B, to the lowest point, while C rolls over half the circumference of A B; and the same point will again traverse the same vertical diameter of A B upwards, while the wheel C rolls over the other semicircle of A B.

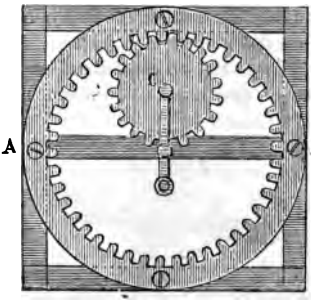

Fig. 261.

Another method of producing a reciprocating rectilinear motion by a reciprocating circular motion is shown in *fig.* 262. The beam A B vibrates on the centre C, and a cord applies itself to a groove in the semicircular arch D E F. This cord is extended over the grooves of two fixed rollers M N; it is evident that as the beam A B vibrates, any point L upon the cord, between M N, will move alternately right and left in a straight line.

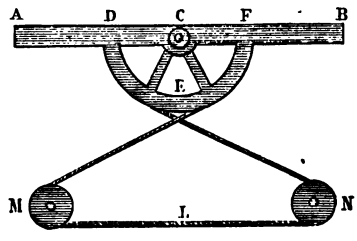

Fig. 262.

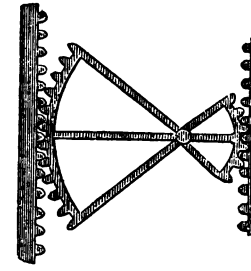

Fig. 263.

By the arrangement shown in *fig.* 263 the alternate circular motion of the unequal toothed sectors will impart reciprocating rectilinear motions to two racks. These motions will be unequal, and will be in the same ratio as the radii of the two circles.

By a mechanical contrivance represented in *fig.* 264, called the lever of La Garousse, a reciprocating circular motion is

made to produce an intermitting rectilinear motion. The lever A B vibrates on the centre C, the arm which ascends drawing up one of the hooks E, while the arm which descends passes the other hook E′ from one tooth to another of the vertical beam F G. By the continuance of this operation, the beam F G is elevated by an intermitting motion.

In *fig.* 265 an expedient is represented by which a similar effect is produced upon a wheel M N. A lever A B vibrates on a

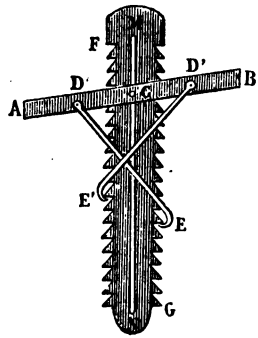

Fig. 264.

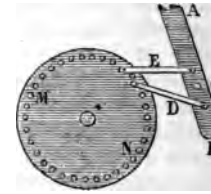

Fig. 265.

centre C, while two arms D and E extend from it, and act upon pins projecting from the wheel M N. The arms D and E are thus alternately pushed forwards and drawn backwards: that which is pushed forwards drives before it one of the pins, and that which is drawn backwards falls from one pin to the other.

267. **Joints.**—It frequently happens in mechanical combinations that it is required to make temporary connections between separate parts, or so to unite them together that they shall admit of having their relative position changed at pleasure. In some cases one, and in others both, of the parts thus connected are susceptible of receiving motions more or less limited in their direction. In some cases the motion of one part necessarily produces some corresponding motion in the other; and in others the motion of one is quite independent of the other, which may be fixed. The class of contrivances which fulfil these purposes are called *joints.*

The universal joint already explained belongs to this class. One of the arms thus connected may be inclined to the other in almost any direction whatever, when the joint is constructed as

in *fig.* 253. But in thus changing its direction no corresponding motion is imparted to the other arm. If, however, a twisting or rotatory motion round its own longitudinal axis be given to either arm, a corresponding rotatory motion round its longitudinal axis will be imparted to the other. In this manner, for example, a screw may be turned by a twisting motion given to an arm having a direction inclined at any angle to the longitudinal axis of the screw.

268. **Gimbals.**—This is a mechanical contrivance, which bears a close relation to the universal joint. An example of it, familiar to everyone, is presented by the apparatus for suspending a ship's compass (*fig.* 266). The object is to keep the suspended body vertical, whatever be the derangements to which the points of suspension are liable.

A brass hoop is supported by two pins, projecting from points of its external surface which are diametrically opposed to each

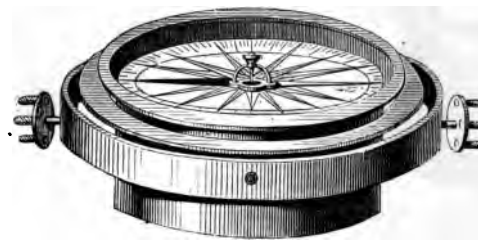

Fig. 266.

other. Another brass hoop is supported within the former, also by two pins projecting from points of its external surface, diametrically opposed one to the other, which play in two holes made in the former hoop at the extremities of that diameter, which is at right angles to the diameter at the extremities of which the first points of support are placed.

A similar mode of suspension is applied to the marine barometer.

269. **Ball and socket.**—The joint thus called has some, but not all, of the qualities of the universal joint. The extremity of one of the arms is attached to a solid ball of metal, and that of the other to a hollow ball, the internal diameter of which is equal to the diameter of the solid ball. If one of these balls were contained within the other, the arm of the solid ball passing through a hole made to fit it in that of the hollow ball,

the two arms would be incapable of any change whatever in their relative position, and the joint so formed would form a rigid and invariable connection. But if the hole in the hollow ball, through which the arm of the solid ball passes, be much larger than the arm, as shown in *fig.* 267, then the arm will have a certain play, the solid ball moving within the hollow ball, and

Fig. 267.

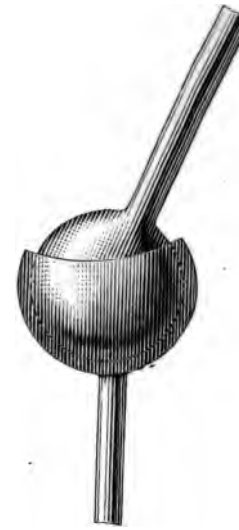

Fig. 268.

the extent of such play would obviously depend on the magnitude of the opening. A joint thus constructed is that which is called the *ball and socket joint.*

This joint is represented in *fig.* 268.

It is evident that the inclination which can be given to the arms will be limited by half the circumference of a section of the socket.

The greatest play which such a joint can have will be produced when the socket exceeds an exact hemisphere by just enough to enable it to retain the ball within it; and it follows, therefore, that the angle under the arms thus connected, when they are most inclined, will be a little greater than 90°.

If either can be fixed, the play of the other will be limited

by the directions of the sides of a cone whose vertex is at the centre of the ball, and whose base is limited by the part of the ball left uncovered by the socket.

The arms connected by this joint are moved independently of each other. Either may be turned round its longitudinal axis, without imparting a corresponding motion to the other.

The socket is usually made of such a magnitude that the ball moves in it with a certain degree of friction sufficient to retain the arms in whatever position may be given to them.

270. **Cradle joint.**—By this joint either arm connected is allowed to move in a given plane round the other arm. The joint is usually formed by a pin passing through the centre of circular discs of metal formed at the end of the arms, having equal diameters, and placed concentrically face to face; sometimes, one such disc is attached to one arm passing between two, which are attached to the other; such a joint is represented in *fig.* 269.

It is by a joint of this kind that the legs of compasses are connected.

271. **Hinge.**—This is, in fact, a variety of the cradle joint. Its construction is rendered so familiar by the lids of boxes, and the suspension of doors, that it will not require further explanation.

272. **Trunnions.**—The method of supporting heavy bodies, which require to be freely movable in a vertical plane, thus denominated, consists of two strong cylindrical pins, projecting from its sides, which rest in semi-cylindrical grooves formed at equal heights, in two pillars, so that a line, connecting the trunnions, shall be horizontal. A body thus supported will be free to assume every possible direction, in a vertical plane at right angles to the line joining the supports of the trunnions.

Fig. 269.

273. **Axles.**—These are, in general, the centres round which wheels revolve; but the wheels may either turn *upon* or *with* the axles.

The wheels of ordinary carriages turn upon their axles, the axles being strong iron bars, extending horizontally and transversely under the body of the vehicle. Their extremities, which pass through the centre of the wheels, have a cylindrical or slightly conical form. The centre part of the wheel in which

the spokes are inserted is called the *nave*, and the central part within it, into which the extremity of the axle passes, is called the *box*. The inner surface of the box has a form corresponding to that of the axle, but is a little larger, so as to admit the play of some lubricating fluid between them, by the interposition of which the actual contact of the metallic surfaces of the axle and the box is prevented, no matter what be the pressure by which they are urged one against the other.

In railway carriages the wheels and axle form one solid piece ; the cylindrical or conical extremities of the axle project outside the wheels, and certain parts called *bearings* rest upon them. In this case, therefore, the extremities of the axle revolve with the wheels under the bearings. Reservoirs of lubricating matter, called the *grease boxes*, are placed immediately over the axles, by which the grease is let continually down upon the axles.

274. **Telescope joint.**—This mode of connection, which is of very extensive use in mechanics, is rendered familiar to everyone by the manner in which the tubes of telescopes and opera-glasses are adjusted. It is used, in general, where a rod or pillar requires to have its length varied occasionally, according to the circumstances in which it is used.

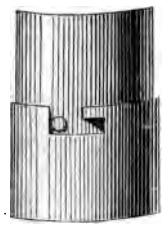

Fig. 270.

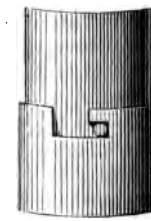

Fig. 271.

275. **Bayonet joint.**—The joint thus called takes its name from the simple and well-known mechanical contrivance by which the handle of a bayonet is fastened on the end of a musket barrel. Its simplicity and efficiency have rendered it of extensive use in practical mechanics. As in the telescope joint, one tube passes within another ; but in this case they are held in a fixed position by means of a pin which projects from the inner tube, and passes through a rectangular opening in the outer tube.

Fig. 270 represents the position of the pin when the inner tube is pressed into the outer, until the pin comes against the

edge of the opening and stops its further progress. In this position, however, the inner tube might be detached from the outer by any force tending to draw it out. To prevent this, it is turned round its axis, until the pin enters the rectangular opening in the outer tube, as shown in *fig.* 271. It cannot be detached then without two successive motions being given to it ; one round, and the other parallel to the axis of the tube.

276. **Clamps and adjusting screws.**—When parts of mechanism usually separated require to be temporarily connected, the object is attained by clamps, which are made in an infinite variety of forms, according to the circumstances in which they are applied. An example of this class of contrivances is shown in *fig.* 272. The ends of the parts to be united being placed one upon the other, are introduced into the rectangular opening shown in the figure, and the screw is then turned until it urges them by a pressure sufficiently strong to hold them together.

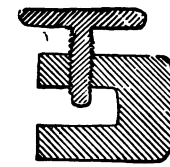

Fig. 272.

277. **Couplings.**—In the practical operation of machinery it is frequently necessary to be enabled to suspend at pleasure, or to recommence, the motion of a wheel or wheels. This is accomplished by a class of contrivances called *couplings.*

When the motion of a wheel is imparted to another by an endless band of leather stretched tightly round them, the motion can be suspended or resumed, at pleasure, by making the band loose, and giving it the requisite tightness by pressing upon it a roller at any intermediate point between the wheels ; so long as the pressure of the roller is maintained, the rotation of one wheel will be imparted to the other ; but the moment the roller is removed, the communication of the motion will be suspended.

In some cases the communication of the motion is discontinued or altered by removing the band from one or other of the wheels by lateral pressure. By this expedient the velocity of the motion imparted may be modified. Thus, if several wheels be fixed side by side, on the same axis, having different diameters, the strap may be shifted from one to another, the velocity of rotation imparted being increased in the same proportion as the diameter of the wheel receiving the motion is diminished.

Couplings are sometimes constructed by providing two wheels upon the same shaft, one turning *upon* and the other *with* the shaft. Let us suppose that the moving power keeps the wheel

which turns upon the shaft constantly in revolution, but does not act upon that which turns with the shaft. In that case, it is evident that no rotation would be imparted to the shaft. Now

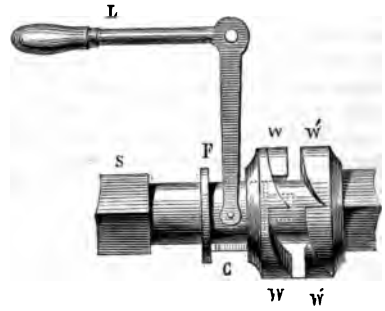

Fig. 273.

let us suppose that the wheel w w (*fig.* 273) is that which turns *upon* the shaft, that attached to it is a collar C embraced by a fork F at the end of a lever L, that the wheel with the collar is capable of sliding longitudinally on the shafts, and that its surface towards s′ is cut into a sort of angular teeth corresponding with similar ones formed in the face of the wheel w′ w′, which turns *with* the axle. When, by means of the lever L and the fork F, the wheel w w is moved towards the wheel w′ w′, so that the teeth are inserted the one within the other, the two wheels thus forming a single one, the wheel w w will impart its motion to w′ w′, and therefore to the shaft s s′.

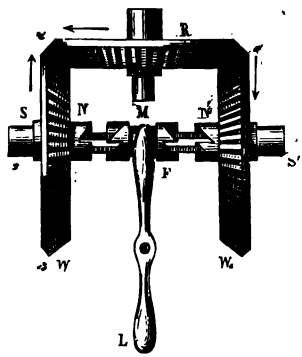

Fig. 274.

On the contrary, when it is desired to suspend the motion of the shaft, the wheel w w is drawn by the lever L F towards s, and disengaged from w′ w′.

A coupling by which the rotation of a shaft can be suspended or reversed at pleasure is shown in *fig.* 274, where w and w′ are two bevelled wheels, both driven by a third bevelled wheel R, and therefore made to revolve in contrary directions. Let us suppose that these wheels w and w′ turn *upon* the shaft

S S′, and carry at their centres two small wheels N N′, the faces of which are cut into angular teeth; let M be two similar wheels similarly cut, presenting their opposite faces towards those of N N′; let these wheels M revolve *with* the shaft, but so as to be capable of sliding upon it; let the fork F, of the lever L F, embrace between its prongs the collar M. By pressing this lever in the one direction or the other, the wheels M may be coupled, either with N or N′, so as to revolve with them, and thus impart to the shaft S S′ a motion in the one direction or the other.

If the lever L F be kept in the vertical position, the wheels M not being connected with either N or N′, the motion will be suspended.

Another method of coupling is represented in *fig.* 275, where

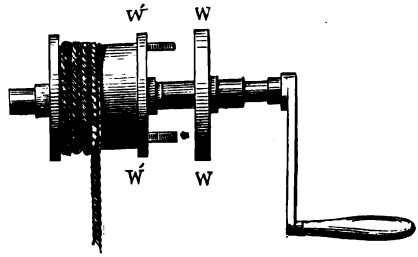

Fig. 275.

the wheel w w has two or more holes in it, corresponding with pins projecting from w′ w′; the former revolves *with* and the latter *upon* the axle; the wheel w w slides upon the axle, so that when pressed towards w′ w′ the pins enter the holes and the wheel w′ w′ revolves with w w.

CHAPTER XVII.

RESISTING FORCES.

278. **Resisting forces.**—A physical agent capable of imparting motion to a quiescent body is called a force. It is evident that such an agent would also be capable of increasing the velocity of a body already in motion if it were applied in the direction of the motion, or diminishing the velocity, or even altogether

destroying the motion, and bringing the body to rest, if it were applied in an opposite direction.

This principle is not, however, convertible: although it follows that an agent capable of imparting motion is also capable of diminishing or destroying it, it does not follow that an agent capable of diminishing or destroying motion is also capable of imparting or increasing it.

Forces of the class to which we now refer are capable of diminishing the velocity of a body in motion, and of bringing it to rest, but they are incapable of imparting motion to a body at rest, or of augmenting any motion it may have. The former class of forces, which are capable of producing or increasing motion, may be described for distinction as *active forces*, and the latter as *passive forces*.

The force of gravity, for example, comes under the former class. A body freely suspended, being disengaged and submitted to the action of gravity, is put in motion, and its motion is continually accelerated as it moves downwards, until it encounters some obstacle which brings it to rest.

If the same body be projected upwards with the velocity with which it strikes the ground, the force of gravity will then gradually diminish its motion until it rises to the height from which it fell, where its motion will altogether be destroyed.

Of the passive or resisting forces, the most important are friction and the resistance of fluid media, such as air or water. All bodies moving at or near the surface of the earth are subject to some, or all, of these forces, and, consequently, all terrestrial motions whatever are liable to constant retardation; and, to be maintained, require the constant agency of some impelling force to repair the loss produced by the resisting forces to which they are exposed.

The smallest attention to the phenomena which form the subject of mechanical inquiries will render manifest the great importance of investigating and comprehending the effects of resisting forces.

In the preceding part of this volume the construction and properties of machinery have been explained, on the supposition that the moving force of the power is transmitted to the working point with undiminished effect. In order to disembarrass the questions of their complexity, and present them in the most simple and intelligible form, machines have been considered as absolutely free from the effects of all resisting forces; surfaces moving in contact have been considered to be perfectly free from friction; axles were regarded as mathematical lines; pivots

as mathematical points; ropes as perfectly flexible; and, in a word, the effect of the moving power has been considered as absolutely undiminished by any resistance whatever, in its transmission through the machinery to the working point.

It is scarcely needful to observe, that none of these suppositions are really true. The surfaces of the machinery which move in contact are never perfectly smooth; axles have always definite thickness, and move in sockets never perfectly polished; ropes have considerable rigidity, and this rigidity is necessarily greater in proportion to their strength. Much has been accomplished, it is true, by a variety of expedients, to diminish these resistances; highly polished surfaces and effective lubricants have been applied to obtain additional smoothness; but still the surfaces in contact continue to be studded with small asperities, which, coming constantly in opposition to each other in their motion, produce considerable resistance, and, robbing the moving power of a great part of its efficacy, transmit it with proportionally diminished intensity to the working point.

To estimate, therefore, correctly the practical effects of any machinery, it is essential that we should calculate the effect of this resistance, and deduct it from that effect of the power which has been computed on the theoretical principles established in the preceding chapters; the residue of effect after this deduction is all that part of the power which can be regarded as practically available.

279. **Friction.**—The effect of friction on a power supporting a weight or resistance at rest is different from its effect when the weight or resistance is moved. In the one case, friction assists; in the other, it opposes the power.

Let us suppose, for example, a power P supporting a weight W, by means of a single movable pulley. From what has been already proved, it is evident that if the power P be half the weight of W, they would be in equilibrium, and the power would keep the weight at rest if the pulley were subject to no friction; and in that case the slightest diminution of the power would cause the weight to descend, and draw the power upwards. But if the pulley be subject, as it always is in practice, to friction, then a small diminution of the power will be resisted by this friction, and the weight will not overcome the power and descend until the diminution of the power shall become so great as to enable the weight to overcome the friction.

It follows, therefore, that when a pulley or any other machine is subject to friction, a less power is sufficient to support a weight at rest than would be necessary if there were no friction; and

the greater the friction is, the less will be the power necessary to support the weight. It is in this case that friction is said to aid the power, when the weight or resistance is supported at rest. But if the power be required, not merely to support the weight, but to raise it, then we shall find that the friction, instead of aiding, opposes the power.

Let us suppose, for example, the power P acting on the weight through the intervention of a single movable pulley, the power being half the weight. In the absence of friction, the slightest addition to the power will cause it to descend, and to raise the weight; but when the machine is subject to friction, then the power will not descend until it shall receive such an addition as will be sufficient to overcome the friction.

This circumstance modifies materially the conditions of equilibrium. Representing by P that amount of the power applied to any machinery whatever, which would keep the weight W in equilibrium in the absence of friction, let f express the addition which must be made to P, in order to enable it to overcome the friction and put the weight in motion; then f will also express the amount by which the power must be diminished, in order to enable the weight to prevail over it and to descend. It is evident, therefore, that any power which is less than $P+f$, and greater than $P-f$, would keep the weight in equilibrium and at rest.

It may therefore be inferred, generally, that when a machine of any kind is used simply to sustain a weight or to balance a resistance, the friction, acting in common with the power, becomes a mechanical advantage. In many instances this resisting force constitutes the entire efficiency of the instrument. Thus, when screws, nails, or pegs are used to bind together the parts of any structure, their friction with the surface with which they are in contact prevents their recoil, and gives them their entire binding power. In the ordinary use of the wedge itself we have another striking example of the mechanical advantage of friction. When the wedge is used for any purpose, as, for example, to split timber, it is urged forward by percussion, the action of the moving power being only instantaneous, and being totally suspended between each successive blow. But for the resisting force of the friction which takes place between the surface of the wedge and the surface of the timber, the wedge would react after each blow, and render abortive the action of the moving power. The friction, therefore, in this case plays the part of a ratchet wheel, preventing the reaction of the wedge, and making good the action of the power.

Notwithstanding the disadvantages which attend the presence of friction in machines, it is an agent eminently useful in giving stability to structures, and in giving efficiency to the movements of almost all bodies, natural and artificial. Without friction, most structures, natural and artificial, would fall to pieces. The stones and bricks used in building owe to the mutual friction of their surfaces a great part of their stability. Manual exertion would become impracticable if no friction existed between the limbs and the objects upon which they act. The friction between the foot and the ground gives a purchase to the muscular force, so as to enable it to produce a progressive motion. Without friction, every effort of the foot to propel the body forward would be attended with a backward action, so that no progressive motion would ensue. The difficulty of moving upon ice, or upon ground covered with greasy or unctuous matter, illustrates this. Without friction we could not hold any body in the hand. The difficulty of holding a lump of ice between two fingers is an example of this. Without friction, a locomotive engine could not propel its load; for if the rail and the tires of the driving-wheels were both absolutely smooth, one would slip upon the other, without affording the necessary purchase to the steam power.

280. **Sliding and rolling friction.**—Friction is manifested in different ways, according to the kind of motion which one surface has upon the other. When one surface slides upon the other in the manner of a sledge, the friction is called sliding or rubbing friction. When a body rolls upon another, so that different points of such body come into successive contact with each other, it is called rolling friction.

The laws which regulate friction are derived exclusively from experiments, independently of theory. There are no simple or general principles from which they can be deduced by mathematical reasoning. It is a matter of regret that, even amongst the best conducted experiments that have been made, considerable discrepancies are observable, and that differences of opinion still prevail between the most competent authorities respecting several particulars connected with the properties and laws of this and other resisting forces.

Although these laws, so far as they are known, depend thus wholly on experiment, yet the general principles of science, as applied to them, are far from being dispensable. They serve as a guide in the selection of the experiments which are best adapted to develop those laws which are the subject of inquiry, as well as to show the inconclusiveness of some experiments on which re-

liance might otherwise be placed, and thus enable us further to deduce from the trustworthy results of experimental inquiries numerous useful practical rules.

There are two methods by which the amount of friction produced when two surfaces are moved one upon another can be ascertained.

1st. The surfaces being rendered perfectly flat, let one be fixed in a horizontal position on a table T T (*fig.* 276), and let the other be attached to the bottom of a box B E, adapted to receive weights so as to vary the pressure.

Let a flexible cord be attached to this box, and being carried parallel to the table, let it pass over a fixed pulley at P, and have a dish suspended to it at D.

If no friction existed between the surfaces, the smallest weight suspended from D would cause the box B E to move with a uniformly accelerated motion along the table towards the pulley ; but the resistance of friction renders it necessary, before motion can take place or be maintained, that the weight D shall

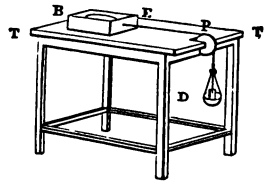

Fig. 276.

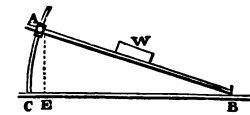

Fig. 277.

be equal to the amount of this friction. If the weight D and the friction be equal, then, the power and the resistance being in equilibrium, the box B E, if put in motion, will move towards the point P with any velocity, continued uniform, which may be imparted to it. If the weight D be greater than the friction, then the motion of the box towards P will be accelerated ; and if the weight be less than the friction, then any motion which may be given to the box will be retarded, and will soon cease altogether.

The determination, therefore, of the weight acting at D, which represents the exact amount of the friction, will depend upon the velocity given to the box in the direction B P being maintained uniform.

2ndly. Let one of the surfaces be attached, as before, to a flat plane A B (*fig.* 277), but instead of being horizontal let it be inclined, and so arranged that the inclination may be varied

at pleasure. The box W being constructed as before, and placed upon the plane, let such an elevation be given to the plane that the box shall be capable of moving down it with an uniform velocity, without acceleration or retardation. If the elevation be greater than this, the motion of the box down the plane will be accelerated, and the component of gravity acting along the plane will be greater than the friction; if it be less, the motion of the box will be retarded, until it ceases altogether, and the action of gravity upon W will then be less than the friction.

That particular inclination of the plane corresponding to the friction of any given surface, which renders the action of gravity equal to the friction, is called the *angle of repose*. According to the principles already explained, it follows that in these cases, if the length of the plane A B represent the total weight W, the gravity down the plane, which is equal to friction, will be represented by the height A E, and the pressure upon the plane will be represented by the base B E, and consequently the ratio of the friction to the pressure will be that of the height A E to the base B E. Experiments conducted according to both these methods have given nearly the same results, which may be summarily stated to be as follows:—

The proportion of the friction to the pressure, when the *quality* of the surface is given, is always the same, no matter how the weight or the magnitude of the surface may be varied, except in extreme cases, when the proportion of the pressure to the surface is very great or very small.

If the surfaces in contact be placed with their grains in the same direction, the friction will be greater than if their grains cross each other. Smearing the surfaces with unctuous matter diminishes the friction, probably by filling the cavities between those minute projections which produce the friction.

The pivots of pendulums or balances are usually made of steel, and rest upon hard polished stones, different surfaces being used for the purpose of diminishing the amount of friction. Brass sockets are generally used for iron axles on the same principle.

281. **Lubricants.**—In the selection of lubricants, those of a viscous nature are selected, in the case of the rough surfaces of softer bodies, and those which are more fluid are applied to the smoother surfaces of harder bodies. Thus, when metal moves upon wood, tallow, tar, or some solid grease is generally used; but when metal moves upon metal, oil is preferred; and the harder and the smoother the metal, the finer the oil. Finely pulverised plumbago is found to be a very efficient agent in

diminishing friction, especially as applied to the axles of carriages and the shafts of machinery.

282. **Rolling friction.**—The friction which attends a rolling motion is very much less than that which would attend a sliding or rubbing motion with the same surfaces and under the same pressure. Hence it is that rollers are used with so much success as an expedient for diminishing friction. A roughly chiselled block of stone, weighing 1,080 lbs., was drawn from the quarry to the surface of the rock by a force of 758 lbs. It was then laid upon a wooden sledge, and drawn upon a wooden floor, the tractive force being 606 lbs. When the wooden surfaces moving upon one another were smeared with tallow, the tractive force was reduced to 182 lbs.; but when the load was, in fine, placed upon wooden rollers three feet in diameter, the tractive force was reduced to 28 lbs.

283. **Sledges.**—Although it is therefore obvious, on general principles, that the friction of sliding considerably exceeds that

Fig. 278.

of rolling, vehicles supported on straight and parallel edges, sliding over the surface of the ground, are often found more convenient than wheel carriages. In climates where snow and frost prevail during the winter, sledges (*fig.* 278), supersede all other carriages. In New York and other cities of North America public vehicles, such as omibuses, hackney coaches, and all other conveyances, are taken off the wheels and placed upon sledges, on which they are drawn along the streets and roads.

At all seasons and in all climates sledges are, for certain purposes, more convenient than wheel carriages; thus, draymen are provided with small ones (*fig.* 279), to take barrels into

narrow streets, where the approach of the dray would be difficult or impracticable.

284. **Use of rollers.**—When heavy weights, such as large blocks of stone, are required to be moved through short dis-

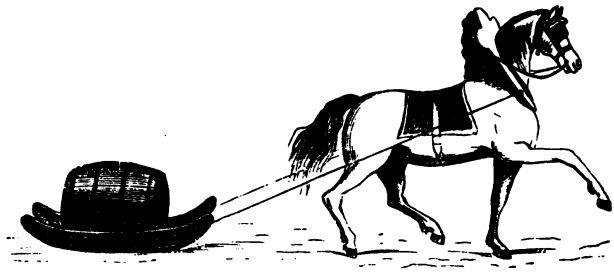

Fig. 279.

tances, the application of rollers is attended with great advantages; but when loads are to be transported to considerable distances, the process is inconvenient and slow, owing to the

Fig. 280.

necessity of continually replacing the rollers in front of the load, as they are left behind by each progressive advancement (*fig.* 280).

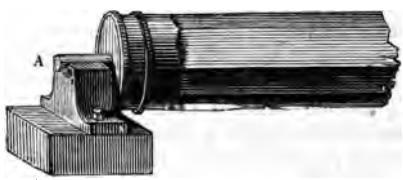

Fig. 281.

The wheels of carriages may be regarded as rollers which are continually carried forward with the load. In addition to

the friction of the rolling motion on the road, they have, it is true, the friction of the axle in the nave; but, on the other hand, they are free from the friction of the rollers with the under surface of the load or the carriage in which the load is transported. The advantage of wheel carriages in diminishing the effects of friction is sometimes attributed to the slowness with which the axle A (*fig.* 281) moves within the box, compared with the rate at which the wheel moves over the road; but this is erroneous. The quantity of friction does not in any case vary considerably with the velocity of the motion, but least of all does it in that particular kind of motion here considered.

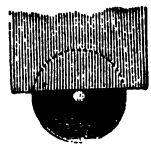

Fig. 282.

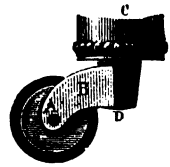

Fig. 283.

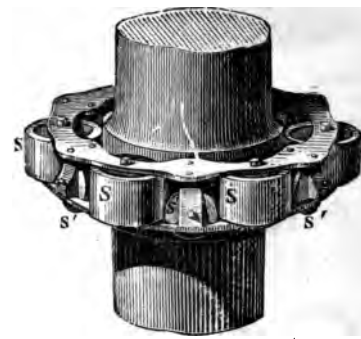

Fig. 284.

Castors (*figs.* 282, 283) placed on the feet of tables and other articles of furniture facilitate their movement from one place to another by substituting rolling for sliding friction.

285. **Friction rollers** (S S, *fig.* 284) are sometimes interposed between an axle and its bearings, to diminish the friction attending their motion one upon the other.

286. **Friction wheels.**—In certain cases where it is of great importance to remove as much as possible the effects of friction, a contrivance called friction wheels or friction rollers is used. The axle of a friction wheel A (*fig.* 285), instead of revolving within a hollow cylinder which is fixed, rests upon the edges of wheels B B, which revolve with it: the mode of contact of the moving parts thus becomes that in which the friction is of least amount.

In carriages the roughness of the road is more easily overcome by large wheels than by small ones; hence we see wheels

of very great magnitude used for carrying beams of timber of extraordinary weight. The animals drawing these, notwithstanding their weight, do not manifest any considerable exertion. The cause of this arises, partly from the carriage-wheels bridging over the cavities in the road, instead of sinking into them, and partly because, in surmounting obstacles, the load is elevated less abruptly.

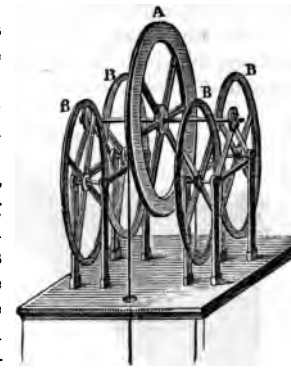

Fig. 285.

287. **Line of draught.**—If a carriage were capable of moving on a road absolutely free from friction, the most advantageous direction in which the tractive force could be applied would be parallel to the road; but when the motion is impeded by friction, as in practice it always is, it is better that the line of draught should be inclined to the road, so that the drawing force may be exerted partly in lessening the pressure on the road, by in some degree elevating the carriage, and partly in advancing the load.

It can be established by mathematical reasoning that the best line of draught, in all cases, is determined by the angle of repose, that is to say, the traces should form an angle with the road equal to the elevation of a plane which would exactly overcome the friction. Hence it appears that the smoother the road, and the more perfect the carriage, and consequently the less the friction, the more nearly parallel to the road the line of draught should be.

In wheel carriages there exists two sources of friction: one which prevails between the tires of the wheels and the road on which they run, the amount of which depends on the quality of the road; and the other which prevails between the axle and the nave of the wheel in which it turns. This latter is sliding friction; but the rubbing surface is small, being the line of contact of the axle with the nave or socket.

From the structure of the axle and the nave, this source of friction admits of being almost indefinitely diminished, by the application of lubricants and other expedients. The other resistance, depending on the action of the tires of the wheels, amounts, on well-paved roads, to about $\frac{1}{70}$th of the load. On

gravelled roads it is $\frac{1}{35}$th; and when a fresh layer of gravel has been laid, it is increased to the $\frac{1}{16}$th of the load. It is found, however, on a well-macadamized road, when in good order, that the resistance does not exceed the $\frac{1}{35}$th or $\frac{1}{40}$th of the load.

288. **Railways.**—The most perfect modern road is the iron railway, by which the resistance due to friction is reduced to an extremely small amount.

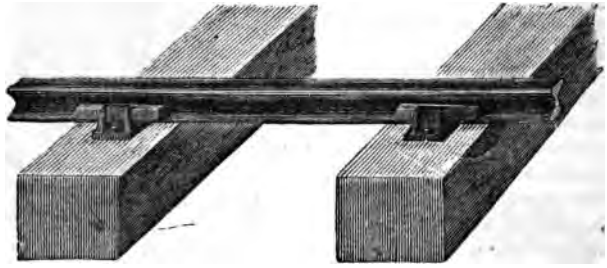

Fig. 286.

The rolling friction of the wheels is in this case diminished by substituting for the surface iron bars called rails, supported upon cross-beams of timber (*fig.* 286), at distances apart corre-

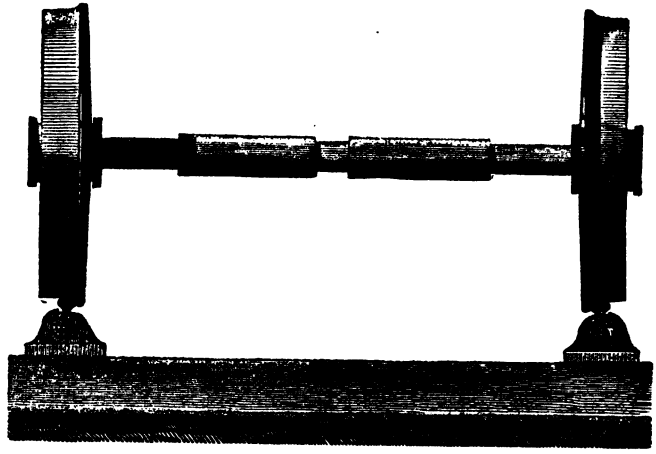

Fig. 287.

sponding to that part of the wheels which are formed with ledges or flanges projecting from their tires. These falling

within the rails (*fig.* 287) confine the vehicle to the rails, the even surfaces of which in contact with the tires produce very little resistance.

Various experiments have been made, with a view to determine this resistance ; but much difficulty arises, owing to the effects of atmospheric resistance being combined with those of friction. Extensive series of experiments made at various times with a view to determine the amount of resistance to railway trains, gave as a general result that this resistance is much more considerable than it had been previously supposed to be ; but that the total resistance to motion depends in a great degree upon the velocity, and probably arises more from the resistance of the air than from friction properly so called.

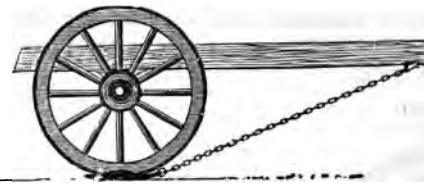

Fig. 288.

289. **Brakes.**—Friction is generally resorted to as the most convenient method of retarding the motion of bodies and bringing them to rest. Expedients of various forms, called *brakes*, have been contrived for this purpose.

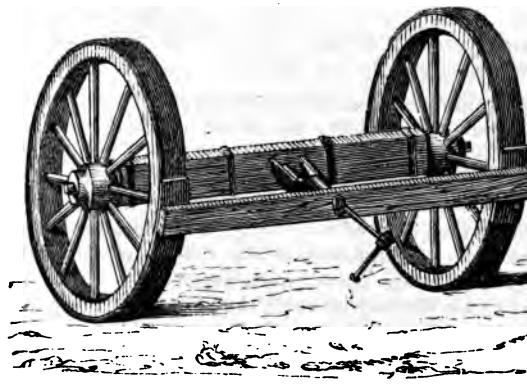

Fig. 289.

The form of brake called a *shoe*, used in travelling carriages, is shown in *fig.* 288.

The brake used in diligences and other heavy vehicles on the Continental roads, which is similar in principle to those used for tram-cars and railway carriages, is shown in *fig.* 289. Surfaces of wood are pressed against the tires of the wheels by a combination of levers pressed by the hand of the guard or conductor.

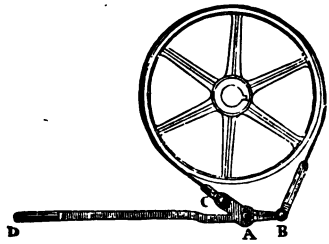

Fig. 290.

A brake used in machinery is shown in *fig.* 290, consisting of a strap tightly drawn upon the tire of the wheel intended to be retarded or stopped.

Friction is sometimes used as a point of resistance, as where a cable is coiled round a post to arrest the progress of a vessel (*fig.* 291).

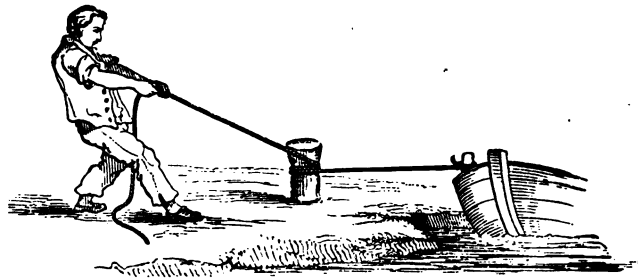
Fig. 291.

290. **Imperfect flexibility of ropes.**—When ropes or cords form a part of machinery the effects of their imperfect flexibility are in a certain degree counteracted by bending them over the grooves of wheels. But although this so far diminishes these effects as to render ropes practically useful, yet still, in calculating the power of machinery, it is necessary to take into account some consequences of the rigidity of cordage, which even by these means are not quite removed.

To explain the way in which the stiffness of a rope modifies the operation of a machine, we shall suppose it bent over a wheel, and stretched by weights A and B, *fig.* 292, at its extremities. The weights A and B, being equal, and acting at C and D in opposite ways, balance the wheel. If the weight A receive an addition, it will overcome the resistance of B, and turn the

wheel in the direction D E C. Now, for the present, let us suppose that the rope is perfectly inflexible, the wheel and weights will be turned into the position represented in *fig.* 293. The leverage by which A acts will be diminished, and will become O F, having been before O C; and the leverage by which B acts will be increased to O G, having been before O D.

But the rope, not being inflexible, will yield to the effect of the weights A and B, and the parts A C and B D will be bent into the forms represented in *fig.* 294. The preponderating weight A still has a less leverage than the weight B, and consequently a proportionate part of the effect of the moving power is lost.

The extent to which the rigidity of cordage affects the motion of machinery has been ascertained by experiment in a still more imperfect manner than the results of friction. Many incidental circumstances vary the conditions, so as to throw

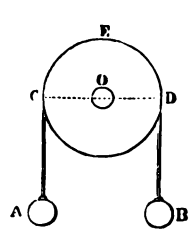

Fig. 292.

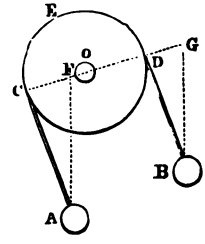

Fig. 293.

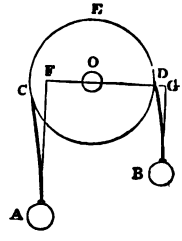

Fig. 294.

great difficulties in the way of such an investigation. Different ropes, and the same ropes at different times, produce extremely different effects, influenced by the circumstances of their dryness or humidity, the quality of their material, the mode in which they are prepared and twisted, &c. These circumstances, it is evident, do not admit of being estimated or expressed with any degree of accuracy. It may, however, be stated generally that the resistance produced by the rigidity of a rope is directly proportional to the weight that acts upon it, and to its thickness. Other things being the same, it is also in the inverse proportion of the diameter of the wheel or axle upon which the rope is coiled; the greater the weights, therefore, which are moved, and the stronger the ropes, the greater will be the resistance proceeding from rigidity; and, on the other hand, the greater the diameter of the wheel or axle on which the rope runs, the less in proportion will be the force necessary to overcome the rigidity.

The resistance from new ropes is greater than from those of the same quality which have been some time in use. Ropes saturated with moisture offer increased resistance on that account.

291. **Resistance of fluids.**—Since all the motions which commonly take place on the surface of the earth are made in the atmosphere or in water, it is of great practical importance to ascertain the laws which govern the resistance offered by these fluids to the motion of bodies passing through them. A body moving through a fluid must displace as it proceeds as much of that fluid as fills the space which it occupies; and in thus imparting motion to the fluid it loses by reaction an equivalent quantity of its momentum.

If a body thus moving were not impelled by a motive force in constant action, it would be gradually deprived of its momentum, and at length brought to rest. Hence it is that all motions which take place on the surface of the earth, and which are not sustained by the constant action of an impelling power, are observed gradually to diminish, and ultimately to cease.

Since the resistance presented by a fluid to the motion of a solid is equivalent to the momentum imparted by the solid to the fluid which it thrusts out of its way in its motion, it follows evidently that, other things being the same, this resistance will be proportional to the density or weight of the fluid. Thus the resistance produced by air is less than that produced by water, in the proportion of the weight of air to the weight of an equal bulk of water.

The resistances are proportioned to the quantity of the fluid which the moving body thrusts from the path; and this again depends upon the form and magnitude of the body, and more especially on its *frontage*.

The resistance which a body encounters in moving through a fluid is greater, therefore, with a broad end foremost, than with a narrow end foremost. A ship would evidently encounter a much greater resistance if it were driven sideways, than if it moved in the direction of the keel. It would also encounter a greater resistance if it moved stern foremost than in the usual direction.

The blade of a sword would be wielded with some difficulty if moved with its flat side in the direction of motion, whereas it is easily flourished when moved edge foremost. Bodies whose foremost ends have the form of a wedge or a point move through a fluid with less resistance than if the pointed ends were cut off, and

they presented a flat surface to the fluid medium, because in their motion they act upon the principle of the wedge, and more easily cleave the fluid. Nature has formed birds and fishes in this manner to facilitate their passage through the air and through the water.

The resistance which a body moving through a fluid encounters increases in a high ratio with its velocity. If the body move with the velocity of one foot per second, it will act in each second upon a column of the fluid whose base is equal to its own transverse section and whose length is one foot, and it will impel such a column with a velocity of one foot per second ; but if the velocity of the moving body be doubled, it will not only drive before it in one second a column of fluid two feet long—that is, double the former length—but it will impel this column with double the former speed.

The resistance, therefore, will be doubled, on account of the double quantity of the fluid set in motion, and again doubled, on account of this quantity receiving double the velocity. The momentum, therefore, imparted by the body to the fluid when the velocity is doubled will be increased in a fourfold proportion.

In the same manner it may be shown that if the velocity of the body be increased in a threefold proportion, it will drive from its path three times as much of the fluid per second, and impart to it three times as great a velocity ; consequently, the momentum which it will give up to the fluid will be nine times that which it imparted moving at the rate of one foot per second.

In general, therefore, it follows that the momentum which the body imparts to the fluid in moving through it, will be increased in proportion to the square of the velocity ; but as the resistance which the body suffers must be equal to the momentum which it imparts to the fluid, it follows that the resistance to a body moving through a fluid will be proportional to the square of its velocity.

292. **Ponderous missiles.**—Hence it follows that missiles lose a less proportion of their momentum in passing through the air, as their weight is increased ; for, according to what has been stated, the resistance which they suffer at a given velocity will be proportional to their transverse section, which in this case is in the ratio of the squares of their diameters ; but as their weight increases as the cubes of their diameters, and their momentum is proportional to their weight, it follows that by increasing their magnitude their momentum is increased in a higher ratio than the resistance they encounter. For example, if two

cannon-balls have diameters in the proportion of 2 to 3, the resistances which they will encounter at the same velocity of projection will be in the ratio of 4 to 9, while their weights will be in the ratio of 8 to 27. The resistance, therefore, presented to the smaller ball will bear to its weight a greater ratio than that encountered by the larger.

293. **Resistance of the air to the motion of falling bodies.**—It has been shown that a body obedient to the action of gravity would descend in a vertical line with a uniformly accelerated motion. Its velocity would increase in proportion to the time of its fall, so that in ten seconds it would acquire ten times the velocity which it acquired in one second; but these conclusions have been obtained on the supposition that no mechanical agent acts upon the body save gravity itself. If, however, the body fall through the atmosphere, which in practice it must always do, it encounters a resistance which augments with the square of the velocity. Now, as the acceleration of gravity remains sensibly constant, while the resistance continually increases, this resistance, if the motion be continued, must at length become equal to the gravitation of the falling body, and, when it does, the velocity of the falling body will cease to increase. It follows, therefore, that when a body falls through the atmosphere, its rate of acceleration is continually diminished; and there is a limit beyond which the velocity of its fall cannot increase, this limit being determined by that velocity at which the resisting force of the air will become equivalent to the gravity of the body.

As the resisting force of the air, other things being the same, increases with the magnitude of the surface presented in the direction of the motion, it is evidently possible so to adapt the shape of the falling body that any required limit may be impressed upon the velocity of its descent. It is upon this principle that parachutes have been constructed.

When a body attached to a parachute is disengaged from a balloon, its descent is at first accelerated, but very soon becomes uniform, and as it approaches the earth, the air becoming more and more dense, the resistance on that account increases, and the fall becomes still more retarded.

The theory of projectiles, which is founded upon the supposition of bodies moving in vacuo, is rendered almost inapplicable in practice, in consequence of the great effect produced by atmospheric resistance to bodies moving with such velocities as those which are generally imparted to missiles. According to experiments and calculations, it has been found that the range

of a 4-lb. cannon-ball, which in vacuo would be 23,226 feet, was reduced to 6,437 feet by the resistance of the air. Hutton showed that a 6-lb. ball, projected with a velocity of 2,000 feet per second, encountered a resistance a hundred times greater than its weight.

CHAPTER XVIII.

STRENGTH OF MATERIALS.

294. **Strength of solid bodies.**—The solid materials of which structures, natural and artificial, are composed, are endued with certain powers, in virtue of which they are capable of resisting forces applied to bend or break them. These powers constitute an important class of resisting forces, and are technically called in mechanics the strength of materials.

Experimental inquiries into the conditions which determine the strength of solid bodies, and their power to resist forces tending to tear, break, or bend them, are obstructed by practical difficulties, the nature and extent of which arise partly from the great force which must be employed in such experiments, but more from the variety of individual physical properties of the bodies upon which such experiments are made.

The object of such an inquiry must necessarily be the establishment of a general law, or such a rule as would be strictly observed if the materials were perfectly uniform in their texture, and subject to no casual inequalities. In proportion, however, as such inequalities are frequent, experiments must be multiplied, so that they shall include cases varying in both extremes, so that the peculiar effects of each may be effaced from the general average result which shall be obtained. These inequalities of texture, however, are so great, that even when a general law has been established by a sufficiently extensive series of experiments, it can only be regarded as a mean result from which individual examples will be found to depart in so great a degree that the greatest caution must be observed in its practical application.

Although the details of this subject belong more properly to engineering than to an elementary treatise like the present, it will nevertheless be a useful illustration of the principles already explained, to give a general view of the most important laws which have been established.

A mass of solid matter may be submitted to the action of a force tending to separate its parts in several ways, of which the principal are :—

I. A direct pull; as when a weight is suspended to a wire or a rope, or when a tie-beam resists the separation of the walls of a structure.

II. A direct pressure or thrust; as when a weight rests upon a pillar, or a roof upon walls.

III. When a force is applied to twist or wrench a body asunder by turning a part of it round a point, or a series of points, within it.

IV. A transverse strain ; as when a beam, being supported at its centre, weights are suspended from its ends, or, being supported at its ends, a weight is suspended from its centre.

When a rod, rope, or wire is extended between forces applied to its ends, and tending directly to stretch it, its strength to resist such force is, other things being the same, in proportion to the magnitude of its section.

Thus, suppose an iron wire stretched by a weight which it is just able to support without breaking. It is evident that a wire having twice the quantity of iron of the same quality in its thickness would support double the weight, because such wire would be in effect equivalent to two wires like the former combined. In the same manner, a wire having three times the quantity of iron of the same quality in its thickness, being equivalent to three wires like the first, would support three times the weight, and so on. Thus the power of bodies to resist a direct pull will be in general in proportion to the area of their transverse section. In practice it is found that when the length is much increased, the strength to resist a direct pull is diminished.

This departure, however, from the general law is explained by the increased probability of casual defects of structure in the increased length ; and, subject to such qualification, it may be stated generally, that the strength of a body to resist tension, or a direct pull drawing from end to end, is in the direct ratio of the area of its section made at right angles to the direction in which it is stretched.

The strength of bodies to resist a direct pull is experimentally estimated by attaching the upper extremity securely to a point of support, and suspending weights to the lower extremity, which are increased gradually until the body under ex-

periment is broken. The weight which breaks it is taken as the expression of its strength. The bodies which have been subjected to experiments of this kind are chiefly metals, woods, and ropes, these being most generally used in structures, in which their capacity for resisting tension is of great importance, as in suspension bridges, iron roofs, cables, cordage, &c.

In the following table are collected the results of the most recent and extensive experiments on this subject. The bodies subjected to experiment are supposed to be in the form of long rods, the cross-section of which measures a square inch. In the second column is given the amount of the breaking weights, which are the measure of their strength. This table is selected from various recent works.

Table showing the strength with which prisms of the undermentioned substances resist a direct pull, expressed in lbs. per square inch of the area of their transverse section.

1. *Metals*:—

Name.		lbs.		lbs.
Steel, untempered	from	110,690	to	127,094
,, tempered	,,	114,794	,,	153,741
,, cast	,,	134,256		
Iron, bar	,,	53,182	,,	84,611
,, plate, rolled	,,	53,920		
,, wire	,,	58,730	,,	112,905
,, Swedish malleable	,,	72,064		
,, English ,,	,,	55,872		
,, cast	,,	16,243	,,	19,464
Silver, cast	,,	40,997		
Copper ,,	,,	20,320	,,	37,380
,, hammered	,,	37,770	,,	39,968
Brass, cast	,,	17,947	,,	19,472
,, wire	,,	47,114	,,	58,931
,, plate	,,	52,240		
Gold	,,	20,490	,,	65,237
Tin	,,	3,228	,,	6,666
,, cast	,,	4,736		
Bismuth, cast	,,	3,137		
Zinc	,,	2,820		
Antimony, cast	,,	1,062		
Lead, molten	,,	887	,,	1,824
,, wire	,,	2,543	,,	3,823

2. *Woods* :—

Name.		lbs.		lbs.
Teak	from	12,915	to	15,405
Sycamore	,,	9,630		
Beech	,,	12,225		
Elm	,,	9,720	,,	15,040
Memel fir	,,	9,540		
Christiania deal	,,	12,346		
Larch	,,	12,240		
Oak	,,	10,367	,,	25,851
Alder	,,	11,453	,,	21,730
Lime	,,	6,991	,,	20,796
Box	,,	14,210	,,	24,043
Pinus sylvestris	,,	17,056	,,	20,395
Ash	,,	13,480	,,	23,455
Pine	,,	10,038	,,	14,965
Fir	,,	6,991	,,	12,876

3. *Cords* :—

Hemp, twisted	
¼ to 1 inch thick	8,746
1 ,, 3 ,,	6,800
3 ,, 5 ,,	5,345
5 ,, 7 ,,	4,860

From this table it appears that the strongest of the bodies for resisting tension is iron, and that the strongest condition of iron is that of tempered steel. In general, metals when cast are less strong than when hammered. Thus, cast iron has not one third of the tenacity of wrought iron. It is also found that metals which are composed of two or more alloyed together are often stronger than any of their components. Thus, brass wire, which is composed of zinc and copper, has greater tenacity than copper wire, although the tenacity of zinc, as appears by the table, is extremely small.

It is also found that the strength of metals is affected by their temperature, being diminished in general as their temperature is raised. Sudden, frequent, and extreme changes of temperature impair tenacity.

The woods are subject to extreme variations, produced in general by the great inequalities which are incidental to them. Thus, the strength of oak varies, as appears by the table, between the limits of 10,000 and 25,000 lbs. It is found that

trees which grow in mountainous places have greater strength than those which grow on plains, and also that different parts of the same tree, such as the root, trunk, and branches, vary in strength within wide limits.

It is found that cords of equal thickness are strong in proportion to the fineness of their strands, and also to the fineness of the fibres of which these strands are composed. It is found also that their strength is diminished by being overtwisted. Ropes which are damp are stronger than ropes which are dry, those which are tarred than the untarred, the twisted than the spun, and the unbleached than the bleached. Other things being the same, silk ropes are three times stronger than those composed of flax.

The strength of many substances is increased by compressing them; this is the case with leather and paper, for example.

Animal and vegetable substances which, being originally liquid, are rendered solid by evaporation, change of temperature, or exposure, often possess extraordinary strength. Examples of this are presented in the gums, glue, varnish, &c. Count Rumford found that a copper plate having a thickness of $\frac{1}{20}$th of an inch, rolled into the form of a cylinder, had its strength doubled when coated with well-sized paper of double its own thickness; and that a cylinder composed of sheets of paper glued together, having a sectional area of one square inch, was capable of supporting a weight of 15 tons for every square inch in its sectional area; and, in fine, that a cylinder composed of hempen fibres glued together had a strength greater than that of the best iron, being capable of supporting 46 tons per square inch of sectional area.

295. **Strength to resist pressure.**—According to the theory of Euler the strength of a column composed of any material and of any prismatic form to resist the crushing force of a weight placed upon it, increases as the square of the number expressing its sectional area divided by the square of the number expressing its height. This law, which is of great practical importance, on account of its application to architectural purposes, is found to be in very near accordance, within practical limits, with the experiments of Musschenbrock and others; and more recently with the still more extensive experiments of Mr. Eaton Hodgkinson made on pillars of wrought iron and timber.

It is necessary to observe here that when the height is reduced to a very small magnitude, the pillar is more easily crushed than would be indicated by this law. Exceptions are

also presented in the case of pillars formed of particular materials; such, for example, as cast iron, which Mr. Hodgkinson found to vary in rather a higher proportion in reference both to the sectional area and to the height.

According to Eytelwein's experiments the strength of rectangular columns to resist compression is directly as the product of the larger side of their section multiplied by the cube of its shorter side, and inversely as the square of their height. This will coincide with the rule of Euler when the pillar is square.

Eytelwein gives the following table of the weights *necessary to crush pillars composed of the materials expressed in the first column, the numbers expressed in the second column being the total crushing weights in lbs. per square inch* :—

1. *Metals* :—

Name.		lbs.	lbs.
Cast iron	from	115,813 to	177,776
Brass, fine	,,	164,864	
Copper, molten	,,	117,088	
,, hammered	,,	103,040	
Tin, molten	,,	15,456	
Lead, molten	,,	7,728	

2. *Woods* :—

Oak	,,	3,860 to	5,147
Pine	,,	1,928	
Pinus sylv.	,,	1,606	
Elm	,,	1,284	

3. *Stones* :—

Gneiss	,,	4,970
Sandstone, Rothenburg	,,	2,556
Brick, well baked	,,	1,092

Mr. Hodgkinson gives the following results of his experiments as to the strength of timber pillars :—

Description of wood	Strength per square inch in lbs.	
	Moderately dry	After subjected to drying process
Alder	6,831	6,960
Ash	8,683	9,363
Bay	7,518	7,518
Beech	7,733	9,363
English birch	3,297	6,402
Cedar	5,674	5,863
Red deal	5,748	6,586
White deal	6,781	7,293
Elder	7,451	9,973
Elm	—	10,331
Fir (spruce)	6,499	6,819
Mahogany	8,198	8,198
Oak, Quebec	4,231	5,982
,, English	6,484	10,058
Pine, pitch	6,790	6,790
,, red	5,395	7,518
Poplar	3,107	5,124
Plum, dry	8,241	10,493
Teak	—	12,101
Walnut	6,063	7,227
Willow	2,898	6,128

The numbers in the first column are the number of lbs. per square inch, deduced from experiments made on cylinders one inch in diameter and two inches in height, with flat sides, the wood being moderately dry. The second column gives the strength of similar pillars of the same woods, which had been subjected to a drying process in a warm place for two months and upwards. The great difference in the strength of the two cases shows in a striking manner the effect of dryness upon the strength of timber.

296. **Strength to resist torsion.**—Neither theory nor experiment has thrown much light on the laws which govern this mode of resistance to the separation of the constituent parts of bodies. The following results, showing the comparative strength of various metals to resist torsion, were obtained from a series of experiments made by Mr. Rennie :—

Table showing the comparative strength of various metals to resist torsion.

Lead	1,000
Tin	1,438
Copper	4,312
Brass	4,688
Gun-metal	5,000
Swedish iron	9,500
English iron	10,125
Cast iron	10,600
Blister steel	16,188
Shear steel	17,063
Cast steel	19,562

It is stated by Mr. Bankes that a square bar of cast iron which would measure an inch is wrenched asunder by an average force of 631 lbs. applied at the extremity of a lever two feet long.

297. **Strength to resist transverse strain.**—Bodies submitted to a transverse strain are usually considered as having the form of prismatic beams, with their lengths at right angles to the direction in which the strain is applied. A prismatic beam is one all whose transverse sections are equal and similar figures, and the character of the beam is determined by the figure of this section. Thus, a cylindrical beam is one whose

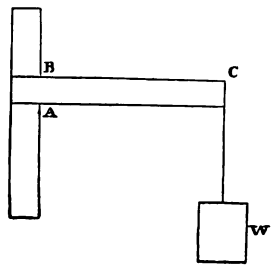

Fig. 295.

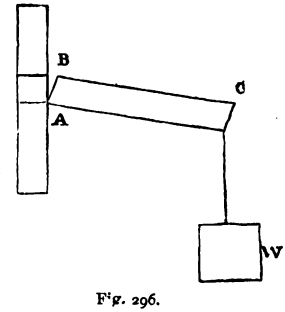

Fig. 296.

transverse section is a circle; a rectangular, one whose transverse section is a right-angled parallelogram; a square, whose transverse section is a square; and so on. The force producing the strain may be resisted by one or by two points of support, in the latter case the weight being placed between these two points.

In the first case, let B C, *fig.* 295, be a prismatic beam, fixed in a wall, or other vertical means of support, at B A, and let a weight W be supported from its end C. We shall, for the present omit the consideration of the weight of the beam itself.

The effect of the weight W produced at B A will be to turn the beam round the point A, as if it were a hinge, as represented in *fig.* 296, and thus to tear asunder the fibres which unite the parts of the body forming its transverse section at B A. Let f be the force corresponding to the unit of surface of this section, let A be the area of the section; then $f \times A$ will express the total force of the body over the whole surface of the section. But if the beam tends to turn round the point A as a fulcrum, this force acts by a leverage at each point corresponding to the distance of such point from A. The total force expressed by $f \times A$ may therefore be considered as having an average leverage, determined by the various distances of all the parts in the section from a horizontal line passing through A.

Now, the point determined by these conditions is the *centre of gravity* of the section of the beam at A B. Let the distance of this centre of gravity from the horizontal line passing through the point A be c; we shall then have the effect of the forces of cohesion by which the body is united in the surface of the section expressed by $f \times A \times c$. Against this the weight W acts with the leverage C B, that is to say, the length of the beam.

Let this length be expressed by L, and we shall have, according to the properties of the lever,

$$W \times L = f \times A \times c,$$

and therefore

$$W = f \times \frac{A \times c}{L}.$$

It appears from this, therefore, that the weight necessary to fracture a beam placed in this manner will be in the direct ratio of the area of the transverse section, multiplied by the height of the centre of gravity above the lowest point of this section, and divided by the length of the beam.

For beams of the same length their strength is proportional to the area of their section, multiplied by the distance of the centre of gravity above the lowest point; and for beams of the same section their strength is inversely as their length.

By the length of the beam in this case is to be understood the distance of the part of the beam at which the weight is suspended from the point of support.

Let us now consider the case in which the body is supported at two points, the breaking weight being placed at some inter-

mediate point. Let such a beam be represented in *fig.* 297, supported at the points D and C, the weight being placed at an intermediate point A. In this case the tendency of the weight is to rupture the fibres in the vertical section of the beam passing

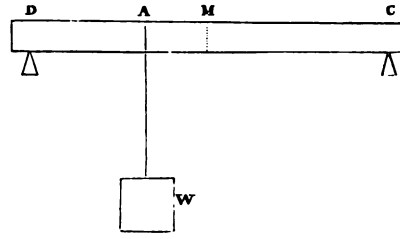

Fig. 297.

through A, and in doing so the beam would break as if a hinge were placed at A, on the upper surface of the section supporting the weight, as represented in *fig.* 298.

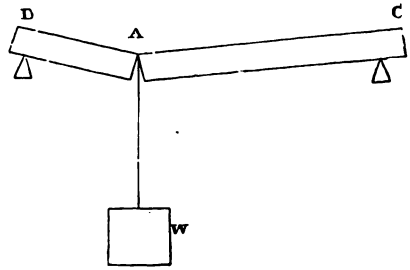

Fig. 298.

The fracture may in this case be considered to be produced by the reaction of the points of support D and C.

To determine the effects of this reaction, we are to consider that, by the properties of the lever, the part of the weight which acts at C is expressed by

$$W \times \frac{D\,A}{C\,D},$$

and the part of the weight which acts at D is expressed by

$$W \times \frac{C\,A}{D\,C}.$$

But the former of these acts upon the section at A with the leverage C A, and the latter with the leverage D A. If each be therefore multiplied by its respective leverage, we find that the effect of each of these actions in producing the fracture at A will be expressed by

$$W \times \frac{D A \times C A}{C D};$$

the total effect, therefore, will be

$$2 W \times \frac{D A \times C A}{C D}.$$

Against this force the strength of the beam acts, and the strength of the beam at the section A is determined and expressed in exactly the same manner as in the former case, except that, in the present case, the average leverage by which the strength of the fibres resists rupture is the distance of the centre of gravity below the highest point A of the section. Expressing this distance as before by c, we shall have the condition of equilibrium at the moment of fracture expressed by

$$2 W \times \frac{D A \times C A}{C D} = f \times A \times c.$$

This may be simplified in the following manner:—

Let M, *fig.* 297, be the middle point of the beam between the two points of support, and let a express D M, half its length; let x express M A, the distance of the point where the breaking weight is placed from the middle point. We shall then have, by well-known principles of geometry,

$$D A \times C A = a^2 - x^2.$$

Hence we shall have

$$2 W \times \frac{a^2 - x^2}{2 a} = f \times A \times c,$$

and consequently we shall have

$$W = \frac{f \times A \times c \times a}{a^2 - x^2}.$$

Hence it follows that if the weight be placed at the middle point between the two points of support, we shall have $x = 0$; and consequently,

$$W = \frac{f \times A \times c}{a}.$$

It appears from this that the weight placed at the centre of a beam, between two points of support necessary to break it, is double that which would be sufficient to break the same beam if

it were supported only at one point, the weight being placed at the other point.

Such are the general practical principles for determining the transverse strength of beams as established by Galileo; and although other practical formulæ have been proposed by later writers, the above have been found in such near accordance with the average results of experiments made upon a large scale, that they have been generally adopted by mechanical writers as the basis of their investigations upon the strength of materials.

Let us now see how the preceding formulæ are modified for beams of particular forms.

If the beam be rectangular, let its breadth be b, its depth d, and its length $2\,a$. The distance of the centre of gravity of its section, therefore, from its upper or its lower surface will be $\frac{1}{2}\,d$. Now, the area of its section will be $b \times d$; hence we have

$$\begin{aligned} \text{A} &= b \times d, \\ c &= \tfrac{1}{2}\,d, \\ \text{L} &= 2\,a; \end{aligned}$$

and consequently we have, if the beam be supported at one end only,

$$\text{W} = f \times \frac{b \times d^2}{4\,a}.$$

In like manner, if the beam be supported at both ends, the weight being placed at the middle, we shall have

$$\text{W} = f \times \frac{b \times d^2}{2\,a}.$$

If the beam be square, its breadth will be equal to its depth, and the breaking weight, where there is but one point of support, will be

$$\text{W} = f \times \frac{b^3}{4\,a};$$

and when there are two points of support,

$$\text{W} = f \times \frac{b^3}{2\,a}.$$

From the general principles here established it is evident that, so long as the quantity of matter composing the beam, and therefore its sectional area, remains the same, its strength will be augmented by any modification of form which will carry its centre of gravity to a greater distance from its lower surface, if the beam have but one, and from its upper surface, if it have

two points of support. Some curious and instructive consequences ensue from this.

Thus, the strength of a rectangular beam, when its narrow side is horizontal, is greater than when its broad side is horizontal, in the same proportion as the width of its broad side is greater than the width of its narrow side. Hence, in all parts of structures where beams are subject to transverse strain, as in the rafters of floors, roofs, &c., they are placed with their narrow sides horizontal, and their broad sides vertical.

If a beam, supported at two points, bear a strain at any intermediate point, a given quantity of matter composing it will have greater strength if it be so formed that its area shall be less in the upper part than in the lower.

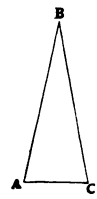

Fig. 299.

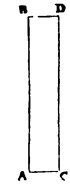

Fig. 300.

Thus, a beam of triangular section, such as A B C (*fig.* 299), will be stronger than a rectangular beam of equal section, such as A B C D (*fig.* 300), because the centre of gravity of the triangle with its vertex upwards will be further from B than that of the parallelogram from B D.

But if the beam be supported at one point only, then the position of the triangle must be inverted.

If a solid and a hollow cylinder of equal lengths have the same quantity of matter, so that their sectional areas shall be equal, then their strength will be proportional to the distances of their centres of gravity from their external surface. But their centres of gravity being at their geometrical centres, it follows that the strength of the solid cylinder will be less than the strength of the hollow cylinder, in the ratio of the diameter of the solid cylinder to the diameter of the external surface of the hollow cylinder.

It appears, therefore, that the strength of a tube is always greater than the strength of the same quantity of matter made into a solid rod; the practical limitation of the application of this principle being, that the thinness of the tube should not

be so great as to cause a local derangement of its form by the application of a strong force.

Innumerable striking and beautiful examples of this principle occur in the organised world. The bones of animals of every species are hollow cylinders, thereby combining strength with lightness. The stalks of numerous species of vegetables which have to bear a weight at their upper end are also tubes, whose lightness is remarkable when their strength is considered. These are intended to resist not only the crushing weight of the ear which they bear at their summit, but also the lateral strain produced by the movement of the air.

The quills and the plumage of birds, and especially of their wings, present a still more striking example of the application of this principle in the animal structure. The surface of the extended wing acting on the air produces a strong transverse strain upon the quill, which has a single point of support near the joint of the wing.

The number expressed by f in the preceding formulæ is the strength of a beam, whose sectional area is the square unit, which in this case is generally taken as the square inch. This number is always the same for the same species of material, but different for different materials. In tabulating the strength of materials obtained from experiment, this is accordingly the number which is taken to designate the strength of each species of substance. When this number f is known, the strength of a beam of any proposed dimensions can be immediately calculated by the formulæ; for it is only necessary to multiply this number f by the number of square inches in the sectional area, and the number of inches in the distance of the centre of gravity of this area from the upper or lower surface, as the case may be, and to divide the product by the length of the beam, and the result of this arithmetical process will give the breaking weight.

It must be understood that this weight, and the force expressed by the number f, are to be expressed in the same unit.

In ascertaining by experiment the average strength of each material, the number which it is important to determine is, therefore, that which is here expressed by f, and which may be considered the unit of strength; because, this being once determined, the strength of a beam of the proposed materials of any proposed denomination can be immediately computed.

This quantity f can be determined by direct experiment. If the amount of the breaking weight upon a beam of given

dimensions be ascertained, we shall know the numbers severally expressed by w, b, d and a in the preceding formulæ; and in this case we shall have

$$f = w \times \frac{2\,a}{b \times d^2}.$$

In practice it will be found that the values obtained for the number f will be subject to considerable variations in different experiments, owing to casual inequalities and defects which affect each particular beam submitted to experiment.

The value of f must, therefore, for each material, be obtained by taking an average of the results of numerous experiments, the casual inequalities being therein made to disappear from the result in proportion to the number and variety of trials.

In the following table is given the results of a series of experiments made by Tredgold upon the transverse strength of prismatic beams. The numbers in the second column represent the values of f in the preceding formulæ, while the numbers in the third column express the number of lbs. weight in a cubic foot of the material under experiment:—

Name	f*	lbs. weight in cubic foot
1. *Metals*:—		
Malleable iron	17,800	475
Hammered iron	—	487
Cast iron	15,300	450
Brass	6,700	586·25
Zinc	5,700	439·25
Tin	2,880	455·7
Lead	1,500	709·5
2. *Woods*:—		
Oak	3,960	52
Fir (red or yellow)	4,290	34·8
Pine (American yellow)	3,900	26·75
Fir (white)	3,630	29·3
Ash	3,540	47·5
Elm	2,340	34

* The numbers given by Tredgold are the values of

$$w \times \frac{L \cdot}{4\,B \times D^2}.$$

We have obtained the numbers given above by multiplying this by 4.

The late Professor Barlow has given a table of the strength of beams to resist transverse strain, from which the following is extracted. The numbers in the second column in this case represent the same number f as those in the second column of the preceding table:—

	$f=$
Teak	9,848
Poon	8,884
English oak, 1st specimen	4,724
,, 2nd ,,	6,688
Canadian oak	7,064
Ash	8,104
Beech	6,224
Elm	4,052
Pitch pine	6,528
New England fir	4,408
Riga fir	4,432
Mar Forest fir	5,048
Larch	4,596
Norway spar	5,896

If the weight producing the strain upon a beam, instead of being concentrated at a single point, as supposed in the cases here investigated, be equally distributed over the whole beam, the power of suspension becomes twice as great as if it were applied at the middle point of the beam.

It has been also shown that each point of the beam has a greater supporting power the nearer it is to either point of support. It is evidently, therefore, advantageous in all structures, in the distribution of the weight upon them, to throw a less quantity at the centre, and to increase the quantity towards the points of support. In this manner of loading, a far greater weight may be placed near the walls than at the centre. In all cases, however, a concentration of weight at a single point is to be avoided. A sheet of ice which would break under the weight of a skater would sustain the same skater if his weight were equally distributed over its surface. In allowing for the weight of the beam itself, this weight may be considered as uniformly spread over its surface.

According to Peschel, the transverse strength of a beam of timber may be greatly increased by sawing down from one-third to one-half of its depth, and driving in a wedge of metal or hard wood until the beam is forced at the middle out of the horizontal line, so as to form an angle presented upwards. It was

found by experiment that the transverse strength of a beam thus cut to one third of its depth was increased one nineteenth; when cut to one half of its depth, it was increased one twenty-ninth; and when cut to three fourths of its depth, it was increased one eighty-seventh.

It follows from the principles which have been explained, that if any structure be increased in magnitude, the proportion of its dimensions being preserved, the strength will be augmented as the squares of the ratio in which it is increased. Thus, if its dimensions be increased in a twofold proportion, its strength will be increased in a fourfold proportion; if they be increased in a threefold proportion, its strength will be increased in a ninefold proportion, and so on. But it is to be considered that, by increasing its strength in a twofold proportion, its volume, and consequently its weight, will be increased in an eightfold proportion, its volume and weight will be increased twenty-seven times, and so on. Thus it is apparent that the weight increases in a vastly more rapid proportion than the strength, and that consequently, in such increase of dimensions, a limit would speedily be attained at which the weight would become equal to the strength, and beyond this limit the structure would be crushed under its own weight. On the other hand, the more below this limit the dimensions of the structure are kept, the greater will be the proportion by which the strength will exceed the weight.

The strength of a structure of any kind is therefore not to be determined by its model, which will always be much stronger relatively to its size. All works, natural and artificial, have limits of magnitude which, while their materials remain the same, cannot be exceeded. Small animals are stronger in proportion than larger ones. We find insects and animalcula capable of bodily activity exceeding almost in an infinite degree the agility and muscular exertion manifested by the larger class. The young plant has more available strength in proportion than the forest tree.

An admirable instance of beneficence in the consequences of this principle is, that children, who are so much more exposed to accidents, are less liable to injury from them than grown persons.

CHAPTER XIX.

MOVING POWERS.

298. **Mechanical agents.**—The natural forces used for the production of mechanical effects are very various. Those applied to move machines, sometimes called *prime movers*, are animal power, water, wind, and steam. Besides these, however, there are several others which have been applied on a more limited scale to particular purposes, or under special circumstances. Some of these would be altogether inapplicable as movers of machinery, because their application and action cannot be rendered sufficiently constant and uniform. Others would be applicable; but their use has been hitherto excluded because of the expense which attends their production.

299. **The dynamical unit.**—Since all moving powers are applied to overcome resistances, and all resistances can be represented by equivalent weights, while the spaces through which they are urged can be represented by corresponding heights, the effect produced by a moving power is always expressed by a certain weight raised a certain height. Thus, for example, if a horse draw a loaded waggon with a force by which the traces are stretched with the same force as if 200 lbs. were suspended vertically from them, and if the horse thus acting draw the waggon over a space of 100 feet, the mechanical effect produced is said to be 200 lbs. raised 100 feet; or, what is the same, 20,000 lbs. raised 1 foot.

In the expression, therefore, of mechanical effects, it must be understood that the dynamical unit adopted is 1 lb. raised 1 foot, and that the effect produced by any moving power or mechanical agent is expressed by an equivalent number of such units. We shall, however, return to this unit in the next chapter.

The great convenience attending this conventional mode of expression is, that the mechanical effects produced in different times, places, and circumstances by different mechanical agents can be immediately compared one with another, and their numerical ratio ascertained.

300. **Machines and tools.**—It happens rarely that a natural mechanical agent is applied immediately to the object on which it is intended to act. In most cases some instrument or expedient is interposed which, when its form is simple and its magnitude small, is called a *tool*, and when greater and more

complex, a *machine*. The line of distinction, however, between tools and machines is not clearly defined; for we sometimes find large and complicated machines, such, for example, as those used for planing, punching, and boring metal, called tools.

The expedient by which an agent acts upon an object, not immediately, but by the intervention of a tool, is a mark of intelligence which has been considered by some as the most conspicuous distinction between man and the lower animals. Many of these display unmistakeable evidence of skill, sagacity, and reasoning under circumstances which will not allow of the supposition of mere instinct; but we believe that no example can be found among the lower animals of the production of any effect upon objects external to them otherwise than by the immediate use of those members or organs with which Nature has furnished them, but never by the intervention of anything analogous to a tool.

301. **Animal power.**—The strength of animals may be employed as a moving power in various ways: the animal may remain without change of position, working by the action of its members, as when a man works at a windlass; or it may advance progressively, transferring its own body, and carrying, drawing, or pushing a load. The mechanical effect produced by such a power is subject to extreme variation, according to the various conditions under which it is exerted; and in an economical point of view it is therefore of extreme importance to determine, with respect to each animal, the circumstances and conditions under which the greatest amount of useful effect can be obtained.

In considering this question, two extreme cases present themselves in which the useful effect altogether vanishes, and between which it continually varies, increasing gradually from nothing to a maximum quantity, and then decreasing again to nothing.

There is a certain amount of load or resistance which requires the entire strength of the animal to sustain, no force remaining for producing motion. Thus, we can conceive a load which a man or horse is just able to support, but unable to move; in his case, therefore, the useful effect is nothing; in other words, no work is done.

We can conceive, on the other hand, a speed of motion so great that with it the animal is unable to carry or draw any load, however small. At all intermediate speeds, however, some load more or less can be carried or drawn, the amount of such load being obviously greater as the speed is less. A horse

can draw a greater load at three miles an hour than at ten miles an hour. The useful effect of the agent being, however, measured by multiplying the load by the speed, it is evident that if either the load or speed be unduly diminished, the useful effect will consequently be diminished, and there is a certain relation or proportion between the load and speed which will give a maximum effect. This relation is not only different with different animals, but varies even with the same animal, according to the different ways in which it acts.

If the force of a man be applied, for example, to the mere elevation of a weight, without any reference to the continuance of its action, it is found that the most advantageous mode of exerting the force is when he endeavours to raise a weight placed between his legs. The greatest weight which can be raised in this case varies from 400 lbs. to 600 lbs., according to the strength of the individual: its average amount, however, is much less, not exceeding 250 lbs.

The force which can be exerted by human strength varies, as has been already observed, extremely, according to the way in which it is applied. Thus, if men work at a pile-engine, such as represented in *fig.* 314, it is found that each can pull with a force sufficient to raise about 144 lbs. weight of the ram 1 foot high, and that working thus they can make about 20 strokes per minute, which, being continued for 4 or 5 minutes, they require to recruit their strength by an interval of repose.

When men work at a capstan, it is found that each can exercise a force of about 28 lbs. on the extremity of the lever which he pushes, and that with this force they can move at the rate of about 10 feet per minute.

When a man works at a winch, of which the arm is from 12 to 14 inches from the centre on which it revolves, it is found that he can exercise a force of about 16 lbs. on the handle, and can make from 20 to 25 turns per minute.

In general, however, human labour produces, in the long run, the greatest effect when it is exercised with frequent intervals of rest; and accordingly the greatest effect is produced when an animal ascends, raising nothing but its own weight, and produces the mechanical effect by that weight itself in descending, so that the animal actually reposes in the interval, during which the mechanical effect is produced.

Thus, for example, if two baskets be connected by a rope which passes over a pulley at the level to which heavy matter is to be raised, the length of the rope being such that when one of the baskets is brought up to that level the other shall have

descended to the level from which the heavy matter is to be elevated, the lower basket being charged, let one or more men get into the upper basket, so as to give it, by their weight, a sufficient preponderance to make it descend, drawing up the lower basket with its load. When the latter has been brought up and discharged at the upper level, the men leave the lower basket and charge it, when men at the upper level get into the upper basket, and again bring up the lower basket as before. Meanwhile, the men who had previously gone down again ascend to the upper level by a ladder or staircase. In this manner the work may be continued without any intermission, the labour of the men consisting exclusively in elevating the weights of their own bodies while ascending the ladder or staircase.

Fig. 301.

It is found by experiments, conducted upon a large scale, that men working in this manner for eight hours a day can produce in each day an effect equivalent to 2,000,000 lbs. raised 1 foot.

The great advantage obtained by this mode of applying animal force will be apparent, when it is stated that the man who can thus produce an effect of two millions of

pounds could not produce, in working at a windlass, a greater effect than a million and a quarter: while, if he acted upon a pile-engine, such as that represented in *fig.* 314, he would produce an effect of only three-quarters of a million.

An apparatus such as that here described is represented in *fig.* 301. It was employed in making the earth-works at Vincennes, near Paris, and was found to be attended with great economy.

Analogous, though not quite identical with this mode of action, are the tread-mill and the ladder-wheel which are represented in *figs.* 172 and 174. In these cases the labour is expended upon the continual elevation of the body, which, however, descends as fast as it is elevated. The daily effect produced by men of the average strength working in this way for eight hours is found to be equivalent to 1,875,000 lbs. raised 1 foot.

Fig. 302.

Thus it appears that this method of employing human force is a little less advantageous than the method of ascent and descent, described above, which produces a daily effect of 2,000,000.

The comparative disadvantage, slight as it is, is explained by the fact that in the one case the labour is intermitting, frequent intervals of repose alternating with those of labour, the most advantageous conditions under which animal force can be employed; while in the other case the labour is continuous.

Spade labour (*fig.* 302) is one of the most disadvantageous forms in which human force can be applied. The spade or shovel is a lever of the first or third kind, according as the one hand or the other applied to its handle is regarded as the fulcrum, the other

being the power, and the earth taken up upon it being the weight. And since that part of the handle included between the two hands is always less than that between the lower hand and the earth, the force exerted is always greater than the weight of the earth raised. Another mode of applying labour of men is shown in *fig.* 303; it is somewhat more advantageous than the tread-mill, and considerably more so than spade-labour.

In all extensive works, such, for example, as the formation of embankments on railways, the labour is executed by contract, the quantity of work being determined by the number of cubic yards of earth excavated and raised to a given height, or transported to a given distance. The number of cubic yards is computed by multiplying the superficial extent of the excavation, measured horizontally, by its average depth.

In the formation of railways, the surveying engineer endeavours, as far as possible, to render the quantity of earth-work in the cuttings, by which the road is carried through elevations, equal to the quantity necessary to form the embankments by which it is carried across the adjacent and intermediate valleys. In this case the quantity of labour is computed by taking the average distance between all the points of the cutting and all the points of the embankment, and assuming that the entire quantity of earth excavated in the cutting must be transported over this distance.

It rarely happens, however, that this exact equilibrium is attainable between the cuttings and embankments. If the earth of the cutting be in excess, the surplus is deposited in the nearest convenient place, forming what is called a *spoil-bank*; and if the earth of the cutting fall short of the quantity necessary for the embankment, the deficiency is made up by excavations made in the ground beside the embankment, called *side-cuttings.*

302. **Horse power.**—Before the invention and improvement of the steam-engine, horses were very extensively used as moving agents; and although their application to machinery is now much less frequent, it is still resorted to, especially in places where fuel is expensive. Much of what has been said of the relative advantage of so applying the labour of men is also applicable to that of other animals; and accordingly one of the most advantageous methods of applying such labour is that represented in *fig.* 173, which is upon the principle of the tread-mill, and in which the animal uses its strength in continually endeavouring to ascend, while, by the nature of the machine, those efforts are continually frustrated by its descent.

. It is found, however, in practice, that the most convenient method of applying horse-power to machinery is by means of a large toothed wheel fixed on a strong vertical axis, and therefore turning horizontally; to the arms of this wheel two or more

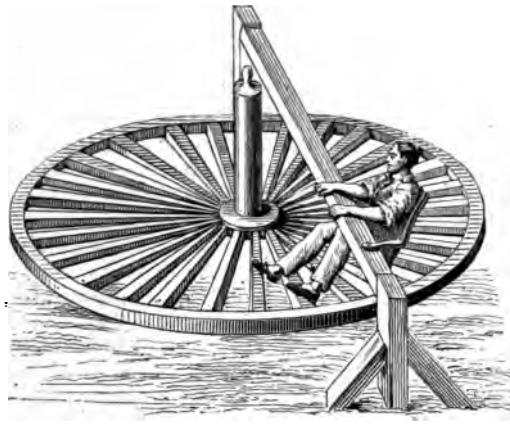

Fig. 303.

horses, called machiners, are yoked in such a manner as, by travelling constantly in a circle of which the axle of the wheel is the centre, and thus pushing against their collars, to make the wheel revolve. The wheel in this case may have the form called

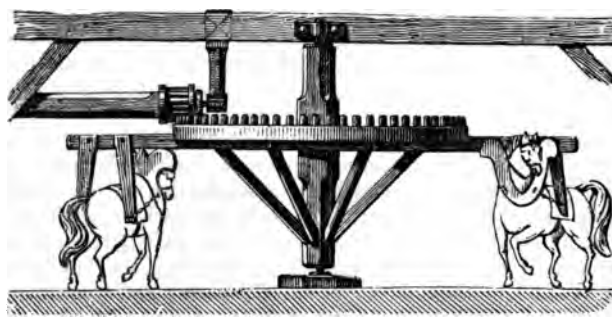

Fig. 304.

a crown wheel, as represented in *fig.* 304, in which case it gives motion to a horizontal shaft by means of a spur-pinion or basket fixed on that shaft on which it acts; or the same object may be

attained, perhaps preferably, by a bevelled wheel acting on a bevelled pinion, as explained in art. 215.

The greatest average maximum force which a horse can exert in drawing is about 900 lbs.; but when he works continuously, the force exerted will be much less than this. A good draught-horse working 6 days a week for 16 hours per day, and travelling 5 miles an hour, will bear a tractive force of about 110 lbs., and his daily labour will be equivalent to about 10,000,000 lbs. raised 1 foot. A horse driving a machine, as represented in *fig.* 304, produces a somewhat less effect than a draught-horse. To prevent the animal from needless fatigue by travelling in too small a circle, the arm to which he is yoked should have a length of not less than 20 feet.

The effect produced by a horse of average strength so working is found to be equal to that of 7 average men working at a windlass.

The average effect produced per second of time by a machiner is equivalent to about 300 lbs. raised 1 foot.

An ox harnessed to a vehicle can exercise a tractive force equal to that of a horse, but he produces less than half the effect, owing to his natural slowness; but as a machiner, where speed is not required, the efficiency of an ox is nearly equal to that of a horse.

An ass of average force acting as a machiner has about one-fourth of the efficiency of a horse.

When it is considered how very variable animal power is, whether we compare one animal of the same species with another, or even the same animal with itself, at different times and under different circumstances, it will be easily understood, that the estimates of different observers of the average force exercised by each species of animal differ considerably; it will, therefore, be useful here to give a few of the most generally received of these estimates.

In the following table are shown the estimates made by the observers named in the first column, of the average speed with which a man can walk without a load, for any continuance :—

Observer	Feet per second	Miles per hour
Schulze	5·37	3·66
Bernoulli	6·56	4·47
Guenyveau . . .	from 6·56 to 9·84	from 4·47 to 6·70

The average powers exerted by a man employed in working a pump, turning a winch, ringing a bell, and rowing a boat are, according to Mr. Buchanan, in the proportion of the numbers 100, 167, 227, and 248. It follows from this that the act of rowing is that in which the force is exerted with most advantage.

According to Mr. Peron, the following is the average relative power exerted by men employed in continuous labour in the countries indicated :—

England	71·4
France	69·2
Timor	58·7
Van Diemen's Land	51·8
New Holland	50·6

According to Tredgold, the average speed with which a horse can travel without a load will vary according to the number of hours per day he works, as shown in the following table :—

Time of March in Hours	Greatest velocity in Miles per Hour	Time of March in hours	Greatest Velocity in Miles per Hour
1	14·7	6	6·0
2	10·4	7	5·5
3	8·5	8	5·2
4	7·3	9	4·9
5	6·6	10	4·6

According to observations made upon the performance of draught horses in Scotland, it appears that the average load, which a single horse can draw at the rate of 22 miles per day in a cart weighing 7 cwt., is a ton.

The following estimates of the relative forces of a man and other animals have been given by the authorities whose names are indicated, Coulomb's estimate of the labour of a man being, in each case, taken as the unit :—

Carrying loads on the back on a level road :—

Horse, according to Brunacci	4·8
,, according to Westermann . . .	6·1
Mule, according to Brunacci	7·6

In drawing loads on a level road with a wheeled vehicle :—

Man with wheelbarrow, according to Coulomb	10·0

In drawing loads on a level road with a wheeled vehicle :—

Horse in four-wheeled waggon, according to Wesermann	175·0
Do. in two-wheeled cart, according to Brunacci	243·0
Mule ,, ,,	233·0
Ox ,, ,,	122·0

Hassenfratz gives the following comparative estimates :—

In carrying loads on a level road :—

Man	1·0
Horse	8·0
Mule	8·0
Ass	4·0
Camel	31·0
Dromedary	25·0
Elephant	147·0
Dog	1·0
Reindeer	3·0

In drawing a load on a level road :—

Man	1·0
Horse	7·0
Mule	7·0
Ass	2·0
Ox	4 to 7·0
Dog	0·6
Reindeer	0·2

303. **Steam-horse.**—Everyone is familiar with the term horse-power as applied generally to steam-engines, as well as to water mills and other machines, but few have a definite notion of its import; horse-power thus applied, is a term merely conventional, having no reference whatever to the actual work of the animal from which its name is taken. A steam-engine which is capable of a mechanical effect per minute equivalent to 33,000 lbs. raised 1 foot, is a steam-engine of 1 horse power; and in general, if the effect produced per minute by any machine, whatever be the power which impels it, be expressed as usual by an equivalent number of lbs. weight raised through a certain space, the number of horse-power which the machine is said to have will be found by dividing the number which expresses its mechanical effect in foot-pounds by 33,000.

304. **Water-power.**—This power is rendered available for the motion of machinery by intercepting its fall or progress by

wheels having buckets or leaves at their circumference, upon which the water acts either by its impact or weight so as to keep them in revolution with a corresponding force; the form and effect of these wheels, however, is explained more fully in the treatise on HYDROSTATICS in this series.

305. **Wind-power.**—The force of the atmosphere in motion is rendered available as a mover for machinery by means of arms, called sails, which are driven round in a plane nearly vertical, giving revolution to a horizontal axis. These machines are also explained more fully under the head of pneumatics in the volume mentioned.

306. **Steam-power.**—When heat is applied in sufficient quantity to water, the liquid is converted into vapour which has all the mechanical qualities of air, being elastic, expansive, and compressible. Its expansive force cannot be said to have any practical limit, but the space through which it acts will decrease in nearly the same proportion as that in which its intensity increases; thus, if it be confined by a resistance equivalent to a pressure of 15 lbs. per square inch, the steam will move the resistance until it has swelled to about 1,700 times the bulk it occupied as water. If the resistance is 30 lbs. per square inch, it will move it, until it has obtained a space equal to about 850 times its bulk as water, and so on.

It follows from this, that when water contained in a vessel such as a cylinder, confined by a movable piston pressing on it with a force of 15 lbs. per square inch, is converted into steam, the piston will be raised through about 1,700 inches for every inch of depth of the water evaporated. A moving force would, therefore, be developed equivalent to a weight of 15 lbs. raised 1,700 inches, or, in a round number, 150 feet; this would be in effect the same as 15 times 150 lbs., or 2,250 lbs. raised 1 foot. It may, therefore, be stated in round numbers, that when a cubic inch of water is evaporated a mechanical force is produced equivalent to that required for lifting a ton weight 1 foot high. It does not matter under what pressure the evaporation is produced, since any increase of pressure will be attended with a proportionate decrease of the space through which the resistance is moved.

The water, however, must in this case be regarded merely as a medium, by which the mechanical effects of heat are evolved. The real moving power is primarily, not the water, but the combustible by which heat is produced, and that combustible being usually pit coal, it becomes a question of the highest economical *importance to* determine the average quantity of coal which is

consumed in the evaporation of a given quantity of water, and in the consequent production of a given amount of work.

The consumption of coal in the evaporation of water in steam boilers, varies considerably according to the circumstances under which the fuel is applied. In Cornwall, however, where the conditions most favourable to economy are strictly observed, it has been found that in experiments conducted under circumstances of the greatest precision, a bushel of coal, that is, 84 lbs. weight, of good quality, has produced a mechanical effect equivalent to 120,000,000 lbs. raised 1 foot. This must, however, be regarded as an extreme experimental result. We may take, perhaps, 100,000,000 foot-pounds as the maximum mechanical effect attainable in regular work by the combustion of a bushel of coals.

Since it has been shown that the average maximum daily work of a man, working under the most favourable circumstances, is 2,000,000 units, and that of a horse 10,000,000, it follows that 1 bushel of coals consumed daily can perform the work of 50 men or 10 horses.

It has been computed, on data such as these, that the materials of which the great pyramid of Egypt is formed, could have been raised from the ground to their actual position by the combustion of something less than 700 tons of coal. Herodotus states that 100,000 men were employed for 20 years in raising this structure.

The Menai bridge consists of about 2,000 tons of iron, placed at a height of 120 perpendicular feet above the water level; its entire weight could have been raised to that height by the combustion of 400 lbs. weight of coals.

A train of carriages weighing 80 tons, and conveying 240 passengers, is drawn from Liverpool to Birmingham and back by the combustion of 4 tons of coke, the cost of which is 5*l*. To carry the same number of passengers daily, in stage coaches, on a common road would require an establishment of 20 coaches and 3,800 horses.

The circumference of the earth measures about 25,000 miles; if it were girt with an iron railway, such a train would be drawn round it in five weeks, by the combustion of about 500 tons of coke.

307. **Springs and weights.**—In a certain sense, springs and weights may be considered as moving powers; they are, in fact, the movers in all watch and clock work. It must be observed, however, that the title of movers cannot be given to them in the same absolute sense as that in which it is applied

to the several powers which we have described above. It has been already explained that when a watch or clock is wound up, the force which is exerted by the hand is expended in coiling up the main spring or elevating the weight, and that the motion of the watch or clock from that moment until it ceases is merely produced by the spring or weight giving back by slow degrees and during a comparatively long interval the force which was exerted by the hand in winding them up. Strictly speaking, therefore, the moving power is in this case the force of the hand, and the spring or weight is merely the depository in which that force is stored up, and from which it is given out until it is completely exhausted, when the clock ceases to move.

308. **Forces produced by heat.**—It is a physical law of high generality that when heat is imparted to or withdrawn from a body, expansion or contraction is produced. The force with which such expansion or contraction takes place has been sometimes used as a moving power or mechanical agent.

Air being eminently susceptible of such expansion and contraction by variation of temperature, attempts have been made, with more or less success, to apply the force of such expansion and contraction as a moving power. One of the more recent and ingenious of these projects is the caloric-engine invented by Captain Ericson as a substitute for the steam-engine. This engine has been constructed on a large scale, and tried experimentally at New York in the propulsion of a vessel. The piston is urged, as in the steam-engine, on one side by the expansive force of heated air, the resistance of the air on the other side being diminished by a great reduction of temperature.

Hitherto the application of such air and gas engines has not been very successful, in a practical sense; they are mostly reported to have operated with a certain efficiency, but under conditions not favourable to economy.

The expansion and contraction of metals has been used with great success in many places as a mechanical agent where great force has been required to be exerted through small spaces; examples of this have already been given.

309. **Mechanical force of congelation.**—Water at a certain point of the thermal scale exhibits a striking exception to the general law of contraction by cold. Just before congelation commences, the reduction of its temperature is attended, not with contraction, but with expansion; and this expansion takes place with irresistible force. When water, having percolated into the fissures and clefts of rocks, is frozen there, the force with which it expands in the process of congelation is such that

the rocks are split asunder, and fragments of enormous weight and magnitude are detached from them.

This force has been sometimes applied artificially for the fracture of large masses of stone.

310. **Electro-magnetic force.**—By expedients which are fully explained in the volume on ELECTRICITY in this series, a mass of soft iron can be rendered instantaneously magnetic, and as instantaneously deprived of its magnetism; and these alterations in its magnetic condition can be made to succeed each other at any desired rate, however rapid. The magnetism imparted to it may also have any desired intensity. It is, therefore, easy to imagine that such a piece of iron will attract another situated at a limited distance from it, with a certain definite force while it is magnetic, and that the moment it loses its magnetism, the iron which it had thus attracted may be made to recoil from it either by its weight, by the reaction of a spring attached to it, or by any other convenient means. A force, acting alternately in one direction and another, of definite intensity, will thus be produced, which may be applied as a moving power in the same manner exactly as that in which the alternate motion of the piston of a steam-engine is applied.

Various attempts have been made to render this available as a general moving power for machinery; but although the desired effect has been in many cases attained, the expense has hitherto so much exceeded that of steam power, that no successful practical result has been attained, except in a few cases. In the workshop of M. Froment, a well-known philosophical-instrument-maker of Paris, electro-magnetism has for some years been used as a moving power for the mechanism by which the divisions are engraved upon the limbs of circles, quadrants, and other similar instruments of precision. The mode of applying electricity to mechanical appliances for producing motion is explained fully in the above-mentioned volume in this series.

311. **Chemical agency.**—A great variety of powerful mechanical effects are produced by the chemical combination or decomposition of bodies. These phenomena, however, are generally developed under circumstances and conditions which render their application as mechanical agents often difficult, and sometimes impracticable. The circumstances under which the mechanical force is thus developed are generally those in which solid or liquid bodies of comparatively small dimensions are suddenly converted into gaseous bodies of great volume, the change being usually attended with a large and intense development of heat. The gases thus evolved at a high temperature,

expanding with prodigious force, drive before them whatever resistances may be opposed to their dilatation.

One of the most familiar examples of this class of phenomena, and that which has been most extensively applied, is gunpowder. This substance is a mixture of nitre, charcoal, and sulphur, all in a very pure state, and in certain definite proportions; thus, in 100 lbs. weight of gunpowder there are generally 75 lbs. of nitre, 15 lbs. of charcoal, and 10 lbs. of sulphur. These proportions are subject to a small variation in different qualities of powder, which need not be noticed more particularly here.

When a spark is applied, the charcoal and the sulphur heated by it, attract the oxygen, which is one of the constituents of the nitre; and, combining with it, form gases, called carbonic oxide, carbonic acid, and sulphurous acid gas; while another constituent of the nitre, disengaged from the oxygen, is the gas called nitrogen, or azote. The result, therefore, of the phenomenon is the instantaneous evolution of a mixture of the four gases above-named; this evolution being attended with such intense heat, that the gases are incandescent, or luminous. It is found that the gunpowder, in this process of explosion, swells into 2,000 times its volume. Count Rumford found that 28 grains of gunpowder, screwed into a cylindrical space within a piece of iron, tore it asunder with a force of 400,000 lbs.

312. **Gun cotton.**—If a piece of common raw cotton, usually called cotton wool, be steeped in a mixture composed of equal measures of sulphuric and nitric acids, and then pressed and dried, it will, to all external appearance, be the same as before; but if it be weighed it will be found to be nearly one half heavier. The change which it has undergone is this; the cotton has lost a quantity of water which was combined with it, equal to about one-third of its weight, but it has entered into combination with such a quantity of nitric acid as to give the whole a weight 50 per cent. greater than that of the original weight of the cotton.

The cotton thus prepared is highly explosive, and the effects of its explosion are explained on the same principle as those of gunpowder. It is considered by chemists that, weight for weight, the force of this substance is greater than that of gunpowder. It explodes also at a lower temperature, and, when of good quality, leaves no perceptible residuum. It is, therefore, natural that, since the process of making it has undergone great improvements and the action of the substance has been thoroughly studied, it has replaced gunpowder in some kinds of fire-arms,

as it has already done to a certain extent in the blasting of rocks.

313. **Capillary attraction.**—The mechanical agency supplied by this principle in the arts, and its use in the vegetable economy, are explained in the volume on HYDROSTATICS in this series.

314. **Perpetual motion.**—There is no mechanical problem on which a greater amount of intellectual ingenuity has been wasted than that which has for its object the discovery of the perpetual motion. Since this term, however, is not always rightly understood, it will be useful here to explain what the perpetual motion is not, as well as what it is.

The perpetual motion, then, which has been the subject of such anxious and laborious research, is not a mere motion which is continued indefinitely. If it were, the diurnal and annual motion of the earth, and the corresponding motions of the other planets and satellites of the solar system, as well as the rotation of the sun upon its axis, would be all perpetual motions.

To understand the object of this celebrated problem, it is necessary to remember that, in considering the construction and performance of a machine, there are three things involved: 1st, the object to which the machine gives motion; 2ndly, the structure of the mechanism; and 3rdly, the moving power, the effect of which is transmitted by the machine to the object to be moved. In consequence of the inertia of matter, the machine cannot transmit to the object more force than it receives from the moving power; strictly speaking, indeed, it must transmit less force, since more or less of the moving force must be intercepted by friction and atmospheric resistance. If, therefore, it were proposed to invent a machine which would transmit to the object to be moved the whole amount of force imparted by the moving power, such a problem would be at once pronounced impossible of solution, inasmuch as it would involve two impracticable conditions; first, the absence of atmospheric resistance, which would oblige the machine to be worked in a vacuum; and secondly, the absence of all friction between those parts of the machine which would move in contact with one another.

But suppose that it were proposed to invent a machine which would transmit to the object to be moved a greater amount of force than that imparted by the moving power, the impossibility of the problem would in this case be still more glaring, for even though the machine were to work in a vacuum, and all friction were removed, it could do no more than convey to the object

the force it receives. To suppose that it could convey more force, it would be necessary to admit that the surplus must be produced by the machine itself, and that, consequently, the matter composing it would not be endowed with the quality of inertia. Such a supposition would be equivalent to ascribing to the machine the qualities of an animated being.

But the absurdity would be still greater, if possible, if the problem were to invent a machine which would impart a certain motion to an object without receiving any force whatever from a moving power; yet such is precisely the celebrated problem of the perpetual motion.

In short, a perpetual motion would be, for example, a watch or clock which would go so long as its mechanism would endure, without being wound up; it would be a mill which would grind corn or work machinery without the action upon it of water, wind, steam, animal power, or any other moving force external to it.

It is not only true that such a machine never has been invented, but it is demonstrable that so long as the laws of nature remain unaltered, and so long as matter continues to possess that quality of inertia which is proved to be inseparable from it, not only in all places and under all circumstances on the earth, but throughout the vast regions of space to which the observations of astronomers have extended, the invention of such a machine is an impossibility the most absolute.

CHAPTER XX.

ON WORK AND ENERGY.

315. **Units of force.**—Let us suppose that we call the quantity of matter contained in a pound avoirdupois the unit of mass. We may then define our unit of force as *a force which, acting upon the unit of mass (a pound) for one second, generates in it a velocity of one foot per second.* This unit is called the *absolute unit of force*, because it is quite independent of locality, and is derived from the one unalterable property of mass.

If, instead of a pound, the matter in a gramme be taken as the unit of mass, and a centimetre as the unit of length, the method of measuring forces by reference to the centimetre-gramme-second units is still that of absolute measure. These

latter units are, however, more employed where the forces are minute, as in electrical measurements. More recently endeavours are made, with every prospect of speedy success, to extend the use of the centimetre-gramme-second unit throughout the whole range of physical research, and we shall, therefore, give a few examples of its application further on. Forces measured in terms of the absolute unit are said to be expressed in 'absolute measure.'

Another method of expressing force is that usually called *gravitation unit* of force and mass. From a great number of experiments, the exact velocity has been ascertained which is generated in a body falling under the attraction of the earth, and freed from the resistance of the air. The conclusion arrived at is that the standard pound weight, so falling in London, would acquire a velocity of 32·1889 feet per second. This number varies with the latitude, according to an ascertained law. In latitude 45°, for example, it has the value of 32·1703. The letter g is generally appropriated for representing the velocity acquired in one second by a body falling from rest in a space devoid of air.

Now in gravitation measure, which has been employed in the preceding articles, we call a force which will support the weight of a pound the unit force. Hence a force which will support 5 pounds is represented by 5. In general, in gravitation measure the unit of mass is the quantity of matter in a body which weighs g pounds, and forces are measured by the weights which they will support.

From what has been said on the value of g, it will be clear that the unit of mass in gravitation measure changes with the locality. Nevertheless it is selected for very cogent reasons. From our assumption, that a mass represented by unity weighs g pounds, it follows that if M be the mass of a body weighing W pounds, this weight will be equal to M g pounds, or

$$M = \frac{W}{g}.$$

Any force may easily be expressed in gravitation measure if the velocity is given which the force can generate in one second in a body of given weight. Thus, suppose that a force F generates a velocity of 46 feet in one second in a body weighing 7 pounds. If this weight of 7 pounds be simply acted on by gravity, it would fall and acquire a velocity of 32·2 feet in the first second. The force then acting would be called a force of 7 pounds, and we shall compare the forces F and 7 by compar-

ing the quantities of motion produced by each in one second. Thus,

$$\frac{F}{7} = \frac{\text{mass} \times 46}{\text{mass} \times 32{\cdot}2} = \frac{46}{32{\cdot}2}$$

$$\therefore F = \frac{7 \times 46}{32{\cdot}2} = 10 \text{ pounds.}$$

Or, generally, if a force F generate a velocity f in one second in a body weighing W pounds, we have

$$\frac{F}{W} = \frac{\text{mass} \times f}{\text{mass} \times g}, \quad \therefore F = \frac{W f}{g}.$$

It follows from the definition, that the force of attraction of the earth on the unit of mass, that is, the standard pound, is expressed in absolute units by the number 32·2, or g. For a force which generates a velocity of g feet per second is g times as great as a force which will generate a velocity of one foot per second. But the attraction of the earth generates a velocity of g feet in the standard pound in one second; hence the force of gravitation acting on the standard pound is g times the unit force, or is expressed by the number $g = 32{\cdot}2$.

Since, on the other hand, the attraction of the earth on the standard pound is, in gravitation measure, expressed by the number unity, it follows that we may pass from the gravitation to the absolute measure of force by multiplying every force expressed in pounds by the number g; and that we may pass from absolute to gravitation measure by dividing every number representing a force by g. Thus, in absolute measure the *unit of force* is $\frac{1}{32{\cdot}2}$ of the attraction of the earth on a standard pound, and may be represented roughly in gravitation measure by about half an ounce weight.

The expression for the momentum of a body is, accordingly, in gravitation units, represented by $\frac{W v}{g}$, whereas in absolute units the same momentum is W v.

316. **The dyne.—The poundal.**—The facts explained in the preceding article may be summarised and extended to the centimetre-gramme unit by the following brief statements:—

1. The unit of force is that force which, acting on the unit of mass for the unit of force, generates unit of velocity. This is called the *absolute unit of force.*

2. The weight of gramme, that is, the force which causes it to fall, may be ascertained by letting it fall freely. If the experiment be made in Britain, it will be found that at the end of

one second the velocity of the gramme will be about 981 centimetres per second.

3. Hence the weight of a gramme, that is, the force which causes it to fall, is represented by the number 981, if the centimetre, the gramme, and the second are taken as the fundamental units.

4. The force which in one second would communicate to one gramme a velocity of one centimetre per second, is the unit of force in the French metric system of units. This unit of force is called a *Dyne*.

5. It is sometimes convenient to compare forces with the weight of body, and to speak of a force of so many pounds weight, or grammes weight. This is called *Gravitation measure*.

6. Although a pound or a gramme is the same all over the world, the weight of a pound or a gramme is greater in high latitudes than near the equator, and therefore a measurement of force in gravitation measure is of no scientific value unless it is stated in what part of the world the measurement was made.

7. In Britain the units of length, mass, and time, are respectively one foot, one pound, and one second. The unit of force is, in this system of units, that force which, in one second, would communicate to one pound a velocity of one foot per second. This unit of force is called *Poundal*.

8. Since the foot is 30·4797 centimetres, and the pound is 453·59 grammes, the Poundal is 13825 Dynes.

317. **Work.**—Wherever motion is produced against some resistance, then work is done. A horse does work when he moves with a loaded cart, or when he gives motion to any kind of machinery. A steam-engine does work when it is used to lift water, or to drive a train of carriages along a railway, or to do in fact any sort of labour for which animals are employed. The idea of work for machinery, or natural processes, is taken from comparison with the working power of men, and we can best illustrate from human labour the most important features of the conception of work. In speaking of the work of machines, and of natural forces, we must, of course, in this comparison eliminate anything in which activity of intelligence comes into play. A man may work with his mind as well as with his body. When he works with his mind he performs intellectual labour; when he works with his body he performs physical labour, or mechanical work, which could be done quite as well by a steam-engine, or any other labouring agent. But it is only the latter kind of work, done by man, which is considered in the present question.

It must be clearly understood, that whenever mechanical work is done there is a pressure or resistance exerted, and a certain space through which that pressure or resistance is exerted. Thus, when a carpenter saws a piece of timber, he applies a pressure to the saw, and this occasions a resistance on the part of the timber to the motion of the saw. Now, whatever may be the pressure applied to the saw, if it were not moved, there would obviously be no work done, and, on the other hand, if the saw were moved without any pressure being applied to it, there would also be no work done. When a man carries a weight up a ladder or staircase, we say that he does work, but if he stood still with his load, then, although he would sustain the same pressure, yet he would not do any work. Hence, in order to have mechanical work performed, we must have not only pressure, but that pressure must be sustained through a certain space.

318. **Further illustrations. Measurement of work.**—The external work of man is of the most varied kind as regards the force or ease, the form and rapidity, of motions used on it, and the kind of work produced. But both the arm of the blacksmith who delivers his powerful blows with the heavy hammer, and that of the violinist who produces the most delicate variations in sound, and the hand of the lacemaker who works with threads so fine that they are on the verge of the invisible, all these acquire the force which moves them in the same manner and by the same organs, namely, the muscles of the arm. An arm the muscles of which are lamed is incapable of doing any work; the moving force of the muscle must be at work in it, and these must obey the nerves, which bring to them orders from the brain. That member is then capable of the greatest variety of motions; it can compel the most varied instruments to execute the most diverse tasks. Just so is it with machines. They are used for the most diversified arrangements. By their agency we produce an infinite variety of movements, with the most various degrees of force and rapidity, from powerful steam-hammers and rolling-mills, where gigantic masses of iron are cut and shaped like wax, to spinning and weaving frames, the work of which rivals that of the spider. Modern mechanism has the richest choice of means of transferring the motion of one set of rolling-wheels to another with greater or less velocity; of changing the rotating motion of wheels into the up-and-down motion of the piston-rod, of the shuttle, of falling hammers, and stamps; or, conversely, of changing the latter into the *former; or it can*, on the other hand, change movements of

uniform into those of varying velocity, and so forth. But one thing is common to all differences, they will need a *moving force*, which sets and keeps them in motion, just as the works of the human hand all need the moving force of the muscles. But the work of the smith, in the previous illustration, requires a far greater and more intense exertion of the muscles than that of the violin player, and there are in machines corresponding differences in the power and duration of the moving force required. These differences, which correspond to the different degree of exertion of the muscles in human labour, are alone what we have to think of when we speak of the *amount of work* of a machine. We have nothing to do here with the manifold character of the actions and arrangements which the machines produce; we are only concerned with an expenditure of force.

The expression 'expenditure of force,' which we use so fluently, indicates not only that in doing work the force applied has been expended and lost, but leads to a further characteristic analogy between the effects of the human arm and those of machines. The greater the exertion, and the longer it lasts, the more is the arm tired, and the more is the moving force for the time exhausted. Now this peculiarity of becoming exhausted by work is also met in the moving forces of inorganic nature. When fatigue sets in, in the case of living agents, recovery is needed, and this can only be effected by rest and nourishment. In the case of inorganic moving forces, when their capacity for work is spent, there is also a possibility of reproduction, although in general other means must be used to this end than in the case of living agents.

From what has been said in the preceding article, we may form some rough idea of the comparative quantity of work which two labouring agents might perform. Thus, if two men were to carry the same weight of material to the same height, then they would do the same work; and if the one were to carry the same weight to twice the height, or half the weight to four times the height, then he would perform twice the work of the other. But it is necessary that we should have some more definite means of estimating work than such methods of calculation afford. Again, from the feeling of exertion and fatigue in our muscles we can form a general idea of what we understand by amount of work; but we must endeavour, instead of the indefinite estimate afforded by this comparison, to form a clear and precise idea of the standard by which we have to measure the amount of work.

As we measure a distance by the number of times that some

unit of length, such as a foot or a yard, is contained in the proposed distance ; or, as we express the weight of any substance by the number of times that some unit of weight, as a pound or an ounce, is contained in the proposed weight ; so we must fix upon some unit whereby to express the amount of labour or work which any agent may perform. Now the measuring unit must always be of the same sort or kind as the thing to be measured ; thus we measure length by a unit of length, surface by a unit of surface, weight by a unit of weight, and so on. The measure of work, therefore, must be some unit of work.

319. **Units of work. The foot-pound. The horse-power.** —The unit of work used in this country, as has already been stated in the preceding chapter, is the labour requisite to raise one pound through the space of one foot. This unit is called a *Foot-pound.* Thus, if a man take a pound weight in his hand and raise it one foot, he will perform one unit of work ; if he raise it two feet he will perform two units of work ; if he raise it three feet, he will perform three units, and so on. Again, if he take a weight of four pounds in his hand, and raise it five feet, he will perform twenty units of work, because in raising one pound five feet, he will perform five units ; therefore, in raising four pounds the same height, he will perform $4 \times 5 = 20$ units. Hence we derive the rule, that in order to obtain the work expended in raising any body, we multiply the weight of the body in lbs. by the vertical space in feet through which it is raised.

Further, as resistances and pressures of every kind may be expressed in pounds, it follows that the one unit of work will measure every kind of work. This will be best illustrated by the following examples.

When considerable distances are measured, we use a higher unit than that of feet or yards. So, in like manner, when the work is very great, it is found more convenient to adapt a larger unit of work. Now Watt estimated that a horse could perform 33,000 units of work (foot-pounds) per minute. This work, therefore, is called a horse-power. Thus, work, done in a given time, may be expressed in units of a horse-power.

1. Required the units of work expended in raising a weight of 60 lbs. to the height of 23 feet.

Units in raising 1 lb. 23 feet $= 23$

60 ,, $= 60 \times 23 = 1,380.$

2. The ram of a pile-engine weighs 7 cwt. and has a fall of 23 feet. Required the units of work performed in raising the *ram.*

Weight of ram 7×112 lbs. $= 784$ lbs.

Units of work in raising 1 lb. 23 feet $= 23$

784 lbs. ,, $= 784 \times 23$

$= 18{,}032.$

3. How many units of work are required to raise 4 cwt. of coals from a pit 70 fathoms deep?

$$4 \times 112 \times 70 \times 6 = 188{,}160.$$

4. If the weight of a man be 140 lbs., and he ascends 56 feet, how many units of work will he perform?

Weight to be raised 140 lbs.

$\therefore$ Work $140 \times 50 = 7{,}000.$

Let now the man descend in a basket. Then he performs the same weight upon a counterpoise, which he had done in ascending.

5. How much work is required to pump 3,000 cubic feet of water from a mine 160 fathoms deep?

Weight of cubic foot 62·5 lbs.

$\therefore$ $62{\cdot}5 \times 3{,}000 \times 160 \times 6 = 180{,}000{,}000.$

6. A horse draws 104 lbs. out of a well, by means of a cord passing over a wheel, moving at the rate of $2\frac{1}{2}$ miles per hour. Find the units performed per minute.

$$\frac{2\frac{1}{2} \times 5280}{60} = 220 \text{ ft.}$$

$$220 \times 104 = 22880.$$

7. A horse-power being $= 33{,}000$ units per minute, how many horses would it take to raise 1 cwt. of coals *per minute* from a pit 200 fathoms deep?

Work done per minute $112 \times 200 \times 6 = 134400$

$$\therefore \text{Horse-power } \frac{134400}{33000} = 4{\cdot}07.$$

8. How many horse-powers are required to raise 2,000 cubic feet of water per hour from a mine 180 fathoms deep?

Weight of water $62{\cdot}5 \times 2000$

Depth of mine 180×6

$$\text{Work per minute } \frac{62{\cdot}5 \times 2000 \times 180 \times 6}{60}$$

$$\text{H. P.} = \frac{62{\cdot}5 \times 2000 \times 180 \times 6}{60 \times 33000} = 68{\cdot}1.$$

9. An engine for winding up loads is moved by two horses. What weight of coals will be raised per hour from a pit 50 fathoms deep.

Work done by the horses per hour $33000 \times 2 \times 60 = 3960000$.

Depth of pit 300 feet.

Work done in raising 1 lb. = 300.

$\frac{3960000}{300}$ = Number of lbs. raised per hour.

= 13200 lbs. or 5·9 tons.

10. In what time will an engine, capable of performing the work of 8 horses raise a ton of materials from a depth of 170 fathoms?

Work of engine $33000 \times 8 = 264000$.

Work in raising 1 ton $= 2240 \times 170 \times 6 = 2284800$.

$$\text{Number of minutes} = \frac{2284800}{264000} = 8{\cdot}6.$$

11. How many cubic feet of water will an engine of 20 horse-power raise per hour from a mine whose depth is 80 fathoms?

Work done per hour $33000 \times 20 \times 60$

Work in raising 1 foot of water $62{\cdot}5 \times 80 \times 6$

$$\text{Cubic feet raised} \therefore \frac{33000 \times 20 \times 60}{62{\cdot}5 \times 80 \times 6} = 1320.$$

12. From what depth will an engine of 5 horse-power raise 4 tons of coals per hour?

Work done per hour $33000 \times 5 \times 60$

Work in raising 4 tons 1 foot $= 2240 \times 4 \times 1$

$$\text{Number of feet} = \frac{33000 \times 5 \times 60}{2240 \times 4} = 1104 \text{ feet.}$$

13. An engine is observed to raise 3 tons of material per hour from a mine 120 fathoms deep. Find the horse-power of the engine, supposing $\frac{1}{5}$th of the work to be lost in transmission.

$$\text{Work done per minute } \frac{2240 \times 3 \times 120 \times 6}{60} = 80640.$$

But only $\frac{4}{5}$ths of the work of the engine go to raise the material.

$\therefore \frac{4}{5} \times 33000$ work done by the engine in the minute.

$$\text{Horse-power} = \frac{80640}{\frac{4}{5} \times 33000} = 3{\cdot}05.$$

14. What must be the horse-power of an engine working 10 hours per day to supply 240,000 gallons of water per day, supposing the water to be raised to an average height of 70 *feet?*

A gallon of water weighs 10 lbs.

$$\text{Water pumped per minute} = \frac{240000}{10 \cdot 60} = 400 \text{ gallons.}$$

$\therefore$ Weight of it = 4000 lbs.

$$\text{H. P.} = \frac{4000 \times 70}{33000} = 8 \cdot 4.$$

15. A forge hammer weighing 2 cwts. makes 60 lifts per minute, each lift being $2\frac{1}{2}$ feet high. Find the horse-power of the engine giving motion to the hammer.

Weight of hammer 224 lbs. = 112 × 2.

Work in each lift $224 \times 2\frac{1}{2} = 560$.

Work in 60 lifts (1 minute) = 560 × 60 = 33600.

$$\text{H. P.} = \frac{33600}{33000} = 1 \cdot 02.$$

320. **Other Units. The kilogrammetre. The erg.**—If we take a kilogramme, as our weight lifted, and the space through which it is raised as one metre, we obtain a unit of work much used by Continental writers, and called a *Kilogrammetre.* But in this unit, as in the case of a foot-pound the force employed is the weight of a pound, it is a kilogramme,—a quantity which, as we know, is different in different places. The weight of a pound expressed in absolute measure is numerically equal to the intensity of gravity, the quantity denoted by g, the value of which in poundals varies from 32·227 at the pole to 32·117 at the equator, and diminishes without limit as we recede from the earth. In dynes it varies from 983·1 at the pole to 978·1 at the equator. Hence, in order to express work in a uniform and consistent manner, we must multiply the number of foot-pounds by the number representing the intensity of gravity at the place. The work is thus reduced to foot-poundals. When work is expressed in foot-pounds the system is that of gravitation measures, which is not a complete system, unless we also know the intensity of gravity at the place.

In the metrical system the unit of work is the *Erg*, which is the work done by a dyne acting through a centimetre. There are 421393·8 ergs in a foot-poundal.

321. **Work done by gravity.**—Gravity is the moving force which is simplest, and which we know best. It acts, for example, in those clocks which are driven by a weight. This weight, fastened to a string, which is wound round a pulley connected with the first toothed wheel of the clock, cannot obey the pull of gravity without setting the whole clockwork in

motion. Now, the weight cannot put the clock in motion without itself sinking; did the weight not move, it would not move the clock, and its motion can only be such a one as obeys the action of gravity. Hence, if the clock is to go, the weight must continually sink lower and lower, and must at length sink so far that the string which supports it is run out. The clock then stops, and the useful effect of its weight is for the present exhausted. Its weight is not lost but altered; it is attracted by the earth as before, but the capacity of this gravity to produce the motion of the clockwork is lost. But we can wind up the clock by the power of the arm, by which the weight is again raised. When this has been done it has regained its former capacity, and can again set the clock in motion.

We learn from this that a raised weight possesses the capacity of performing work, but that it must necessarily sink if it is to perform work; that by sinking this capacity is exhausted, but that by using another extraneous force—that of the arm, also capable of doing work—the activity of the weight can be restored.

The work which the weight has to perform in driving the clock is not very great. It has continually to overcome the small resistances which the friction of the axles and teeth, as well as the resistance of the air oppose to the motion of the wheels, and it has to furnish the force for the small impulses and sounds which the pendulum produces at each oscillation. If the weight is detached from the clock the pendulum swings for a while before coming to rest, but its motion becomes each moment more feeble and ultimately ceases entirely, being gradually used up by the various obstacles just enumerated. Hence there must be a moving force, like the weight, which must be continually at work to keep the clock going.

322. **Accumulated work.**—When a force acts upon a body capable of moving freely it will set it in motion; the inertia of the body presents in this case the resistance which is overcome by the force, and thus work is done in impressing velocity upon a body. So also a body in motion can only be brought to a state of rest by the action of force, and work is done during the destruction of motion. If we were to raise a body through a certain height and allow it to fall, it would acquire in falling a velocity dependent on the height through which it had been raised. If now the direction of motion could be suddenly reversed, the weight would rise to the exact height from which it fell, and in doing so it would clearly perform work. We infer from this that the velocity acquired in falling is a measure of the work done in

lifting the body, and is produced by the action of a definite force, namely, the weight of the body, acting through a definite space, namely, the height through which the body has been raised. But we may extend the conclusion by saying that any other body moving with any given velocity might have acquired it in like manner by being subjected to the action of the force of gravity while falling through a determinate height, and thus we say generally that work is stored up in a body in motion, and that the measure of this work is the height through which the body must be lifted in order that by falling it may acquire the velocity with which it is actually moving.

323. **Measure of Accumulated Work.**—Suppose a body of weight w to be moving with a velocity of v feet per second. Then if h is the height through which a body must fall in order to acquire a velocity of v feet, we know from art. 113, p. 110, that

$$v^2 = 2gh, \text{ or } h = \frac{v^2}{2g}.$$

But the work done in lifting a body of weight w through a length h is wh, hence

$$\text{the work done} = wh$$
$$\ldots = \frac{wv^2}{2g}$$

if we substitute for h its value.

For example, let a bullet leave the barrel of a gun with a velocity of 1,000 feet per second, the weight of the bullet being an ounce. Then the work stored up in the bullet is, by the preceding formula—

$$\frac{wv^2}{2g} = \frac{\frac{1}{16} \times (1{,}000)^2}{2 \times 32{\cdot}2} = 970{\cdot}5 \text{ foot-pounds.}$$

In this case linear velocity has only to be considered. But there can be no motion of any kind, whether it be of translation or rotation, without the action of force in overcoming resistance. The inertia due to mass is ever present, and work can be stored up in a rotating wheel as certainly as it can be accumulated in the ponderous head of a steam hammer. Thus, a heavy rotating body, such as the fly-wheel of an engine, may be considered as a reservoir into which the work of the engine can be poured just as water is poured into a vessel. The work stored up in a body rotating with a given velocity may be thus measured.

Let us suppose a body of weight w to move in a circle of radius r with a given velocity v. A line drawn from the body to the centre of the circle will rotate round the centre with an angular velocity which we call ω, thus $v = \omega r$.

But the work stored up in the body is

$$\frac{W v^2}{2g} = \frac{W \omega^2 r^2}{2g} = \frac{\omega^2}{2} \times \text{Mass} \times (\text{Radius})^2.$$

We conclude from this expression, that the work stored up in the separate parts of a body moving with a given velocity depends upon the square of the distance of each part from the axis. Thus, a pound weight at a distance of 3 feet from the axis of rotation has 9 times as much work stored up in it as the same weight at a distance of 1 foot from the axis, the angular velocity being the same in both cases.

324. **Energy.**—The term *energy* is restricted in its mechanical application to one definite meaning, namely, the *capacity of doing work*. A body possesses energy when it is capable of doing work; thus the raised weight of the clock, in the previous illustration, possesses energy, for we can obtain work from the weight when it descends. The energy which exists in the raised weight is called *potential energy*, the word potential expressing the fact that the energy is in existence, and that it has been imparted to the weight by the act of raising it. The energy may or may not be called into action, but it exists; the power of performing work may lie dormant for any length of time, but we know that the action will begin when the weight is released and allowed to descend.

Every portion of matter which is at rest, but is nevertheless capable of doing work, possesses potential energy. Potential energy, however, must be distinguished from the energy possessed by bodies in actual motion; to this energy the name of *Kinetic energy* has been given. A body in motion, as we have seen, has work stored up in it, which work it must yield up before it can be reduced to rest. The expression 'Kinetic energy' is thus applied to the energy existing in any moving body. Potential energy has also been called *energy of position*, while to Kinetic energy the names 'actual energy,' and 'energy of motion' have also been applied.

325. **Forms of Energy.**—In the case of the weight, which sets a clock in motion, it is necessary, from time to time, to wind up the weight, and thus to accumulate work in it, by means of which it possesses energy.

The case is somewhat different when Nature herself raises the weight, which then works for us. She does not do this with solid bodies, at least not with such regularity as to be utilised; but she does it abundantly with water, which, being raised to the top of mountains by meteorological processes, returns in

streams from them. The gravity of water we use as moving force, the most direct application being in what are called overshot wheels, one of which is represented in the treatise on HYDROSTATICS, in this series, p. 296; *fig.* 205. Along the circumference of such a wheel are a series of buckets, which act as receptacles for the water, and, on the side turned to the observer, have the tops uppermost; on the opposite side the tops of the buckets are upside down. The water flows at the top into the buckets of the front of the wheel, and below where the mouth begins to incline downwards, it flows out. The buckets on the circumference are filled on the side turned to the observer, and empty on the other side. Thus the former are weighed by the water contained in them, the latter not; the weight of the water acts continually on only one side of the wheel, draws this down, and thereby turns the wheel; the other side of the wheel offers no resistance for it contains no water. It is thus the weight of the falling water which turns the wheel and furnishes the motive power. But it can at once be seen that the mass of water which turns the wheel must necessarily *fall* in order to do so, and that though, when it has reached the bottom, it has lost none of its gravity, it is no longer in a position to drive the wheel, if it is not restored to its original position, either by the power of the human arm or by means of some other natural force. If it can flow from the mill-stream to still lower levels it may be used to work other wheels. But when it has reached its lowest level, the sea, the last remainder of the moving force is used up, which is due to gravity—that is, to the attraction of the earth, and it cannot act by its weight until it has been again raised to a high level. As this is actually effected by meteorological processes, it will at once be observed that these are to be considered as sources of the energy possessed by the head of water.

Water power was the first inorganic force which man learnt to use instead of his own labour, or that of domestic animals. According to Strabo, it was known to King Mithridates of Pontus, who was also otherwise celebrated for his knowledge of nature; near his palace there was a water-wheel. Its use was first introduced among the Romans in the time of the first Emperors. Even now we find water mills in all mountains, valleys, or wherever there are rapidly-flowing, regularly filled brooks and streams. We find water-power used for all purposes which can possibly be effected by machines. It drives mills which grind corn, saw-mills, hammers and oil-presses, spinning-frames and looms, and so forth. It is the cheapest of all motive

powers, it flows spontaneously from the inexhaustible stores of nature; but it is restricted to a particular place, and only in mountainous countries is it present in any quantity; in level countries extensive reservoirs are necessary for damming the rivers to produce any amount of water-power.

Before passing to the discussion of other motive forces, an objection must be answered which may readily suggest itself.

326. **Mechanical Powers.**—There exists, as we know, numerous machines, systems of pulleys, levers, and cranes, by the aid of which heavy burdens may be lifted by a comparatively small expenditure of force. We have all of us often seen one or two workmen hoist heavy masses of stones to great heights, which they would be quite unable to do by applying their own muscular strength directly; in like manner, one or two men, by means of a crane, can transfer the largest and heaviest chest from a ship to the quay. Now, it may be asked, if a large, heavy weight had been used for driving a machine, would it not be very easy, by means of a crane or a system of pulleys, to raise it anew, so that it could again be used as a motor, and thus acquire motive power, without being compelled to use a corresponding exertion in raising the weight?

The answer to this is, that all these machines, in that degree in which for the moment they facilitate the exertion, also prolong it, so that by their help no motive power is ultimately gained. Let us assume that six labourers have to raise a load of six hundredweight, by means of a rope passing over a single pulley. Every time the rope is pulled down through six feet, the load is also raised through six feet. But now, for the sake of comparison, let us suppose the same load hung to a block of six pulleys, as represented in *fig.* 193, page 241. A single labourer would now be able to raise the load by the same exertion of force as each one of the six put forth. But when he pulls the rope through six feet, the load only rises one foot, for the length through which he pulls the rope over A is uniformly distributed in the block over six ropes, so that each of these is only shortened by one foot. To raise the load, therefore, to the same height, the one man must necessarily work six times as long as the six together did. But the total expenditure of work is the same, whether six labourers work for a sixth part of an hour or one works for an hour.

If, instead of human labour, we introduce the work of a weight, and hang to the block a load of 600, and at the end of the rope which passes over at A, where otherwise the labourer works, a weight of 100 pounds, the block is then in equilibrium,

and without any appreciable exertion of the arm, may be set in motion. The weight of 100 pounds sinks, that of 600 rises.

Without any measurable expenditure of force, the heavy weight has been raised by the sinking of the smaller one, and we shall at the same time observe that the smaller weight will have sunk through six times the distance that the greater one has risen. But a fall of 100 pounds through six feet is just as much 600 foot-pounds as a fall of 600 pounds through one foot.

327. **Levers.**—The action of levers in all their various modifications is precisely similar. Let *a b*, *fig.* 305, be a simple lever, supported at *c*, the arm *c b* being four times as long as the

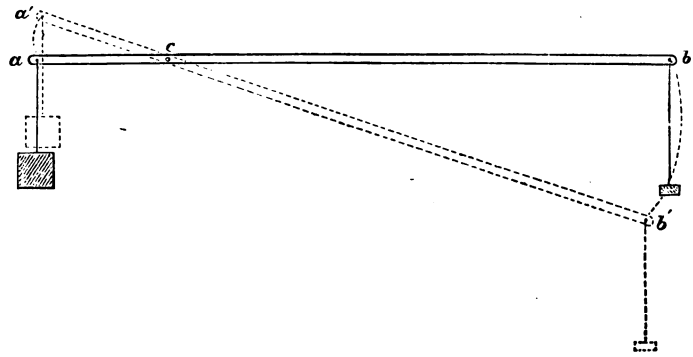

Fig. 305.

other arm *a c*. Let a weight of one pound be hung at *b*, and a weight of four pounds at *a*; the lever is then in equilibrium, and the least pressure of the finger is sufficient, without any appreciable exertion of force, to place it in the position *a' b'*, in which the heavy weight of four pounds has been raised, while the one pound weight has sunk. But here, also, we shall observe that no work has been gained, for while the heavy weight has been raised through one inch, the lighter one has fallen through four inches; and four pounds moved through one inch is, as work, equivalent to the product of one pound through four inches.

Most other fixed parts of machines may be regarded as modified and compound levers; a toothed-wheel, for instance, as a series of levers, the ends of which are represented by the individual teeth, and one after the other of which is put in activity, in the degree in which the tooth in question seizes, or is seized by, the adjacent pinion. Take, for instance, the common crab-winch, and suppose the pinion on the axis of the

barrel of the winch to have twelve teeth, and the toothed wheel seventy-two teeth, that is, six times as many as the former. The winch must now be turned round six times before the toothed-wheel and the barrel have made one complete turn, and before the rope which raises the load has been lifted by a length equal to the circumference of the barrel. The workman thus requires six times the time, though to be sure only one-sixth of the exertion which he would have to use if the handle were directly applied to the barrel. In all these machines, and parts of machines, we find it confirmed that in proportion as the velocity of the motion increases its power diminishes, and that when the power increases the velocity diminishes, but that the amount of work is never thereby increased.

In the overshot mill-wheel, described above, water acts by its weight. But there is another form of mill-wheel, called the undershot wheel, in which it only acts by its impact, as represented in the volume on HYDROSTATICS, page 294, *fig.* 204. These are used where the height from which the water comes is not great enough to flow on the upper part of the wheel. The lower part of undershot wheels dips in the flowing water which strikes against their float-boards and carries them along. Such wheels are used in swift-flowing streams which have a scarcely perceptible fall, as, for instance, on the Rhine. In the immediate neighbourhood of such a wheel, the water need not necessarily have a great fall if it only strikes with considerable velocity. It is the velocity of the water, exerting an impact against the float-boards, which acts in this case, and which produces the motive power. Windmills, which are used in the great plains of Holland and North Germany to supply the want of falling water, afford another instance of the action of bodies having velocity. The sails are driven by air in motion, that is, by wind. Air at rest could just as little drive a windmill as water at rest a water-wheel. The driving force depends here on the velocity of moving masses.

A bullet resting in the hand is the most harmless thing in the world; by its gravity it can exert no great effect; but when fired and endowed with great velocity it drives through all obstacles with the most tremendous force.

If we lay the head of a hammer gently on a nail, neither its small weight nor the pressure of our arm is quite sufficient to drive the nail into wood; but if we swing the hammer and allow it to fall with great velocity, it acquires a new force, which can overcome far greater hindrances.

These examples teach us that the bodies moving with a cer-

tain velocity possess a store of motive force. In some works on mechanics, and in the earlier part of the present work, velocity, in so far as it is motive force, and can produce work, that is, kinetic energy, is still often called *vis viva*. The name is not well chosen; it is too apt to suggest to us the force of living beings. In these cases, also, it may be seen, from the instances of the hammer and of the bullet, that velocity is lost as such, when it produces working power. In the case of the water-mill, or of the windmill, a more careful investigation of the moving masses of water and air is necessary to prove that part of their velocity has been lost by the work they have performed.

328. **The pendulum.**—The relation of velocity to working power is most simply and clearly seen in a simple pendulum,

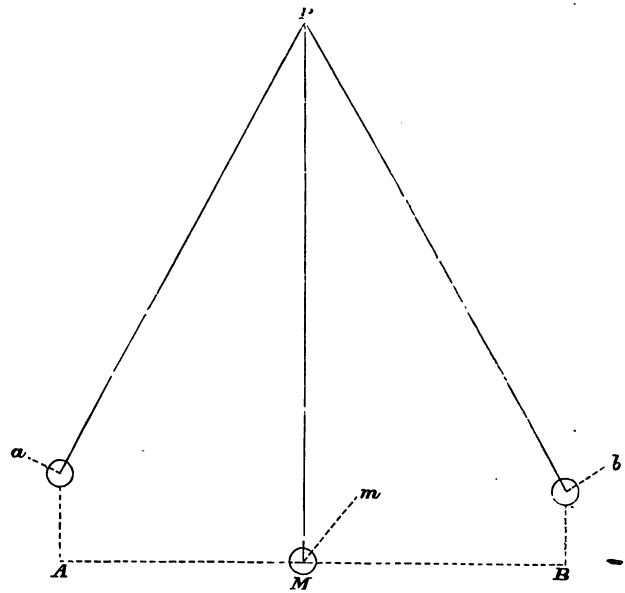

Fig. 306.

such as can be constructed by any weight which we suspend to a cord. Let M, *fig.* 306, be such a weight, of a spherical form; A B a horizontal line drawn through the centre of the sphere; P the point at which the cord is fastened. If, now, we draw the

weight M on one side towards A, it moves in the arc M *a*, the end of which, *a*, is somewhat higher than the point A in the horizontal line. The weight is thereby raised to the height A *a*. Hence the arm must exert a certain force to bring the weight to *a*. Gravity resists this motion and endeavours to bring back the weight to M, the lowest point which it can reach.

Now, if after we have brought the weight to *a*, we let it go, it obeys this force of gravity and returns to M, arrives there with a certain velocity, and no longer remains quietly hanging at M as it did before, but swings beyond M towards *b*, where its motion stops as soon as it has traversed on the side of B an arc equal in length to that on the side of A, and after it has risen to a distance B *b* above the horizontal line, which is equal to the height A *a*, to which our arm had previously raised it. In *b* the pendulum returns, swings the same way back through M towards *a*, and so on, until its oscillations are gradually diminished, and ultimately annulled by the resistance of the air and by friction.

We see here that the reason why the weight, when it comes from *a* to M, does not stop there, but ascends to *b*, in opposition to the action of gravity, is only to be sought in its velocity. The velocity which it has acquired in moving from the height A *a* is capable of again raising it to an equal height B *b*. The velocity of the moving mass M is thus capable of raising this mass ; that is to say, in the language of mechanics, of performing work. This would also be the case if we had imparted such a velocity to the suspended weight by a blow.

From this we learn further how to measure the working power of velocity—or, what is the same thing, the *vis viva* of the moving mass. It is equal to the work, expressed in foot-pounds, which the same mass can exert after its velocity has been used to raise it, under the most favourable circumstances, to as great a height as possible. This does not depend on the direction of the velocity ; for if we swing a weight attached to a thread in a circle, we can even change a downward motion into an upward one.

The motion of the pendulum shows us very distinctly how the forms of working power hitherto considered—that of a raised weight and that of a moving mass—may merge into one another. In the points *a* and *b*, *fig.* 306, the mass has no velocity ; at the point M it has fallen as far as possible, but possesses velocity. As the weight goes from *a* to *m*, the work of the raised weight is changed into *vis viva* ; as the weight goes *further from m to b* the *vis viva* is changed into the work of a

raised weight. Thus the work which the arm originally imparted to the pendulum is not lost in these oscillations, provided we may leave out of consideration the influence of the resistance of the air and of friction. Neither does it increase, but it continually changes the form of its manifestation.

329. **Elasticity.**—We may now pass to other mechanical forces, those of elastic bodies. Instead of the weights which drive our clocks, we find in time-pieces and in watches steel springs which are coiled in winding up the clock, and are uncoiled by the working of the clock. To coil up the spring we consume the force of the arm ; this has to overcome the existing elastic force of the spring as we wind it up, just as in the clock we have to overcome the force of gravity which the weight exerts. The coiled spring can, however, perform work ; it gradually expends the acquired capability in driving the clockwork.

If I stretch a cross-bow and afterwards let it go, the stretched string moves the arrow ; it imparts to it force in the form of velocity. To stretch the cord my arm must work for a few seconds ; this work is imparted to the arrow the moment it is shot off. Thus the cross-bow concentrates into an extremely short time the entire work which the arm had communicated in the operation of stretching ; the clock, on the contrary, spreads it over one or several days. In both cases no work is produced which my arm did not originally impart to the instrument, it is only expended more conveniently.

The case is somewhat different if by any other natural process we can place an elastic body in a state of tension without having to exert the arm. This is possible, and is most easily observed in the case of gases.

330. **Expansive force of gases.**—If, for instance, a firearm loaded with gunpowder be discharged, the greater part of the mass of the powder is assorted into gases at a very high temperature, which have a powerful tendency to expand, and can only be retained in the narrow space in which they are formed by the exercise of the most powerful pressure. In expanding with enormous force they propel the bullet and impart to it a great velocity, which we have already seen is nothing else but a form of work. In this case the work is gained without an exertion of the arm. But, on the other hand, something is lost ; the constituents of the gunpowder have changed into chemical compounds, from which they cannot without great trouble be restored to their original condition. Here, then, a chemical change has taken place, under the influence of which work has been done.

By the aid of heat, applied to gases, elastic forces are produced on a far greater scale. As the most simple instance we may take atmospheric air. In the treatise on HEAT, in this series, page 80, *fig.* 35, an apparatus is represented used by Regnault for measuring the expansive force of heated gases. The air-thermometer represented on page 304, *fig.* 86, of that volume, acts on precisely the same principle. It consists essentially of a globe of glass, containing dry air, which can be placed in a vessel and heated. The globe is connected with a U-shaped tube, which contains mercury, and the limbs of which can be placed in communication with one another. If the mercury is in equilibrium in the tube, when the globe is cold, it rises in one leg when the globe is heated. If, on the contrary, when the globe is heated equilibrium be artificially restored, the liquid will be drawn up in the leg nearer to the globe, as the latter becomes cold again. In both cases liquid is raised and thereby work done.

In steam-engines the same effect is constantly produced on the largest scale, although in this case, in order to keep up a continual disengagement of compressed gas from the boiler, the air of the globe in the preceding experiment is replaced by water, which, by the application of heat, is gradually changed into steam. Steam, so long as it remains as such, is an elastic gas which endeavours to expand exactly like atmospheric air. And instead of the column of liquid which was raised in the last experiment, the machine is caused to drive a solid piston which imparts its motion to the other parts of the machine. We see, then, that heat can produce mechanical work. In the cases which have been discussed previously, it has been seen that the amount of energy produced by a given measure of a physical process is always accurately defined, and that the further capacity for work of the natural forces is either diminished or exhausted by the work which has been performed. With heat it is precisely the same, as shown in the chapter on the dynamical theory of heat, in the treatise on HEAT in this series.

331. **Transmutation of Energy. Law of Conservation of Energy.**—During the last twenty or thirty years a new universal law of all natural phenomena has been recognised which possesses not only an extraordinarily extended range, but constitutes a general connection between natural phenomena of all kinds, even of the remotest times and most distant places. This law is called the *law of conservation of energy.* It is not absolutely new; for individual domains of natural phenomena

it was enunciated by Newton and Daniel Bernoulli; and Rumford and Humphry Davy have recognised distinct features of its presence in the laws of heat. The possibility that it was of universal application was first stated by *Dr. Julius Robert Meyer*, a German physician living in Heilbronn, in the year 1842, while almost simultaneously with, and independently of him, *James Prescott Joule*, a manufacturer, living in Manchester, made a series of important and difficult experiments on the relation of heat to mechanical force, which supplied the chief points in which the comparison of the new theory with experience was still wanting.

Energy is defined as capacity of doing work. Now to understand the law in question let us repeat as briefly as possible some examples of energy, which have been treated in detail in the preceding articles.

A clock, when wound up, has energy, by the lowering of its weight, to give its parts motion, overcoming their friction, &c., until it has run down.

The clock will not start, perhaps, without help—without a tap given to the pendulum; but the capacity, the amount of change that can be produced, does not depend on this, but clearly on the amount of winding.

A coal, burning in the air, has energy to convert itself into the products of combustion, water, carbonic acid gas, &c., producing heat during the operation.

Here, again, the coal will not begin its action without help, a lighted match for instance. But the match burns itself out, and the coal burns itself out; and the energy of the latter is, *cæteris paribus*, proportional to the quantity of coal.

A cannon ball in motion has energy, if so directed, to pierce a target, or to raise itself in the air, &c., and the energy is the greater the greater the initial velocity; and there exists an exact relation between its initial state and its total capacity for work.

We can see clearly enough that the burning coal and the cannon ball impinging on an obstacle are producing change in the condition of surrounding objects, while their own 'energy' is thus being spent: the heated room and the broken target exhibit 'external work' as the result. And we have seen that this is true also of the clock running down, and in all cases of an exhaustion of power in the original agent, which is not compensated for.

But internal change may be conspicuously going on in a body or system of bodies, while external effect is quite or almost inappreciable.

Thus, a pendulum set swinging in the exhausted receiver of an air-pump (with due precautions taken in the suspension) will go on oscillating for days perhaps, and without perceptible flagging for some thousands of oscillations together.

And we can imagine, though not actually produce, a perfect vacuum, and an arrangement for support or suspension without friction or stiffness; under which circumstances this action may be supposed to continue for ever. The Planetary Motions are another more nearly perfect instance. The Earth, for instance, changes its distance from the Sun, alternately approaching and receding from it, with no slackening that we have yet been able to detect by the nicest observations.

It must, however, be carefully noted that in every case this 'internal work' is alternate in character.

The bob of the pendulum is always changing its condition, passing from one extreme, characterised by the maximum of elevation with no motion, at the moment when it is passing from its upward to its downward course, to the other extreme of maximum motion at the lowest point the bob can reach. And in all intermediate stages it has less or more of elevation. We may then distinguish between the 'capacity for raising itself,' which it has in virtue of its state of motion (like that of the cannon ball), and the 'capacity for giving itself motion,' which it owes to its elevation (as the clock can run itself down); and we may describe this continual action as consisting in the alternate exhaustion and renewal of each of these two kinds of energy.

With this explanation, then, it is equally true of 'internal' action, that is, of the relative and mutual changes of the parts of what we consider as one system—as of 'external,' where the action may extend over an indefinite range; that (so far as our examples go) each and every energy, every capacity or power of work, exhausts itself as it does its proper work.

Now the Physical Truth which the succeeding articles of this chapter are intended to expound is this:—

Every kind of change which takes place in bodies and systems of bodies, whether due to their internal or to their mutual actions, can be represented numerically by reference to one certain standard unit of change, or work done, in such wise that the result of the passage of any system from one condition to another may be calculated by mere additions and subtractions, even when we do not know how the change came about. And if such result be positive—that is, more additive than subtractive—the system has gained so much total energy (or power to

do work of some kind) at the expense of some other system ; if negative—that is, more subtractive than additive—then it has, so to say, parted with so much energy and handed it over to some other system. A change which adds energy to the system is 'work done upon it' ; when the energy is found to have been diminished it has 'done work' on some other system. Thus, winding up the clock is doing work upon it, exhausting our own muscular energy in the operation, giving motion to the cannon ball exhausts the power of the gunpowder. All this will be now clear from the previous statements and illustrations.

332. **Changes in Inorganic Nature.**—The principal *changes* which occur in inorganic nature—that is, in every thing around us not referable to animal or vegetable life—may be, conveniently for our purpose, roughly classed and enumerated as follows :—

Motions and changes of position of bodies ; either as of wholes retaining the relative position of their parts, or among their component parts. These may be called Dynamical Changes.

Changes of temperature ; or, Thermal changes.

Changes in the internal condition of a body : as expansion, change from solid to liquid, or liquid to gaseous, which are, in a narrow sense of the word, called Changes of Physical Condition.

Changes of Electric Condition.

Changes in the way of Composition and Decomposition of the Chemical elements, of which all bodies are made up—that is, Chemical Changes.

When we speak of Dynamical, Thermal, Physical (or internal), Electric, and Chemical Energy, it will now be understood, that in general we refer to capacities for work of some kind which the bodies or systems possess in virtue of their condition as to motion and position, temperature, &c., respectively.

For example, a piece of coal may be in motion, and so act dynamically, like the cannon ball ; this would be Dynamical Energy. But it can also burn, and in burning may do various kinds of work—heat a room, or convert ice into water, &c. ; whatever the nature of the work done in burning, the energy is chemical.

333. **Transmutatory Series.**—Let us now imagine, for the sake of illustrating our law, the following series of physical phenomena and events, following in the order as given by Mr. D. D. Heath, in his exposition of Energy :—

I. A quantity of water is at rest in an elevated reservoir with a sluice gate.

II. The sluice opened, allowing the water to fall (vertically, or in a sloping channel), gaining velocity as it loses height.

III. The falling water encountering a water-wheel, whereby it loses some of its own motion, and imparts motion to the wheel and machinery connected therewith.

IV. Heat produced by collision or friction, both where the water clashes on the wheel, and wherever there are rubbing surfaces; and most sensibly at the grinding surfaces of the mill-stones.

V. The mill-stone working in a chamber of water, heating it up to 'boiling point.'

VI. When this point is reached, the water will not rise any more in temperature, but will gradually pass into steam.

VII. The steam so generated works a steam-engine.

VIII. Which works an electric machine—that is, it turns a glass cylinder which, as it turns, rubs on one side against a properly prepared cushion (the rubber), and on the other passes close in front of some metallic spikes connected with a large metallic surface (the 'prime conductor'). Heat is produced as in all cases of rubbing; but, besides this, the prime conductor becomes 'electrically charged,' that is, it will exhibit all the ordinary properties of an electrified body.

IX. If a metallic knob project from the conductor and the hand be held near it, a spark of fire will flash between the knob and the hand, and a shock be felt.

If, instead of this, a thin wire be carried from the conductor to moist earth, the wire may become sensibly heated. In either case the conductor is discharged, to be charged again by the action of the steam-engine.

X. If the discharging wire, or a wet thread, is carried, with a slight break, through a drop of water mixed with a little sulphuric acid, on its way to the earth, it will show less heat, but will to a small extent decompose the water into its two component elements, the gases oxygen and hydrogen.

XI. This decomposition will be so small in amount in any practicable case, that the series would close here. But we can imagine the experiment performed on so gigantic a scale as to make it sensible; or, another process of a less familiar kind may be adopted, in which no ordinary friction (as of the rubber and glass) occurs, and therefore the purely electric effect is greater.

XII. The oxygen and hydrogen thus set free may be re-composed—that is, the hydrogen gas burned in the oxygen—and steam again produced, which might work a pump, and so

help to replenish the reservoir, the emptying of which has been the cause of all these changes, which all will cease when the reservoir is emptied or the sluice gate shut.

The proposition involved in the law of conservation of energy, then is, that we may represent numerically, (1) the power to produce changes which the water in the reservoir has in virtue of its quantity and its height above the mill-wheel; (2) how much of the power due to motion the water gains as it loses height in falling; (3) how much in motion, and how much in heat it gives to itself and to the mill-wheel when it clashes on it; (4) how much heat goes to turn water into steam; (5) what the steam loses in working the steam-engine; (6) how much the steam-engine is retarded by working the electric machine and producing heat and decomposing the water.

And further, that the sum of all the effects so produced by the falling of a given quantity of water down the given channel will be equal to the loss of energy, or capacity to continue to produce such changes, caused in the reservoir of water by its gradual emptying.

If nothing were lost or otherwise disposed of on the way, and we could hand over to the final pump all the energy taken from the water in the reservoir, it would just restore the water to the reservoir at the same rate at which it parts with it; and there would thus be 'perpetual motion' kept up in the complete system—the same ideal state as in our imaginary pendulum.

But this is, in fact, impossible. Heat is necessarily produced all through the machinery where there is clashing or rubbing, which goes off to surrounding bodies and contributes nothing to boiling the water and working the steam-engine. And, even if we suppose all clashing of unequal motions avoided, and all surfaces 'perfectly smooth,' and no radiation of heat from the surface of the vessel containing the grindstones, &c., all which conditions may be more or less approximated to, though they cannot be attained, still the working of the steam-engine and of the electric machine necessarily and essentially (not as it were accidentally) requires the production of heat that must be thrown away (not destroyed, but taken out of the circuit of operations we are considering); in the one the heated steam which works the piston must be either cooled in the condenser or suffered to escape into the air; and in the other the friction of the glass and rubber must heat their surfaces as well as produce electrical charge. Some considerable part, therefore, of the original energy of the system, which we may consider as stored up in

the reservoir, must leave the system in the form of heat, and does not contribute to keep the system going perpetually.

And it is worth observing, too, that even if we could get what is called a perfectly 'conservative system,' one of which our pendulum would be the simplest form, if we could absolutely get rid of friction and air—it would be only self-sacrificing; could do nothing, could produce neither heat nor any other action beyond its own system.

334. **Problems on work and energy.**—The following problems will further illustrate the principles of work and energy explained in this chapter:

1. What must be the effective horse-power of a locomotive engine which moves uniformly at 30 miles per hour on a level rail, the weight of the train being 50 tons, and the friction 8 lbs. per ton.

$$\text{Resistance of friction} = 8 \times 50 = 400.$$

$$\text{Distance moved over in feet per minute} = \frac{30 \times 5280}{60}$$

$$= 2640.$$

$$\text{Work of friction per minute } 400 \times 2640.$$

$$\therefore \text{ Horse power } \frac{400 \times 2640}{33000} = 32.$$

2. At what rate, in miles per hour, will a train of 80 tons be drawn by an engine of 70 horse power (friction as before).

$$\text{Work done by the engine } 70 \times 33000 \times 60.$$

$$\text{Work done by the train in 1 mile} = 8 \times 80 \times 5280.$$

$$\text{Number of miles} = \frac{70 \times 33000 \times 60}{8 \times 80 \times 5280} = 41.01.$$

3. An engine of 40 horse power moves with a maximum speed of 35 miles upon a level rail, what is the weight of the train?

$$\text{Work of engine } 40 \times 33000 \times 60$$

$$\text{Work done by weight w} \qquad 8 \times 35 \times 5280 \times w$$

$$\text{Weight} = \frac{40 \times 33000 \times 60}{8 \times 35 \times 5280} = 53{\cdot}5 \text{ tons.}$$

4. A carriage becomes suddenly detached from behind a train and is observed to pass over a space of 1320 metres in 264 seconds before it stops. What was the velocity of the train when it became detached, and what is the ratio of the retarding force to the weight of the carriage?

$$a)\ s = \tfrac{1}{2} v t$$

$$\therefore v = \frac{2s}{t} = \frac{2 \times 1320}{264} = 10 \text{ metres.}$$

b) $v = ft$

$$= \frac{P}{W} g t \therefore \frac{P}{W} = \frac{v}{gt} = \frac{10}{9{\cdot}81 \times 264}$$

$$= \frac{1}{260}.$$

5. A ball weighs 25 lbs. and is projected from a gun with a velocity of 1000 feet against a wooden target, into which it penetrates 2 feet. What is the amount of resistance of the target?

$$\frac{W v^2}{2 g} = \text{Resistance} \times \text{space}$$

$$= \text{Resistance} \times 2$$

$$\frac{25 \times (1000)^2}{4 \times 32} = \text{Resistance}$$

$$= 195{,}312{\cdot}5 \text{ lbs.}$$

6. A railway carriage of weight = 110 cwt. is to be moved by two men, each pressing against it with a force of 30 lbs. If the friction is $\frac{1}{250}$ of the weight, what work is required to produce a velocity of $3\frac{3}{4}$ feet, and through what space will the men have to push it, in order to attain this velocity?

$$\frac{110 \times 112}{32} = \text{Mass} \quad \therefore \frac{110 \times 112}{32} \times (3\tfrac{3}{4})^2 = \text{work required.}$$

Force of men = 60 lbs.

$$\text{Friction} = \frac{110 \times 112}{250}.$$

$$\text{Hence constant force} = 60 - \frac{110 \times 112}{250} = F.$$

$$\text{But } F s = \frac{M v^2}{2}$$

$$\therefore \left(60 - \frac{110 \times 11}{250}\right) s = \frac{110 \times 112}{32} \cdot (3\tfrac{3}{4})^2 \text{ which gives } s.$$

7. A train, weighing 100,000 lbs., has a velocity of 20 feet when the steam is shut off.

$$\text{Hence its energy} = \tfrac{1}{2} \frac{100000}{32} (20)^2 = F s.$$

$$= 640000 \text{ foot-pounds.}$$

Suppose that the train stops after passing over 6000 feet,

$$F\ 6000 = 640000$$

$$\text{Hence the resistance} = F = \frac{640000}{6000} = 106\tfrac{2}{3}.$$

8. A train of 80 tons ascends an incline which has a rise of 2 feet in every 100 feet, with a uniform speed of 15 miles per hour; what must be the effective horse-power of the engine? Friction = 8 lbs. per ton.

Speed of train per minute 1320 feet.

Weight of train 179200 lbs.

Rise of plane = $\frac{2}{100}$ in one foot.

$$\text{Rise in 1320 feet} = \frac{1320 \times 2}{100} \text{ feet.}$$

$$= 26{\cdot}4 \text{ feet.}$$

The whole is raised 26·4 feet in opposition to gravity.

Work due to gravity $179200 \times 26{\cdot}4 = 4730880$.

Work due to friction $8 \times 80 \times 1320 = 844800$.

Total work $4730880 + 844800 = 5575600$.

$$\text{Horse-power} = \frac{5575600}{33000} = 168{\cdot}9.$$

9. A train of 100 tons ascends an incline ($\frac{1}{6}$ in 100). Find maximum speed when the effective horse-power = 80. Friction, 8 lbs. per ton.

$$\text{Rise in 1 foot} = \frac{1}{600} \text{ foot.}$$

$$\therefore \text{ in } x \text{ feet} = \frac{x}{600}.$$

Weight of train in lbs. $= 100 \times 2240$.

$$\text{Work due to gravity } 100 \times 2240 \times \frac{x}{600}.$$

$$\text{,, friction } 8 \times 100 \times x.$$

Work done by engine $= 80 \times 33000 \times 60$.

$$\therefore \left(100 \times 2240 \times \frac{x}{600}\right) + 100 \times 8 \times x = 80 \times 33000 \times 60.$$

$$x = 25.5 \text{ miles.}$$

10. A train has a velocity of 40 feet at the time when the steam is shut off. Supposing the weight of the whole to be 250 tons, during what time and over what space will it move before it comes to rest, if the resistance of the air and friction is together $\frac{1}{200}$ part of the moved weight?

Generally, we have

$$\text{Acceleration} = \frac{P}{W} g = f.$$

v = velocity acquired in time $t = \frac{P}{W} g\, t.$ $\left(P = \frac{v \cdot W}{g \cdot t}\right)$.

s = space passed over in time $t = \frac{P}{W} \cdot \frac{g}{2} \cdot t^2$.

If a body has a velocity c, and its weight be W, and during t seconds a force P acts in opposite direction, then its velocity after t seconds—

$$v = c - \frac{P}{W} g t \quad . \quad . \quad . \quad . \quad (1)$$

$$s = c t - \frac{P}{W} \cdot \frac{g}{2} t^2 \quad . \quad . \quad . \quad (2).$$

Hence the motion comes to an end if

$$c = \frac{P}{W} g t,$$

which gives for the time $t = \frac{W \cdot c}{P \cdot g}$.

Hence substituting this in (2), the whole space over which the body passes before it comes to rest is

$$s = c \cdot \frac{c \cdot W}{P \cdot g} - \frac{P}{W} \cdot \frac{g}{2} \cdot \frac{W^2 \cdot c^2}{P^2 \cdot g^2}$$

$$= \frac{c^2 W}{P \cdot g} - \tfrac{1}{2} \frac{c^2 W}{P \cdot g} = \frac{c^2 \cdot W}{2 \cdot P \cdot g}.$$

Applying this to the present case,

original velocity = 40
W = 250 tons
P = $\frac{1}{120} \cdot 250$ = $1\frac{1}{4}$,,

$$t = \frac{250 \cdot 40}{\frac{5}{4} \cdot 32} = 250 \text{ seconds.}$$

$$s = \frac{40^2 \cdot 250}{2 \cdot \frac{5}{4} \cdot 32} = \frac{40 \cdot 40 \cdot 250 \cdot 4}{2 \cdot 5 \cdot 32} = 5000 \text{ feet.}$$

11. A body of mass 12, rests upon a smooth horizontal surface; what momentum will be imparted to it in 5 seconds by a force equal to the weight of 18 lbs. And how long must a force in absolute measure of 48 act in opposition to its motion in order to bring it to rest?

(α) Velocity in one second = $\frac{18}{12} g = \frac{3}{2} \cdot 32$;

in 5 seconds = $5 \cdot 3 \cdot 16 = 240$;

Momentum $12 \times 5 \times 3 \times 16 = 48 \cdot 60 = 2880$.

(β) A force of 1 produces in mass 1 a velocity of 1 ft. in 1 sec.

48 ,, 1 ,, 48 ,,

,, ,, 12 ,, $\frac{48}{12} = 4$,,

$\therefore 4x = 240$, $x = 60$ seconds.

12. A weight of 30 lbs. moves with a velocity of 100 feet per second, when it is subjected to a uniformly retarding force of 96

in absolute measure. How far will it move in the first 5 seconds? How far until it stops? and how long will it move until then?

$$\text{The acceleration is } \frac{\frac{96}{32}g}{30} = \frac{1}{10}g.$$

$$\therefore s = 5 \times 100 - \tfrac{1}{2} \times \tfrac{1}{10} \times 32 \times 25$$
$$= 500 - 8 \times 5 = 460.$$

$$\text{Since } 0 = v - \tfrac{1}{10} g t$$
$$\therefore 100 = \tfrac{32}{10} t \text{ or } t = 31\tfrac{1}{4} \text{ seconds.}$$

$$\text{Hence space passed over until it stops} = s$$
$$= 100 \times 31\tfrac{1}{4} - \tfrac{1}{2} \times \tfrac{1}{10} \times 32 \times (31\tfrac{1}{4})^2.$$

13. A ball weighing w lbs. is projected on a horizontal plane with the velocity v per second. What space, s, will the ball move over, before it comes to rest, if f = coefficient of friction.

$$\text{The retarding force} = f w.$$

$$\text{Work destroyed by it} = f w s.$$

$$\text{Energy in the ball} = \frac{w v^2}{2g}.$$

$$\text{When the ball stops } \frac{w v^2}{2g} = f w s$$

$$\therefore s = \frac{v^2}{2gf}.$$

In order to find the time t, before the ball stops, it is easily done by considering that the mean velocity of the ball throughout is $\frac{v}{2}$, and dividing the space by this mean velocity, we obtain t,

$$\frac{\frac{v^2}{2gf}}{\frac{v}{2}} = t = \frac{v}{gf}.$$

14. A railway train, weighing T tons, has a velocity of v_1 feet per second, when the steam is turned off; what distance will it pass over, and in what time, in order to have a velocity v, if the coefficient of friction is p lbs. per ton?

(α) Work lost $= p$ T s,

which is equal to the work in the train when the velocity is v,

$$\therefore p \text{ T } s = \frac{2240 \text{ T } (v_1^2 - v^2)}{2g}, \quad s = \frac{2240 (v_1^2 - v^2)}{2pg}.$$

(β) Mean velocity $= \frac{v_1 + v}{2}$,

$$t = \frac{s}{\frac{v_1 + v}{2}} = \frac{2s}{v_1 + v}$$

$$= \frac{2240\,(v_1 - v)}{pg}.$$

15. *Work done in overcoming the resistance of friction.*—It has already been stated, that the principal resistance to be overcome by a body moving horizontally on a rough surface is *friction*. The experimental laws are:

(1) The resistance of friction, on a given surface, is always a certain proportional part of the weight of the body.

(2) The resistance is independent of the extent of the rubbing surfaces.

(3) It is not affected by any change in the rate of the body's motion.

Let P be the resistance of friction if it just moves a weight w resting on a horizontal surface. For the same rubbing surfaces P is always a definite proportional part of w; if this is f, then, putting F the resistance of friction for P,

$$F = f\,w.$$

f is the coefficient of friction, and its value depends on the nature of the rubbing surfaces. Thus, if the coefficient of friction is $\frac{1}{30}$, the force required just to move a ton along a horizontal road is

$$\tfrac{1}{30} \cdot 2240 = 74\tfrac{3}{4} \text{ lbs.}$$

Similarly, if 80 lbs. just pull 2240, the coefficient is $\frac{80}{2240} = \frac{1}{28}$.

Since the resistance of friction is $f\,w$, the units of work done by overcoming this resistance through s feet,

$$\text{are } \therefore \text{ Units} = f\,w\,s.$$

Thus: required the work performed per minute in sustaining a body of w lbs. at a uniform rate of m miles per hour on a horizontal plane, whose coefficient of friction $= f$.

$$\text{Resistance of friction} = f\,w.$$

$$\text{Space moved over in feet per minute} = \frac{m \times 5280}{60} = 88\,m.$$

$$u = \text{units of work} = f \cdot w \cdot 88 \cdot m.$$

$$\text{If } w \text{ were unknown, } \therefore w = \frac{u}{88 \cdot f \cdot m}.$$

16. A body moves on a rough horizontal surface with a velocity of 50 feet per second, coefficient of friction $= \frac{1}{8}$. How long and how far will it move?

$$t = \frac{\text{Momentum}}{\text{Resistance}} = \frac{w\,v}{g} \div \frac{w}{8} = \frac{50 \cdot 8}{32} = 12\tfrac{1}{2}$$

$$s = \frac{\text{Energy}}{\text{Resistance}} = \frac{w\,v^2}{2\,g} \div \frac{w}{2} = \frac{2500 \times 8}{64} = 312{\cdot}5.$$

17. A train moves up an incline rising 1 in 240 with a speed of 30 miles an hour, the friction being $\frac{1}{30}$. Find the space it will move over when the steam is shut off.

$$\frac{w\,v^2}{2\,g} = \frac{w\,x}{30} + \frac{w\,x}{240} \text{ which gives the space } x.$$

18. A body of weight w falls down a rough plane inclined to the horizon at an angle a. Coefficient of friction μ. Find the space passed over in t seconds.

$$\text{Pressure on plane} = w \frac{\text{base}}{\text{length}} = w \cos a$$

$$\therefore \text{ friction} = \mu\, w \cos a.$$

The pressure down the plane $= w \sin a$.

Whole force down $w \sin a - \mu\, w \cos a$.

$$\text{Acceleration} = g(\sin a - \mu \cos a)$$

$v = f t \quad s = \frac{1}{2} f t^2$ become

$$v = g(\sin a - \mu \cos a)\, t \qquad s = \tfrac{1}{2}\, g(\sin a - \mu \cos a)\, t^2$$

Suppose that after t seconds the body runs again up the plane; then velocity after t seconds,

$$g(\sin a - \mu \cos a)\, t.$$

$$\text{Energy} = \frac{w \cdot g^2 (\sin a - \mu \cos a)^2\, t^2.}{2\,g}$$

Work done against friction $w \cdot \cos a \cdot \mu \cdot x$.

Work done in lifting the weight $= w \cdot x \sin a$,

from which the space it will rise =

$$x = \frac{g\,t^2 (\sin a - \mu \cos a)^2}{2(\sin a + \mu \cos a)}.$$

19. *Work done by machines.*—Let A B C represent an inclined plane, A B = base, A C = length, and B C = height of plane, then work done on the inclined plane = weight of body × vertical height raised.

If W is raised on the plane by P descending vertically downwards, then P descends a length A C, W rises a length B C,

$\therefore$ Work in raising W up the plane = P · A C;

but P : W :: B C : A C

$$\therefore \text{Work} = \frac{\text{W} \cdot \text{B C}}{\text{A C}} \cdot \text{A C} = \text{W} \cdot \text{B C}.$$

If f the coefficient of friction, then

Work done in moving W from A to C

= work due to friction + work due to gravity

$= f \cdot W \cdot AC + W \cdot BC.$

If the body moves down the plane,

Work done in moving W from C to A $= f \cdot W \cdot AC - W \cdot BC.$

Thus: A horse can just pull with a force of L lbs. What weight, W, will he be able to pull up, and also down the hill, if the coefficient of friction $= f$, and the inclination of the hill is a rise of e feet in 100?

Work done over 100 feet by horse = 100 L.

Work due to resistances over 100 feet $= 100f \cdot W \pm W \cdot e$

$$\therefore 100\,L = 100fW \pm We.$$

$$W = \frac{100\,L}{100f \pm e}.$$

20. The length of an inclined plane is 100 ft.; a weight of 50 lbs. rests upon it. If the coefficient of friction is $\frac{1}{25}$, what must be the height of the plane when the weight is just on the point of sliding?

Work done by gravity $= 50 \times x$ (x = height)

,, friction $\frac{50}{25} \times 100 = 200$

$$50 \times x = 200$$

$$x = 4 \text{ ft.}$$

The angle of friction is the angle of inclination when the body is on the point of sliding.

21. The length of an inclined plane is 100 feet, height 0·2 feet, weight of body 90 lbs. Coefficient of friction $\frac{1}{30}$. What pressure is required to move the body down the plane?

Resistance of friction $= \frac{1}{30} \cdot 90 = 3$ lbs.

Work due to friction $= 3 \times 100 = 300$

Work due to gravity $= 90 \times \cdot 2 = 18$

Work to be done $= 300 - 18 = 282.$

Work done by force $= P \times 100 = 282 \therefore P = 2 \cdot 82$ lbs.

22. The length of a wedge is 1·5 feet. Thickness = ·2 feet. Pressure upon it = 100 lbs.

When the pressure applied has moved through the length, the work applied is

$$= 100 \times 1 \cdot 5,$$

The work done is $R \times \cdot 2$, R being the resistance to be overcome.

$$\therefore R = \frac{100 \times 1 \cdot 5}{\cdot 2}$$

$$= 750 \text{ lbs.}$$

23. In a compound screw the length of the lever is 1·5 feet, the distance between the threads of the larger or hollow screw is $\frac{3}{4}$ inch, that between those of the smaller screw $\frac{1}{2}$ inch. If 30 lbs. pressure be applied to the lever, what will be the pressure upon the press-board?

$$\underbrace{30 \times 2 \times 1{\cdot}5 \times 3{\cdot}1416}_{\text{Work applied} = 282{\cdot}74} = W\underbrace{\left(\tfrac{3}{4} - \tfrac{1}{2}\right)}_{\frac{1}{48}\text{ foot.}}$$

$$W = 13571{\cdot}5.$$

24. In a compound wheel and axle, the diameter of the larger cylinder is ·7 feet, of the smaller ·5 feet. The handle is 2 feet long, and a weight of 400 lbs. is to be raised. Find the power.

The cord during each revolution is shortened by the difference of the circumferences, and the weight is raised through half this difference.

Work done by power $= 2 \times 2 \times 3{\cdot}1416 \times P$

Work done by weight $= \dfrac{{\cdot}7 \times 3{\cdot}1416 - {\cdot}5 \times 3{\cdot}1416}{2} \times 400.$

$$4P = \frac{{\cdot}7 - {\cdot}5}{2} \times 400. \qquad \left\{ \begin{array}{l} 2R \times P = \dfrac{D-d}{2} W \\ W = \dfrac{4RP}{D-d}. \end{array} \right.$$

The equations on the right hand give the general solution, if R is the length of the handle, D and d the diameters of the larger and smaller portion of the cylinder respectively.

25. In a system of two movable pulleys, each weighing 3 lbs. if a power of 100 lbs. is employed, find W by the principle of work.

If P descends 1 foot, the first pulley rises $\frac{1}{2}$, the second $\frac{1}{4}$ of a foot.

Work done by $P = 100 \times 1$

,, by pulleys and weight $= W \cdot \frac{1}{4} + 3 \times \frac{1}{4} + 3 \times \frac{1}{2}$

$$100 = \frac{W}{4} + \frac{3}{4} + \frac{3}{2}, \quad \therefore W = 391.$$

BOOK THE FOURTH.

ILLUSTRATIONS OF THE APPLICATION OF MECHANICAL PRINCIPLES IN THE INDUSTRIAL ARTS.

CHAPTER XXI.

CRANES.

335. **Cranes** are pieces of mechanism usually consisting of combinations of toothed wheels and pulleys, by means of which materials for building are raised to the stage where the builders are engaged — goods elevated from vessels to the quays or wharves, and loaded on the waggons by which they are to be transported, or, on the contrary, transferred from the quays or wharves into the vessels—and for all similar purposes.

Fig. 307.

336. **Movable cranes.**—One of the most simple forms of movable cranes commonly used for building in France is shown in *fig.* 307.

It consists of a sort of strong triangular ladder, at the top of which is a fixed sheave C, over which the rope attached to the object to be elevated passing is carried down to the cylindrical axle T, upon which it is rolled by means of bars inserted in holes, as shown in *fig.* 169.

This ladder is inclined more or less from the vertical by means of a rope C D, which is

carried to some fixed object at a distance, to which it is at tached.

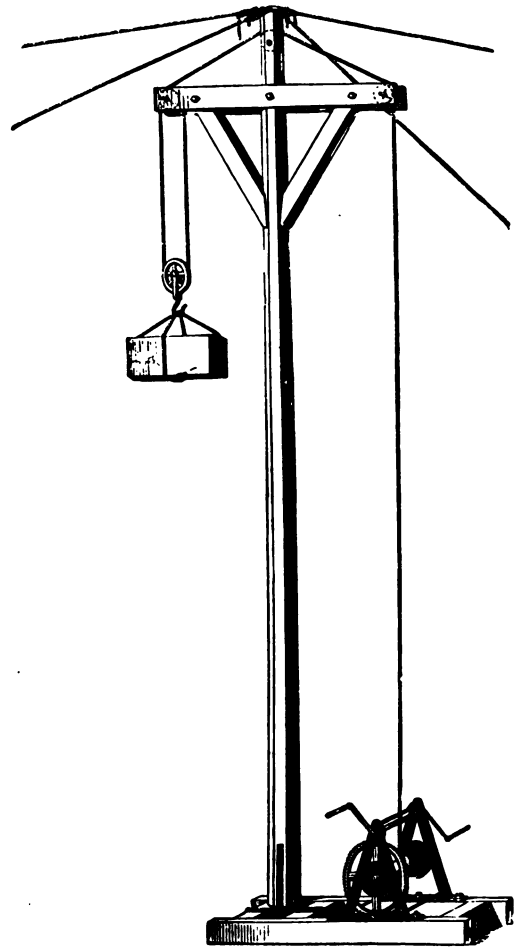

Fig. 308.

Another form of movable crane, also used for building purposes, is shown in *fig.* 308, which consists of a shaft driven by a combination of toothed wheels and a pair of winches. Upon

the shaft a rope is coiled, which, being carried up, is passed over three sheaves, and then, passing downwards, is attached to the object to be raised.

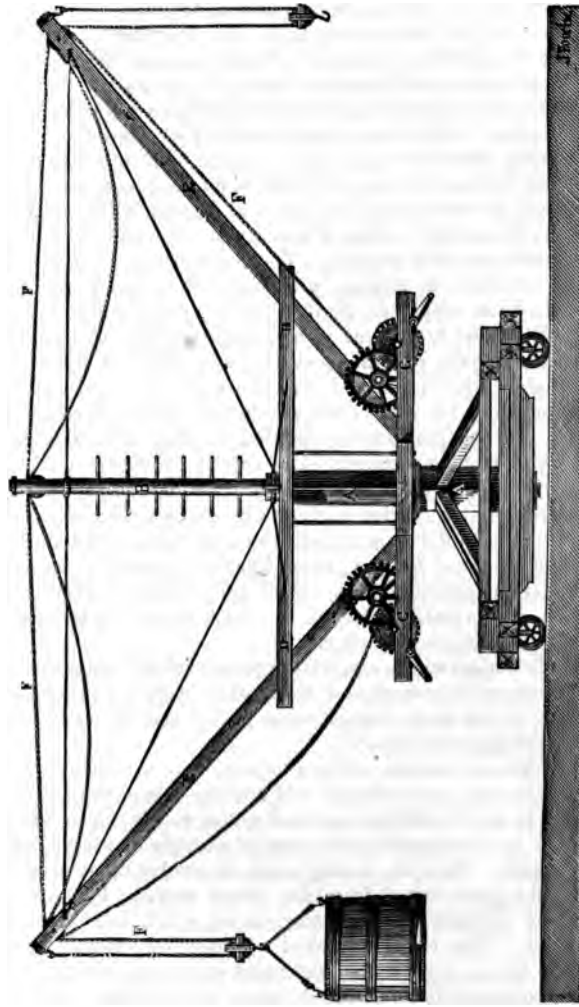

Fig. 309.

337. **Movable cranes for manufactories and railway stations.**—Another form of movable crane, much used in large

engineering establishments and railway stations, is shown in *fig.* 309. The figure is drawn on a scale of 1 inch to 12 feet.

The stand upon which the apparatus is fixed is supported on four small wheels, which, in some cases, are made to run upon rails like those of a railway waggon. It is evident, from what has been explained in Chapter V., Book II., that the apparatus must be so constructed that the centre of gravity of it and its load must be in a vertical line which shall fall within the quadrangle formed by the lines which join the four wheels.

A strong pillar of wood, A A, is secured upon the base of the apparatus by diagonal ties. In this a circular vertical hole is perforated, in which the round vertical beam B which supports the crane is inserted, so that it can revolve horizontally without much resistance from friction. The horizontal beams C C and D D are attached to collars, which also turn freely round A, while they are supported by it. The diagonal pieces E E are attached to these horizontal pieces, and also tied by iron rods to the central axis, both in horizontal and diagonal directions. Two winders with systems of wheelwork attached are fixed at C and C, by which the axles upon which the ropes are coiled are turned. In the position represented in the figure, the right-hand winch alone is worked. The rope F coiled upon its axle passes over a sheave at the top of the right-hand diagonal beam E, from which it is carried horizontally over a sheave at the top of the vertical pillar B, and thence to a sheave in the top of the left-hand diagonal beam E, from whence it passes downwards and under the movable pulley which supports the weight to be elevated, and is finally attached to a hook or ring at the top of the left-hand diagonal beam E.

The left-hand winch C and its apparatus of wheelwork acts in a similar manner over another series of sheaves in juxtaposition with the former upon the movable pulley suspended from the right-hand diagonal beam E.

338. **Fixed cranes.**—When cranes are stationary, as is usually the case upon wharves and landing-places, they are constructed in a more efficient manner, giving greater efficacy to the power by the interposition of a greater number of wheels, axles, and pulleys. They are usually made to revolve upon a centre, so that the goods raised from the vessel may, by lowering the crane, be brought over the waggon on which they are to be deposited. This is accomplished by inserting a round piece of metal, P Q (*fig.* 310), at the lower part of the apparatus, into a hole of corresponding magnitude made in the wharf. The level of the wharf being R Y, the crane, when it turns in the hole,

will carry the object elevated in a circle, of which Q X is the radius, round P Q, the centre or axis.

A crane thus constructed is represented, with all its accessories, in *fig.* 311, upon a scale of 1 to 100, or an inch to 8 ft. 4 in. The wheelwork is shown upon a larger scale and more evidently in *fig.* 312.

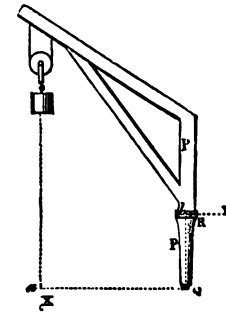

Fig. 310.

The weight is attached to a single movable pulley suspended from the upper end of the crane. The rope which is carried over the fixed sheave, and along the inclined arm of the crane to the axle A (*fig.* 311), is stretched by a force equal to half the weight raised. Upon the axle A is fixed a toothed wheel B, which is driven by a pinion C (*fig.* 312). Upon the axle of this pinion is fixed another large toothed wheel D, which is driven by a pinion E. Upon the axle of this pinion E is fixed another large toothed wheel F. The two axles of the wheels D and F being upon the same level, as shown in the figure, one conceals the other.

Two winches G and H work an axle upon which two pinions L and K are fixed, which are capable of being moved through a small space right and left horizontally by a lever M. These pinions are formed to engage in the wheels D and E, and can be thrown into or out of connection with their wheels by the lever M. As shown in the figure, both are out of connection with D and F, and the crane is inoperative. If we suppose them both in connexion with the wheels, the pinion L, driven by the winch H, will drive F, and consequently E, which will drive D. At the same time the pinion K, driven by the other winch G, will drive D. Thus, the wheel D will receive at once the effect of the two powers applied to the winches G and H. This wheel D will drive the pinion C fixed upon its axle, which will drive B, and consequently will cause the axle A on which the rope or chain is coiled to revolve, and thus to raise the object to be elevated.

After what has been explained respecting the relation of the power and weight, it will be evident that the efficiency of such an apparatus will altogether depend on the relative dimensions of the wheels and pinions. But in all cases it is apparent that the weight must bear a very large ratio to the power. A crane constructed upon this principle by M. Cavé, the well-known engine-maker at Paris, has been erected in the government

dockyard at Brest, in which the wheels have the following number of teeth:—

The wheel B, 66 teeth.
The pinion C, 11 teeth.
The wheel D, 54 teeth.
The pinion E, 9 teeth.
The wheel F, 54 teeth.
The pinion K, 9 teeth.
The pinion L, 9 teeth.

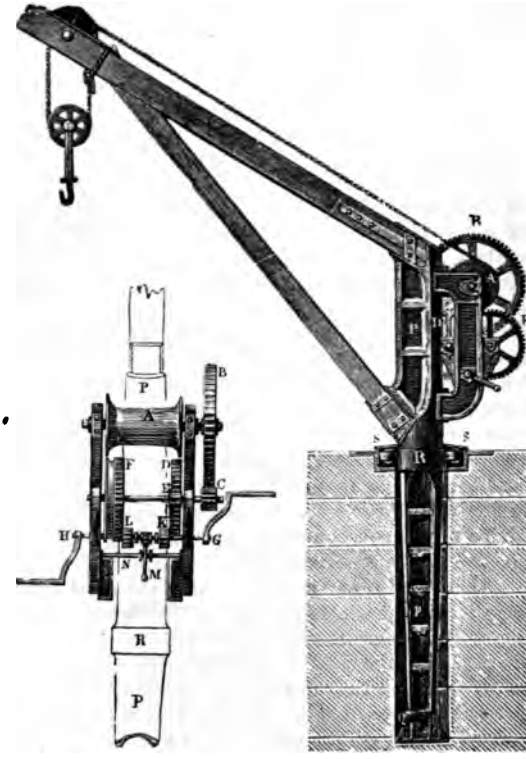

Fig. 312. Fig. 311.

By this crane a power of 20 lbs. applied to each of the winders G and H will be sufficient to raise 25 tons.

Either of the pinions K or L may be put into connection with

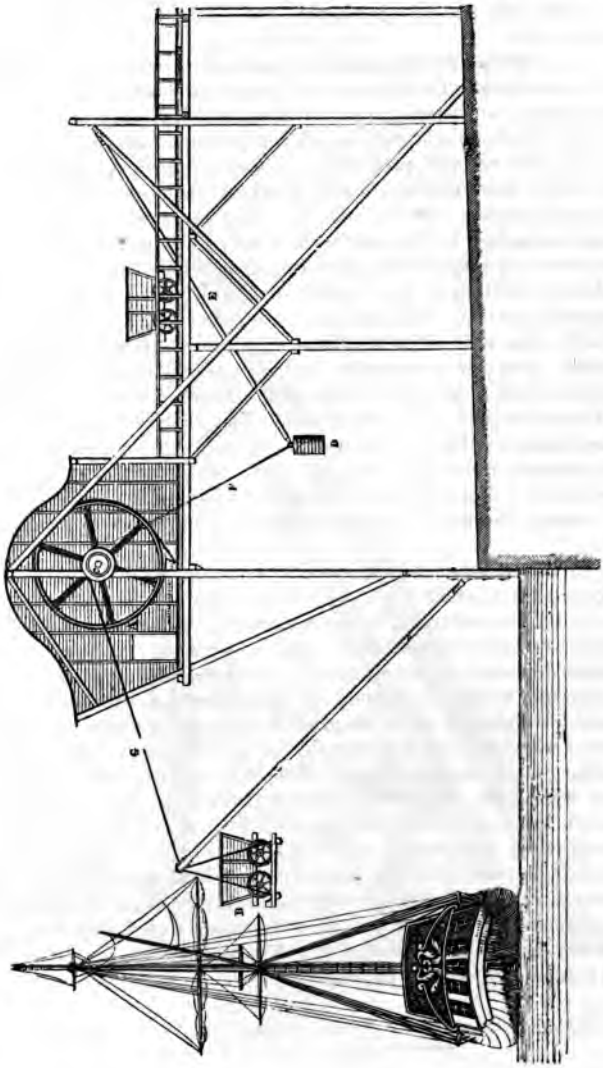

Fig. 313.

the wheels without the other, so that, when the weights to be elevated are not too great, the crane may be worked by one winch only.

339. **Drops for loading and unloading vessels at quays and wharves.**—To facilitate the process of loading and unloading vessels, an ingenious contrivance, which is very nearly self-acting, has been adopted, which is represented in *fig.* 313.

An iron railway, properly supported on the level of the quay or wharf, is continued a few feet beyond its edge, so as to overhang the water. To the end of a long lever which turns on a centre attached to the quay wall, nearly level with the deck of the vessel, is suspended a platform, upon which the waggon containing a portion of the matter with which the vessel is to be freighted stands. The end of this lever is connected by a rope G with the axle of a windlass C, established on the wharf. Coiled upon the same axle, but in the contrary direction, is another rope F, attached to the end of another lever E, to which a counter-weight D is suspended. The waggon B loaded preponderates a little over the weight D, and by this preponderance it descends upon the deck of the vessel, suitably placed to receive it. When the contents of the waggon are discharged in the vessel, the weight D preponderates, and draws the waggon up.

In this manner the weight itself of the goods to be loaded becomes the moving power by which the process is executed.

If, on the contrary, it be required to unload a vessel, the windlass C must be worked by some power adequate to raise the weight deposited in the waggon to the level of the wharf. No advantage would be gained in that case by increasing the counter-weight D, so as to give it a preponderance over the loaded waggon, since a power should in that case be applied to let the empty waggon down. But if it could be so managed that while one cargo was being unloaded another could be loaded, the work would be executed with a very small power, by adjusting the counter-weight D so as to balance, as nearly as possible, the weight of the loaded waggon both in ascending and descending. In that case power need only be applied sufficient to overcome the friction and other resistances of the machinery, and any small difference that might exist between the effect of the loaded waggon and the counterpoise.

CHAPTER XXII.

ENGINES WORKING BY IMPACT.

340. **Pile-engines.**—This class of machine is an example of the application of the momentum acquired by a heavy mass falling from a certain height as a mechanical agent.

Fig. 314.

Heavy weights are raised by the moving power to a greater or less height, from which they are let fall on the object upon which they are intended to act. The forms and magnitudes of such weights, and the mechanism by which they are elevated and detached, vary according to the purposes to which the machines are applied.

The form of machine properly called a Pile-engine is applied to drive piles into the ground, upon which bridges or other structures are intended to be erected.

One of the most simple and primitive forms of such a machine is represented in *fig.* 314, where D is the pile to be driven into the earth, the top of which is bound by a ring of iron to prevent it from being split or flattened by the blows ; A is the weight let fall upon it, which is usually a large block of cast iron called a ram, having two ears cast upon its sides, which play in vertical grooves formed in the two uprights C C, by which the ram is guided in its fall, and prevented from deviating from the vertical line. In this case the ram is elevated by human labour, the extremity of the rope being connected with several independent cords, each of which is pulled by a man ; the rope passes at the top of the framing over a fixed sheave B, from which it descends, and is tied to a ring at the top of the ram.

When the ram is elevated to the top of the framing, the men let go the ropes simultaneously by a signal. It is very important that the ropes should be all let go at the same instant, since if some men held on after others had let go they would be dragged up, and grave accidents would ensue. Where such machines are used, the men are accustomed to accompany their work by a sort of chant which regulates the moment of dismissal.

A more complex pile-driver is shown in *fig.* 315, where the rope which raises the ram, instead of being directly drawn by human power, is worked by a windlass having two winches, A A, by means of which two or four men can raise a ram of enormous weight.

The axle B, turned by the winches, has upon it a pinion E which works in the teeth of a large wheel attached to the drum upon which the rope is coiled. When the ram is raised by the windlass to the top of the framing, the pinion E is disengaged from the teeth of the large wheel by means of a lever C D E, which turns horizontally on a centre D, and is terminated in a fork at E, which, laying hold of the axle B, moves it horizontally so as to make it slide in its bearings, and thus to withdraw the teeth of the pinion E from those of the great wheel. The wheel being thus detached, the ram would be free to fall upon the

pile, but in that case it would draw the rope after it with a

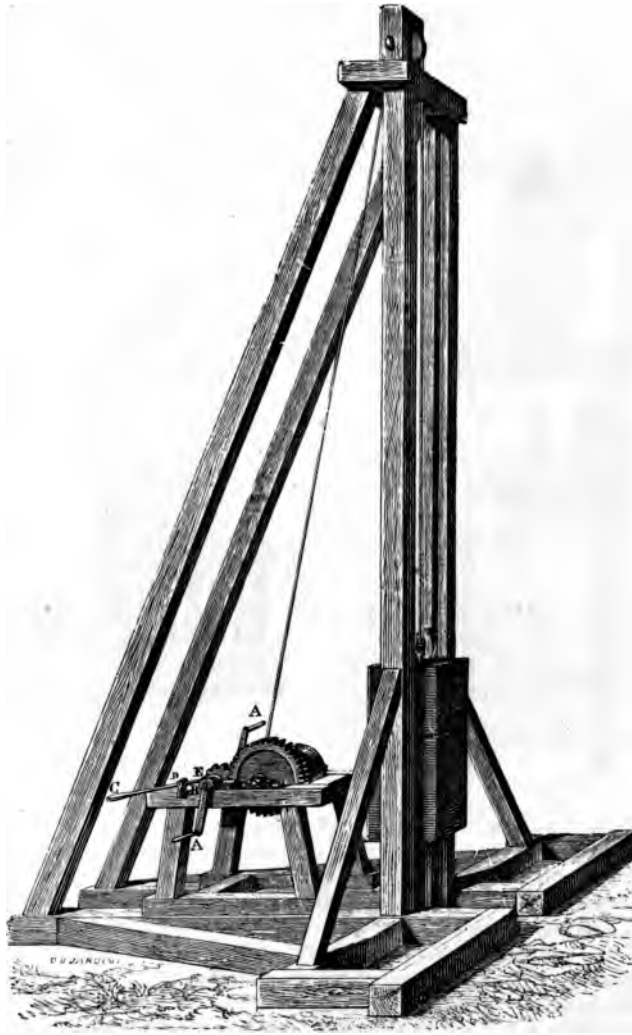

Fig. 311.

force and velocity which would soon wear it out; this is prevented by an expedient which is represented in *fig.* 316.

The rope, instead of being immediately attached to the head of the ram G, is attached to a piece of mechanism consisting of a sort of pincers H O K, the two arms of which play upon centres and are pressed outwards by the springs I I, which keep the points of the hooks K in close contact. The lower surfaces of these hooks are curved, so that, when they descend upon the ram, the ring forces itself between them, the springs I I yielding to the pressure. When the upper part of the ring has passed

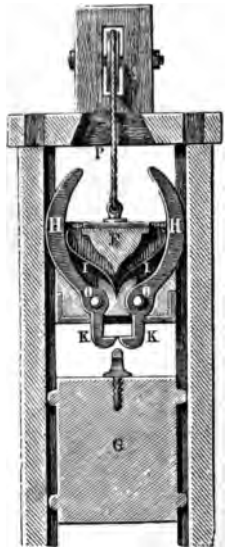

Fig. 316.

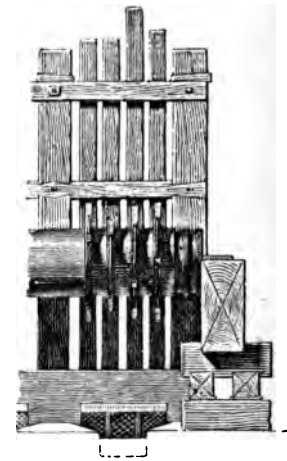

Fig. 317.

above the edges of the hooks, the latter are forced into the hole in the centre of the ring by the reaction of the springs, the ring then resting upon the upper surface of the hooks. The apparatus is thus firmly connected with the ram G, so that the whole can be raised by the windlass. When it is elevated to the top of the framing, the tops of the curved handles H H pass into an opening with oblique sides, which, pressing upon them, force the handles together, the springs I I yielding to the pressure; the hooks K K are thus separated, and the ring attached to the *ram being* no longer supported by them, the latter falls upon

the pile. Meanwhile, the block to which the apparatus H O K is attached, slightly preponderating over the resistance of the rope, slowly descends, and the hooks K K again engage themselves in the ring so that the ram can be again elevated by the windlass.

341. **Stamping-engine.**—One of the most extensively useful forms which engines constructed upon this principle assume,

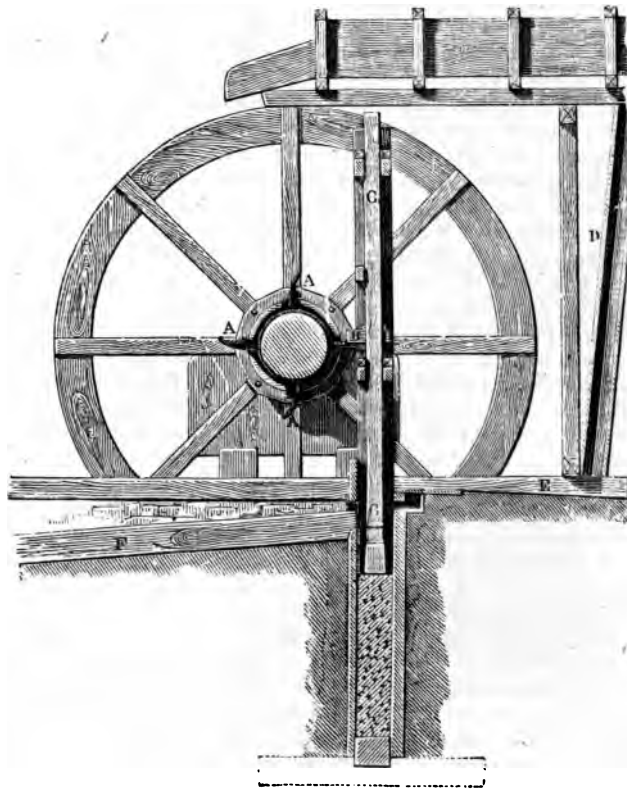

Fig. 318.

is that which is applied to the pounding of ore raised from mines. The raw ore consists, as is well known, of parts which are more or less metallic, mechanically mixed with earthy, stony, and non-metallic matter, from which it is separated by

impulse, restoring to it the force of which it deprived it while previously sliding upon its concave side.

The explanation here given of the action of this form of escapement is well calculated to render the conditions which all escapements should fulfil intelligible. These arrangements are primarily directed to the regulation of the movement of the wheelwork, so as to secure its uniformity. This will obviously be accomplished provided that the escapement, whatever be its form, lets a tooth of the wheel pass for each oscillation of the balance wheel. But owing to the friction of the axis of the balance wheel, and of the pallets on the teeth of the escapement wheel, and the resistance of the air, the range of its oscillations

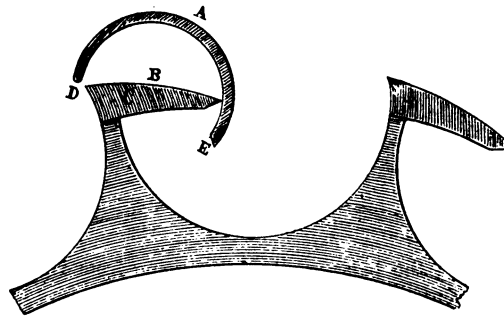

Fig. 364.

would be gradually diminished, so that at last it would not be sufficient to allow the successive passage of the teeth of the escapement wheel, and the watch would stop unless some adequate means be provided by which the balance wheel shall receive from the mainspring through the escapement wheel as much force as it thus loses. All escapements accomplish this by the peculiar forms given to the edges of the pallets and the teeth of the escapement wheel. In the present case, the object is attained by making the edge D round and the edge E inclined, and by giving to the teeth the form shown in the figure.

This form of escapement supplies a sufficiently good regulating power for the best sorts of pocket watches, and is attended with the advantages of allowing the works to be compressed within a very small thickness. It is the form most commonly used in the French and Swiss watches.

379. **Duplex escapement.**—The form of escapement used in the best English-made watches consists of an escapement

or water. When the cams descend, and strike the end of the handle B, the hammer A is raised, and as the motion is rapid, it

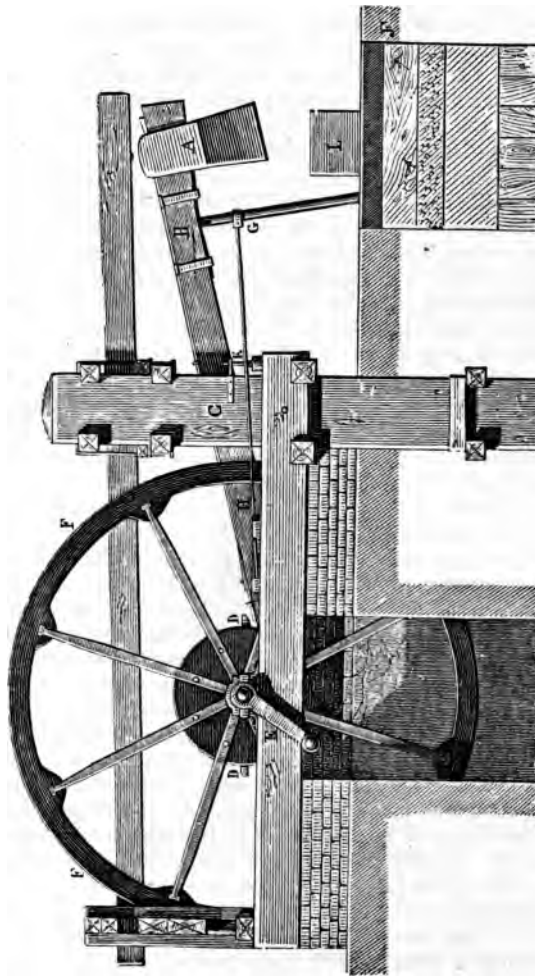

Fig. 319.

is jerked upwards against a horizontal beam of wood above it, by the elasticity of which it is repelled, and descending falls

upon the mass of red-hot iron to be forged, which is previously laid upon the anvil L. Since the axle driven by E encounters no resistance, except while the cams act upon the handle of the hammer, the machine would be unduly accelerated in these intervals of suspension of the resistance, were it not that a heavy fly wheel F F is mounted on the axle, which, receiving the force of the moving power, becomes a depository thereof, and combines with the moving power in acting upon the hammer when the next cam encounters the handle.

A workman holds a long iron rod, to the end of which the mass of red-hot iron to be forged is attached, and turns it between stroke and stroke of the hammer, from one position to another, so that it can be beaten in any desired manner.

A rod of iron, G, is governed by a lever H G, by which it can be brought at will under the handle of the hammer, so as to prevent it from falling on the anvil when it is desired to remove one piece of iron already forged and to replace it by another. During this process the axis continues to be driven, and the cams still act upon the hammer, but the hammer only plays between the rod G and the beam above it. When it is desired to recommence the forging on another piece of iron, the rod G is withdrawn and the hammer again allowed to fall.

The horizontal beam, which reacts by its elasticity on the heel of the hammer, adds nothing to its force, for if it were not present, the hammer would rise to a greater height, and would acquire an increase of force in its fall exactly equal to that which it receives from the reaction of the beam, or, more strictly speaking, a little more, the beam not being perfectly elastic. The advantage of the beam is that it makes the hammer return to the anvil in shorter intervals, and consequently to give a greater number of blows to the iron in a given time.

343. **The coining press.**—The first process in the fabrication of coin consists in the production of the proper composition of metal of which the coin is to be made, for the silver and gold used for coining, when absolutely pure, do not possess the requisite hardness,—a quality which is, however, easily imparted to them, by alloying them with a small proportion of baser metal. When thus properly alloyed, and formed into ingots or bars, they are submitted to the process of rolling, so as to be reduced to the form of plates of a regulated thickness,—an object which is attained with great precision by forcing them between rollers of tempered steel, placed at a distance one from the other equal to the thickness of the intended plate of precious metal.

The plates thus produced are then submitted to the action of circular punches of hard steel, by which circular discs are punched from them of the exact size of the coins to be made. These discs, which are called blanks, are then submitted to the coining press.

This press has been designed in various forms at different mints, but is generally constructed upon the principle of the screw.

The characters and figures to be raised upon the coin are first engraved with the greatest precision in sunk characters, or '*intaglio*,' upon circular surfaces of hardened steel. These are called *dies*, and two of them are of course required for each coin. These dies are placed in the press, one at its base, with its engraved face upwards, and the other attached to the end of a rod driven by the screw, with its engraved face downwards.

Fig. 320.

The blank is then placed upon the lower die, and the upper die is driven down upon it by the screw with a force sufficient to drive the metal of the blank into all the sunk characters of the two dies, in the same manner exactly as wax is forced into the sunk characters of a seal by the pressure of the hand.

Such being the general principle of the coining press, the details of its mechanism, which vary much in different places, may be illustrated by those of the press represented in elevation in *fig.* 320, the parts being shown on a larger scale in *figs.* 321 and 322.

The lever by which the screw is driven is a long metal bar

c c, having heavy knobs at the ends, and strong bunches of ribbon or cord, which the men who drive the press lay hold of,

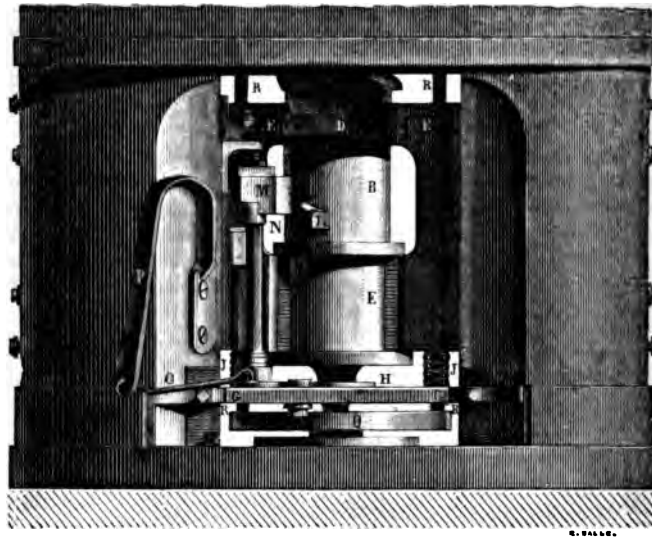

Fig. 321.

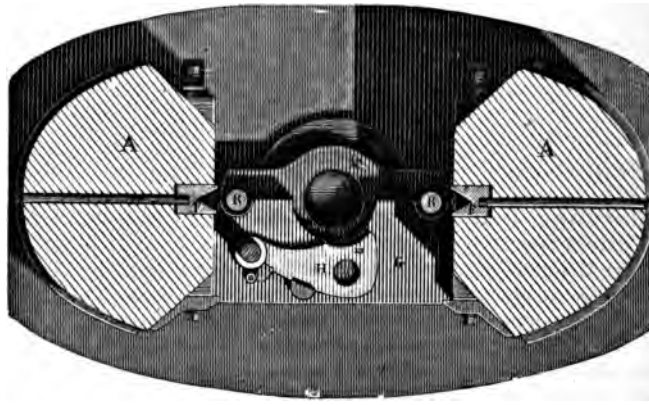

Fig. 322.

to impart the necessary motion to the screw. The masses c c, receiving thus from the hand a considerable velocity, acquire a

great momentum, the effect of which, transmitted to the upper die attached to the screw, is augmented in the proportion of the circumference described by the knobs C C, to the intervals between the threads; a proportion which, it is obvious, is always very great.

The base plate of the press, with the circular cavity to receive the lower die, is shown in *fig.* 322. Upon this plate is attached a piece H, which is moved horizontally by the mechanism M N L (*fig.* 321), so that when the screw rises after making the impression upon the blank, the piece H turns upon a joint at its extremity, moves towards the blank with a rapid jerk, throwing out the coined piece, and dropping into its place another blank to be coined by the next stroke of the press.

There are various other practical details in this mechanism, which it will not be necessary here to discuss, the general principle being rendered sufficiently clear from what has been stated above.

In some still more recent coining presses the screw action has been replaced by a mechanism constructed upon the principle of the knee-lever explained in art. 202. One of these presses, of the most recent construction and improved form, is represented in *fig.* 323, being that which was contructed by M. Thonnelier for the mint at Paris, where it is now worked.

The working shaft which imparts motion to the machine carries a large fly-wheel Z to equalise its motion. Upon this shaft a short crank G is connected with a lever H by a rod F. The lever H is made to vibrate on the centre *a* by the motion of the crank, and a corresponding oscillatory motion is imparted to the short arm a *b*, which is at right angles to the long arm H. This short arm is connected with a piece O, which carries the upper die, by the rod I, and the knee-joint at *b* transmits to the upper die an intense pressure, upon the principle already explained in art. 202.

This action differs essentially by substituting pressure for a blow, and is consequently more favourable to the preservation and durability of the mechanism.

The rod I acts upon the piece O by a ball resting in a corresponding socket, and this piece O is pressed upwards against the ball by heavy counter-weights N acting upon the lever O J *c* through the intervention of the lever M and the vertical rod L. The lower die is placed in a cavity in the upper part of the fixed block Q, and the blanks are placed upon it and removed, after being coined, by mechanism which does not differ much in principle from that already described, and which is worked by the

levers s′ and s, the latter of which receives an oscillating motion by a pin *f*, which is moved along a sinuous groove formed in a

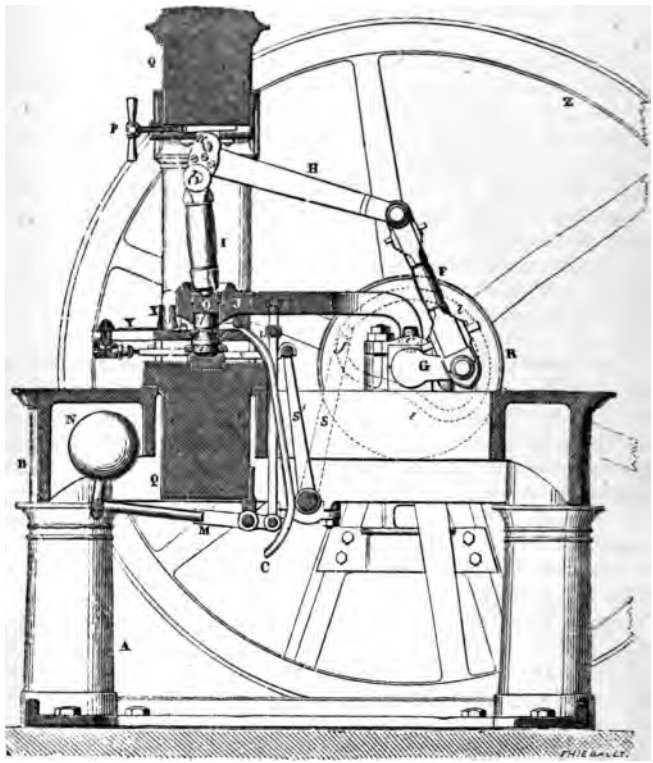

Fig. 323.

central plate fixed upon the main shaft, and turning with the fly-wheel. This groove is indicated in the figure by a dotted line.

CHAPTER XXIII.

BORING, PLANING, SAWING, AND SCREW AND TOOTH CUTTING ENGINES.

344. **Boring tools.**—All instruments of this class, great and small, present examples of continued rectilinear, produced by continued circular motion. The point of the boring tool is driven continually in one direction by means of a circular motion imparted to a disc or lever attached to it.

The common bow-drill (*fig.* 324) presents a familiar example of this.

Fig. 324.

The bit and brace (*fig.* 325) presents another example. The handle is turned rapidly, while the bit is pressed with its point upon the object to be bored.

The form of the point of the bit varies with the material to be bored, and the magnitude and shape of the hole. Different forms of bits are shown in *figs.* 326-332.

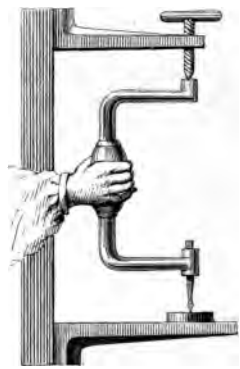

Fig. 325.

Fig. 326.

Fig. 327.

Fig. 328.

Fig. 329.

Fig. 330.

Fig. 331.

Fig. 332.

For heavier work, drilling and boring machines, with self-acting mechanism, and driven by steam- or water-power, are constructed on a large scale. Machines of this kind are used for drilling the holes in the end plates of large tubular boilers, in the heads and flanges of steam cylinders, and other heavy work. The principle is the same as that of the ordinary hand-drill, the details of the mechanism for imparting the motion, for changing the position of the drill or of the work, or both, and the scale of the machine, only being different.

345. **Planing machine.**—This machine presents an example of the production of a continuous rectilinear by a continuous circular motion. The purpose to which it is applied is to render the surfaces of pieces of metal perfectly plane.

The block or plate of metal to be planed is firmly attached to a bed B (*fig.* 333), which is connected with a screw S S′, which extends longitudinally from end to end of the machine, and which is turned by a winch A H, or other similar contrivance. The screw plays in nuts attached to the bottom of the bed B; and as it turns in sockets in the ends S S′ of the frame, and has therefore no progressive motion, it must impart a progressive motion to the frame B, and consequently to the plate of metal to be planed. By each revolution of the screw the metal to be planed is thus advanced through a space equal to the interval between the threads. This motion may be rendered as slow as is necessary by regulating the motion of the winch A H. A frame D is fixed horizontally in grooves in the upright pieces E E′. In this frame is inserted a screw F′, worked by a winch G G′. This screw carries a piece I, in which is inserted a cutter C, formed of hardened steel, presenting its point in a direction

contrary to that in which the metal to be planed is moved. This cutter is raised and lowered at pleasure by a winch L. The piece I which carries the cutter is moved transversely, by means of the winch G G′ turning the screw F′.

When, by means of the winches G′ and L, the position of the cutter C is properly adjusted, the surface of the metal is moved slowly against it, by turning the winch H. The cutter thus acting detaches from the metal a narrow shaving, and forms in it a very narrow and shallow groove.

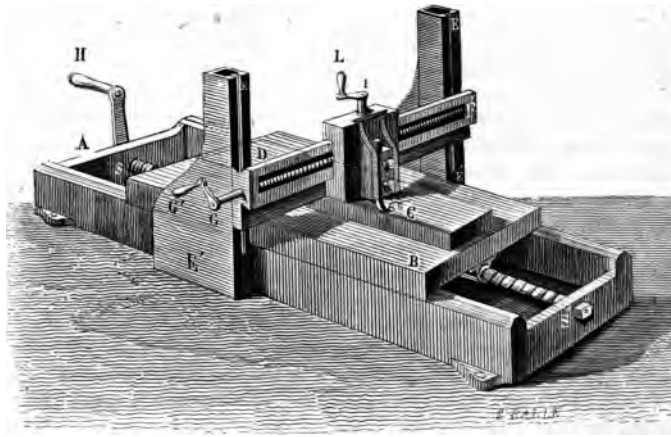

Fig. 333.

When the entire length of the metal has thus been drawn under the cutter, the action of the machine is suspended ; the cutter is raised by the winch L, and the metal advanced towards the end S′ of the frame by turning the winch H in the proper direction.

The cutter C is now advanced transversely through a very small space towards E, by turning the winch G′. It will be understood that this motion is susceptible of any required degree of minuteness, since an entire revolution of the winch G′ would advance the cutter through no greater space than the interval between the threads of the screw F′, the threads of which may be made fine, and the winch G′ may be turned only through a small fraction of an entire revolution. The machine is now prepared to recommence its operation ; and, by turning H continuously, another groove will be cut in the surface of

by the intermediate pinions and wheels *f*, F, *g*, G, *h*, H, and *i*. The wheel I drives the pinion *k*, upon the axle of which is fixed the regulator K. This regulator is a fly, a side view of which, upon a larger scale, is shown in *fig.* 371.

The pinion, which is made to revolve by the wheel W, gives a motion of rotation to the fly A A′ B′ B, which consists of a thin rectangular plate of metal, along the centre of which the prolongation M L of the axle of the pinion is attached. The flat surfaces of the fly, revolving more or less rapidly, strike against the air, which resists them with a force which increases in the

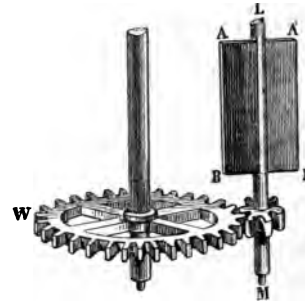

Fig. 371.

proportion of the square of the velocity of the rotation. Thus, if the velocity of rotation be increased in a twofold ratio, the resistance to A A′ B′ B is increased in a fourfold ratio; if the rotation be increased in a threefold ratio, the resistance is increased in a ninefold ratio, and so on. It is evident, therefore, that by this very rapid increase the resistance to the motion of the train must soon become equal to the descending force of the weight, and then the motion will become uniform; for if it were to increase, the resistance would exceed the force of the weight, and would slacken the rate of motion; and if it were to decrease, the resistance being less than the force of the weight, the latter would accelerate the motion. In either case the motion would immediately be rendered uniform.

Projecting from the face of the wheel H (*fig.* 372) there is a small pin which rests upon the end *m* of a lever *m n*, which turns upon the centre *n*. The lever *m n*, when in this position, stops the motion of the striking train. Behind the same lever *m n*, and projecting from it, there is another piece, which in the position represented in the figure rests in a notch of the wheel *o p*, lying behind the striking train, and indicated

steel, having the teeth of a saw formed upon its edge, which is fixed upon a shaft to which a very rapid motion of revolution is imparted by the moving power. The saw in this case remaining stationary in its position, the object to be divided by it is urged against its teeth, with a regulated pressure, and constantly advanced, as the incision is made. When it is applied to divide wood, for example, the blade of the saw, being vertical, rises through a slit in a level table, at which the operator stands, and upon which he places the block of wood to be cut, pressing it gently against the teeth of the saw, and advancing it continually until it is completely divided.

Such saws are used for squaring the ends of the iron rails used in railways. The saws adapted for this purpose are usually about 40 inches in diameter and 1-10th of an inch thick, turning at the rate of about 900 revolutions per minute, or 15 per second. The iron rail is then applied to them red hot, and the lower part of the saw is immersed in a trough of water to keep it cool.

Straight saws worked by machinery consist of an oblong rectangular blade, like that of a common knife mounted upon a rectangular frame, with screw adjustments, for the purpose of giving the necessary tension to the blade. When these are applied to divide timber into boards or planks, they are placed vertically, and are moved with an up and down motion, exactly similar to that given to them when worked by hand in a saw pit. Instead, however, of advancing against the timber, as in the latter case, the timber being stationary, the timber is advanced against the saw, the saw being stationary. Saws worked in this way make from 120 to 150 strokes per minute, and cut at the rate of from one-tenth to half an inch per stroke, according to the softness or hardness of the wood.

To render intelligible this important application of the saw, we shall here describe one of the most improved forms of the saw mill adapted to cut veneering from mahogany and other precious ornamental woods.

The moving power, as is customary in mills for all purposes, imparts revolution to a shaft or shafts, which are carried along the rooms under the ceilings in which the machines are established. Upon these shafts over each machine a drum A (*fig.* 334) is fixed, which revolves with the shaft. This drum is connected by an endless band with the axle B of a fly-wheel E E, and is kept tight by a bar D C fixed upon the centre D, and pressing against it with its weight by a roller C. When it is desired to suspend the motion of the machine, it is only necessary to

raise the bar D C, when the band becoming slack, the motion of A will not be imparted to B.

A connecting rod G F is attached by a pin F to one of the arms of the fly-wheel E E, at a distance from the centre equal to half the stroke intended to be given to the saw. The other end G is attached in like manner by a pin to the frame of the saw, which is movable in horizontal grooves, so that by each revolution of the fly-wheel the pin G, and with it the saw, is moved

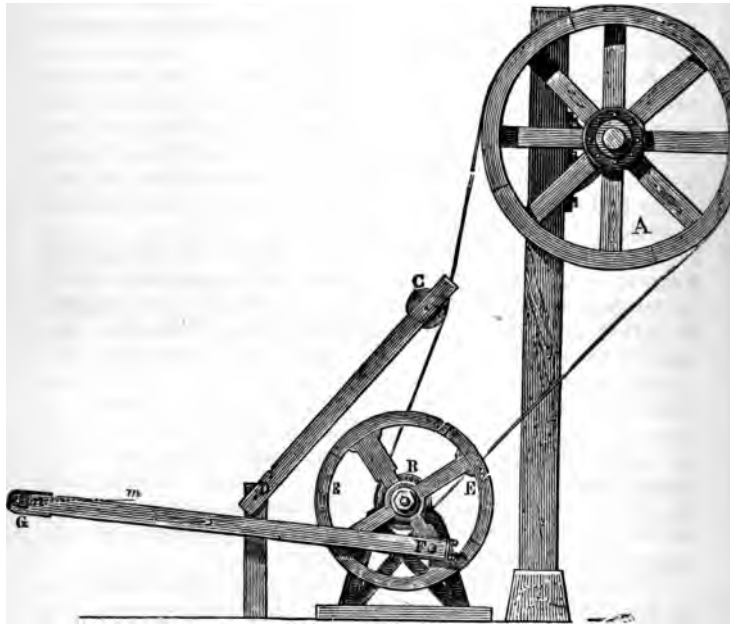

Fig. 334.

backward and forward through the space *m n*, which is equal to twice the distance of F from the centre of the fly-wheel.

A view of the sawing machine, driven by the rod F G, is given in *fig.* 335, where the end G of that rod appears broken off. One side of the horizontal frame which carries the saw is shown at H H, the blade of the saw being behind this, and therefore not visible in the figure. A rod of strong wire is seen in front of H H, which passes through blocks at each end of the frame, *and is* secured by nuts screwed upon it, by means of which the

rod is shortened so as to increase the tension of the blade of the saw. This blade is kept exactly vertical, the teeth being downwards, by a plate of iron, K, the lower edge of which is levelled towards the saw.

According to what has been explained, the saw receives an alternate motion right and left through a space equal to *m n* (*fig.* 334). If, therefore, the wood to be cut be pressed upwards

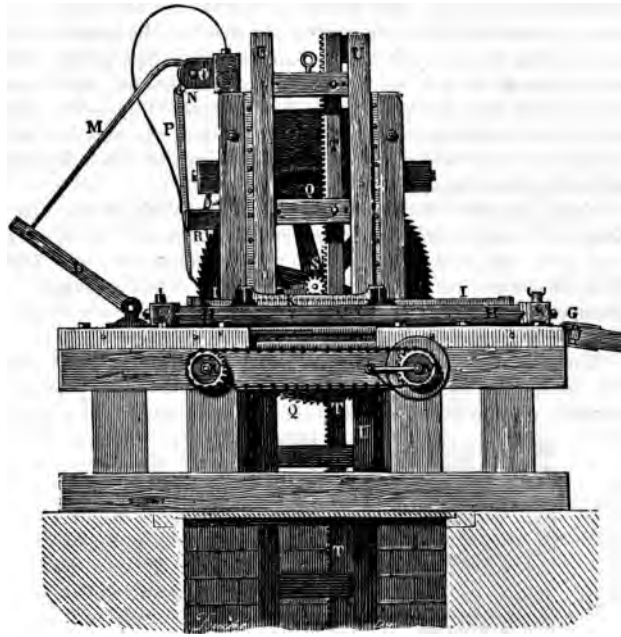

Fig. 335.

against its teeth in a suitable manner, the desired separation will be effected. We shall now explain how this is accomplished.

In *fig.* 335 there is a vertical frame U U fixed behind the saw, to which a rack T T is attached. In the teeth of this rack a pinion S, fixed upon the axle of a rachet wheel Q Q, works. In the teeth of this rachet wheel are engaged two catches : one R, playing on a centre attached to the frame, and urged against the teeth by a spring ; and the other fixed upon the lower end of a bar P which turns on the centre N, and which is urged against the teeth by a spring which appears in the figure. A lever L

works upon a centre attached to the sliding frame of the saw, and which, therefore, moves right and left with it. This lever imparts a corresponding motion to M, and the latter to N. By this means the short lever N moves P alternately upwards and downwards with each stroke of the saw. When P is raised, it draws the hook at its lower end upwards from the tooth of the ratchet wheel in which it was engaged; and when it has risen to the limit of its play, the spring presses it over the next tooth. When P descends, the hook presses the tooth of the wheel down; and, causing the wheel to move through a small space, the other catch R passes to the next tooth. In this manner with each stroke of the saw the ratchet wheel is advanced one tooth; and the pinion S receiving a corresponding motion, the rack T T, and the frame U U to which it is attached, are raised through a corresponding small space.

Now, between this frame U U and the blade of the saw a space sufficiently wide is left for the introduction of the block of wood from which the sheets of veneering are to be cut. That block, therefore, being so introduced, is glued to the frame U U, its upper edge at the commencement of the operation being a little below the teeth of the saw.

The framing which carries the saw is traversed by two screws V V, connected by an endless chain, both of which receive a common motion from one handle which appears in the figure, attached to the right-hand screw. By means of these screws the frame U U, with a block of wood attached to it, can be moved through any desired space to or from the saw. In the commencement of the operation the upper edge of the block is by this means brought under the teeth of the saw, in such a position that a sheet of veneering of the requisite thickness shall be cut from it.

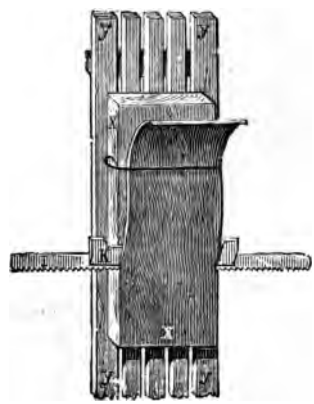

Fig. 336.

These arrangements being made, and the machine being put in motion by throwing the bar D C (*fig.* 334) on the band, the work proceeds, and the saw detaches a sheet of veneering. The manner in which the saw thus operates is shown more conspicuously *in fig.* 336, where x x is the block of wood, and *y y* some bars,

by means of which it is fastened to the frame U U (*fig.* 335). It will be remembered that by each stroke of the saw, the frame U U, and with it the block X X, is raised through a small height by the action of the levers L M N P. The mechanism is so adjusted that this height shall be equal to the depth of the wood through which the saw can make an incision at each stroke. The bevelled bar K K separates the sheet of veneering from the block, so as to prevent the saw from being locked between them; and as this sheet is too thin to support its own weight, it is sustained by a loop of wire, a little below the top of the block, as shown in the figure.

In the practical operation of the machine, the drum A revolves at the rate of about one revolution per second, and if its diameter be 5 times that of the axle B, the latter will revolve 5 times per second, and the same number of strokes will be made per second by the saw. If the wood from which the veneering is cut be mahogany, the depth cut by each stroke will be about the fiftieth of an inch; and such a saw will be capable of detaching about 400 square feet of veneering per day.

After each sheet of veneering is cut off, the action of the machine is suspended by raising the bar D C, and the frame U U is then lowered by disengaging the two catches from the ratchet wheel, until the upper edge of the block X X is brought below the teeth of the saw, and the screws V V are then turned until the upper edge of the block is brought sufficiently within the teeth of the saw to detach another sheet of veneering of the necessary thickness.

It is found that the thickness of wood converted into sawdust by this process is about the fiftieth of an inch, and if the thickness of the sheet of veneering be also the fiftieth of an inch, which it may be, and generally is, it is evident that 50 per cent. of the wood will be wasted in the process.

It is obvious that in the case of highly valuable woods, which are precisely those that are cut in the thinnest sheets, it is highly desirable to avoid a waste so extensive, and various attempts have been accordingly made from time to time to substitute the plane for the saw in the cutting of veneering, so that the wood would be detached in thin shavings without any waste at all. It cannot be said, however, that as yet these attempts have been attended with any practical success.

347. **Screw-cutting engine.**—This engine affords another example of the reciprocal production of circular and rectilinear motion. It is represented in *fig.* 337.

The wheel A imparts rotation to the cylindrical rod S S′, upon which it is desired to cut a thread. A toothed wheel B is fixed

upon the rod S S′, and revolves with it. This wheel imparts revolution to a smaller toothed wheel C, which is fixed upon the end of a fine screw by which a progressive motion is imparted to the piece H, in which a cutter D is inserted. It is evident that the proportion between the progressive motion imparted to the cutter D, and the rotatory motion of the rod S S′, is regulated by the proportion of the wheels B and C, and the thread of the screw driven by C.

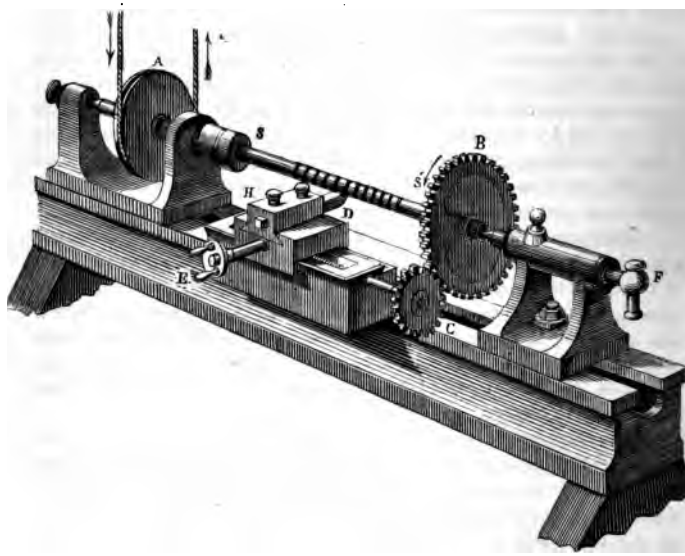

Fig. 337.

According to this combination, while the cutter D moves slowly from S to S′, the rod S S′ revolves, and it is evident that the incision made by the cutter in the rod, will be of a spiral or helical form, and that the interval between the threads of the screw thus produced will be equal to the space through which the cutter D advances during one revolution of the rod S S′.

The depth of the incision made by the cutter is regulated by a screw E, which advances or withdraws the cutter at pleasure.

348. **Engine for cutting the teeth of wheels.**—This engine furnishes an example of the combination of a continuous *with* an intermitting circular motion. It is represented in *fig. 338.*

The disc A, upon which the teeth are to be cut, is fixed upon a rod C, which is turned by a disc B, the circumference of which is divided into equal intervals by holes, into which a point projecting from the end of a lever D, can be inserted so as to render the disc B, and consequently A, stationary. The teeth are cut in A, by means of a circular cutter E, driven by a band F, receiving its motion from any moving power. The piece carrying the cutter is moved longitudinally by a screw driven

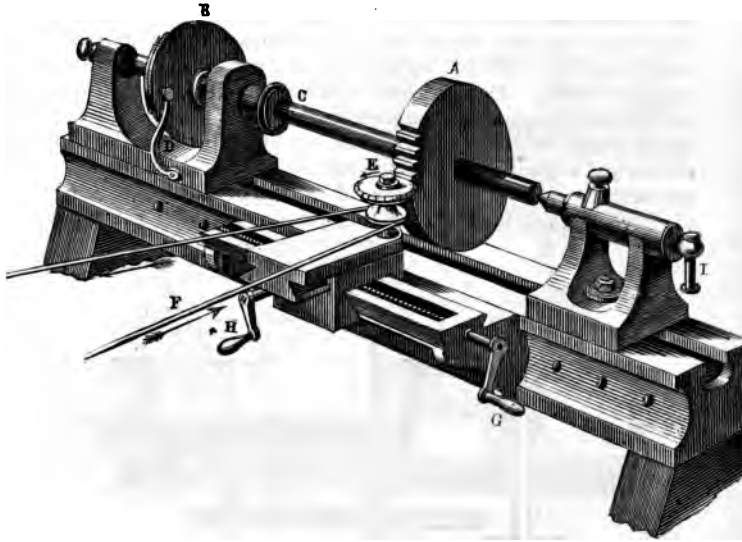

Fig. 338.

by a winch G, and it is moved to and from the wheel to be cut by a screw driven by the winch H.

The form of the teeth is determined by the shape of the cutter E, and their depth by the extent to which the incisions are continued by acting on the winch H.

The wheel A is placed upon the shaft and removed from it by means of the lever I, by which the shaft C can be removed from the frame at pleasure.

CHAPTER XXIV.

FLOUR MILLS.

349. **Process of grinding.**—The process of converting grain into flour is executed by passing the grain between two cylindrical stones, having a common vertical axis, smooth horizontal surfaces, and mounted so that the distance between these surfaces is less than the thickness of the grain. The lower stone is stationary, and the upper kept in revolution round its axis, with a regulated velocity, by some moving power, such as wind, water, or steam. The grain is let fall in small and regulated quantities through an opening which surrounds the axis in the upper stone; it descends thus into the space between the two stones, and while it is broken between their surfaces, its particles are carried round by the rotation of the upper stone, and being slightly affected by the central force produced by the rotatory motion, it recedes gradually from the centre as it revolves, each particle describing a sort of spiral. The particles, at length arriving at the edge, fall into a groove made to receive them, from which they are discharged through openings provided for the purpose into a proper receptacle.

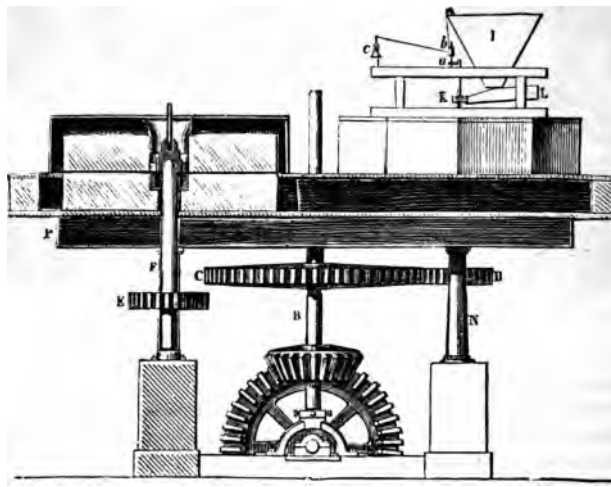

Fig. 3_9.

Such being the general principle upon which flour mills are constructed, their details and mode of operation will be better understood by reference to *figs.* 339 and 340.

The horizontal shaft A (*fig.* 340) receives a motion of rotation from the moving power, whatever it may be, outside the mill; it imparts by the bevelled wheels a motion of rotation to the vertical shaft B, upon which a large toothed wheel C is fixed; this wheel works into small toothed wheels D and E (*fig.* 339), fixed upon two vertical shafts N and F. These wheels D and E are capable of moving through a limited space with a sliding motion of the

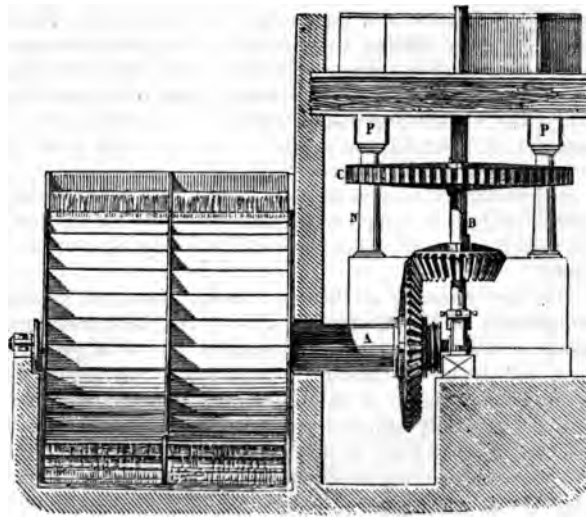

Fig. 340.

shafts N and F, so as to be thrown into or out of connection with the great wheel C at pleasure. The moving power may thus be made to impart revolution to either or both of the shafts N and F. These shafts pass respectively through the centres of two pairs of mill-stones, that through which F passes being shown in section in the figure, and that through which N passes being surrounded by its octagonal wooden casing, on the top of which is placed the apparatus by which grain is supplied to the stones.

The shafts pass through holes in the centre of the lower stones, in which they turn freely, without moving these stones,

and the space around them is stopped up with leather so as to prevent the grain from falling between. The upper stone is fixed upon the top of the shaft and turns with it; a space, however, being left open between the shaft and the stone to let in the grain. The grain is supplied from a funnel-shaped box I, with an opening in the bottom, of regulated magnitude, through which it falls into the inclined spout L, which receives a slight oscillating motion right and left, from pieces which project from the rod K. By this motion the grain is shaken out, and falling little by little through the central opening in the upper stone, passes between the stones, where it is ground.

Mill-stones have been generally formed of simple blocks of stone, but these having been found to be often defective, a better class of stones have been produced by cementing together, by means of plaster, chosen pieces of stone of good quality, the whole being bound together by hoops of iron. The diameters of the mill-stones, which were formerly from 5 to 6 feet are now generally reduced to 3½ feet.

Each pair of stones is capable of grinding about forty-two imperial bushels in twenty-four hours, and it is found that the best velocity for the stones is about seventy revolutions per minute.

The mere grinding of the grain is not, however, all that is accomplished in the flour mill. The grain, as is well known, consists of the material which constitutes the flour, properly so called, and the husky envelope, which forms the bran, so that the immediate product of the mill-stones is a mixture of flour and bran, generally called '*whole meal.*' To produce fine flour from this, it is necessary to separate the bran from the flour, and this process, which may be executed with a greater or less degree of perfection, according to the fineness of the flour intended to be produced, is accomplished in the mill by machinery driven by the same power which moves the mill-stones. For this purpose, proper shafts are connected with the main shaft driven by the power, and their motion is conveyed to those parts of the mill in which is an apparatus consisting of wire sieves of more or less fineness. The whole meal, as it comes from the stones, is passed over a series of these sieves, through the meshes of which the flour falls, the bran being retained.

CHAPTER XXV.

CLOCK AND WATCH WORK.

350. **Time-measurers a necessity of civilised life.**—After the supply of the absolute necessaries of physical existence—food, clothing, and lodging—one of the first wants of a society emerging from barbarism, is the means of measuring and registering time. In civilised society, all contracts for labour, and for all kinds of service, are based upon time. Even in the cases of the highest public functionaries, and where the service rendered is purely social and intellectual, still it is regulated, limited, and compensated with relation to time. Time-measurers or chronometers were therefore among the earliest mechanical and physical inventions.

Although nature has supplied visible signs to measure and to mark the larger chronometric units, such as days, months, and years, she has not furnished any corresponding measures of the lesser units of hours, minutes, and seconds. There are no visible marks on the firmament by passing from one to another of which the sun can note the hours, still less are there any signs for minutes or seconds. These subdivisions are therefore merely artificial and conventional, and, to measure and mark them, artificial motions must be contrived.

351. **Sun-dials.**—Rough approximations were first made to the chief divisions of the day, by observing the apparent motion of the sun from rising to setting. Thus the direction of the meridian, or of the south, being once known, and marked by some fixed and visible object, the time of noon was known by observing when the sun had this direction. The hours before and after noon were roughly estimated by the position of the sun between noon and the times of its rising and setting. Greater precision was given to this method, by erecting a wand or gnomon, the shadow of which would fall upon a level surface, in a direction always opposite to that of the sun. Thus, after sunrise, the shadow would be inclined towards the west, the sun being then towards the east. From the moment of sunrise until noon, the shadow would move continually nearer and nearer to the direction of the north, and at noon it would have exactly that direction. From noon to sunset the shadow would be more and more inclined towards the east.

It is evident, however, that such a dial would not afford

uniform indications at all seasons of the year, so that the hour-lines of the shadow determined in spring, for example, would not show the same hours in winter as in summer. Without much astronomical knowledge, it is easy to be convinced of this. At the equinoxes, the sun rises and sets at six o'clock, and at the east and west points precisely; and, therefore, at these seasons, the six o'clock hour-lines of such a dial would be for the morning due west, and for the evening due east. But on the first day of summer (June 21), the sun rises and sets at points of the horizon very much north of the east and west points, and at six o'clock in the forenoon and afternoon its bearing is north of the east and west points.

A dial so constructed at any given place would be useless as a time indicator. To render it useful it would be necessary that the shadow of the style should fall in the same directions at the same hours at all seasons of the year. Now, to attain this object, the style must be not vertical, but must be directed to the celestial pole. It is easy to comprehend that in that case a plane passing through the style and the sun would always be carried round the style with an uniform motion by the diurnal motion of the sun, and that at all seasons this plane would at the same hours have the same position.

It is for this reason that the gnomon of sun-dials is placed at such an inclination with the plate of the dial, that when the dial is properly set the gnomon will be directed to the north pole of the heavens, and being so placed, its shadow will fall upon the same lines of the dial at the same hours, whatever be the season of the year.

352. **Position of gnomon.**—It is evident, therefore, that dials must be differently constructed for places which have different latitudes. It is shown in astronomy that the elevation of the celestial pole is equal to the latitude of the place, and consequently the inclination of the gnomon of a sun-dial must be also equal to the latitude of the place where the dial is intended to be set. It follows, therefore, that a dial constructed for London would not be suitable for York, Newcastle, or Edinburgh.

The position of the plate of the dial upon which the shadow of the gnomon is projected is quite unimportant. All that is really important is the direction of the gnomon, which must always be that of the celestial pole, whatever be the position of the plate of the dial. Thus the plate of the dial may be either horizontal, vertical, or oblique. Its position will depend upon the place where it is to be erected. If it be in an open space,

as in a garden or field, having a clear exposure on all sides, it will be generally most convenient to make it horizontal; and hence, in such cases, it is usual to fix it upon the top of a column of three or four feet high, so that it may be easily observed by a person of ordinary height standing near it. Sometimes it is convenient to place it upon the wall of a building, such as a church. A wall with a southern exposure is in that case the most convenient; but to indicate the hours of the early morning in the spring and summer, an eastern exposure would be required, and to indicate those of the late evening, a western exposure would be necessary.

Where these vertical dials are erected, it is therefore frequently the practice to establish them at the same time on different walls of the same building.

Whatever be the position of the plate of the dial, the position of the hour-lines upon it is a matter of mere technical calculation, for which the formulæ and principles of spherical trigonometry are necessary, but which is not attended with any difficulty.

It must, however, be observed, that generally the hour-lines are inclined to each other at unequal angles, as may be seen by inspecting any ordinary sun-dial. There is one, and one only, position which could be assigned to the plate of the dial, such that the hour-lines would make equal angles with each other. That position would be at right angles to the gnomon, and a dial so constructed would be suitable to any place, whatever be its latitude. All that would be necessary would be to set it so that the gnomon would be directed to the celestial pole. The sun, however, would shine upon the upper or north side of it during the spring and summer, and on the lower or south side during the autumn and winter. It would, therefore, be necessary that it should be marked on both sides with hour-lines, and that a gnomon should be fixed on both sides.

The name dial is derived from the Latin word *dies*, a day, and the invention and use of the instrument as a time indicator is very ancient. According to Herodotus, the invention came to Greece from Chaldæa. The first dial recorded in history is the hemisphere of Berosus, who is supposed to have lived 540 B.C.

353. **Clepsydra.**—The first attempts to measure time by motions artificially produced, consisted in arrangements by which a fluid was let fall in a continuous stream through a small aperture in the pipe of a funnel, the time being measured by the quantity of the fluid discharged. The CLEPSYDRA, or

water-clock, of the ancients, was constructed upon this principle. This and the sun-dial were the only instruments contrived or used by the ancients for the measurement of time.

Clepsydras were contrived by the Egyptians, and were in common use under the reign of the Ptolemys. In Rome, sun-dials were used in summer and clepsydras in winter. These instruments, though subject to very obvious defects, were nevertheless, when skilfully used, susceptible of considerable accuracy, as may be easily conceived when it is stated that, before the invention of clocks and watches, they were the only chronometric instruments used by astronomers. The chief sources of their irregularities were the unequal celerity with which the fluid is discharged, owing to its varying depth in the funnel and its change of temperature.

354. **Sand-glass.**—The common hour-glass (*fig.* 341) comes under this class of chronometric instruments, but is the most imperfect of them. Nevertheless, for certain purposes, it is even now, advanced as we are in the application of science to the arts, still found the most convenient chronometer. The process of ascertaining a ship's rate of sailing or steaming by means of the log affords an example of its use. One man holds the reel from which the line runs off, while another holds the sand-glass and gives the signal when the sand has run out. The number of knots run off from the reel is then the number of miles per hour in the rate of the vessel. The intervals between the knots, the quantity of sand in the glass, and the aperture through which it falls, are so adapted to each other as to give this result.

Fig. 341.

355. **Mercurial time-measurer.**— Notwithstanding the great perfection to which the art of constructing chronometers has attained, an apparatus was not long since proposed by the late Captain Kater for the measurement of very small intervals of time, fractions of a second, for example, which is a modification of the clepsydra. A quantity of pure and clean mercury is poured into a funnel with a small aperture at its apex, so that a stream of the quicksilver shall fall through it. The flow is rendered uniform by keeping the mercury in the funnel at a constant level. The apparatus is intended in scientific researches

to note the exact duration of phenomena, and it is so managed that the stream issuing from the funnel is turned over a small receiver at the instant the phenomenon to be observed commences, and is turned away from it the instant the phenomenon ceases. The mercury discharged into the receiver is then accurately weighed, and the number of grains and parts of a grain it contains being divided by the number of grains which would be discharged in a second, the number of seconds and the parts of a second which elapsed during the continuance of the phenomenon is found.

356. **Clocks and watches.**—For the purposes of civil life, as well as for the more precise objects of scientific research, all these contrivances have been superseded by clocks and watches, which are now so universal as to constitute a necessary article of furniture in the most humble dwellings, and a necessary appendage of the person in all civilised countries.

All varieties of this most useful mechanical contrivance include five essential parts.

I. A moving power.

II. An indicator, by whose uniform motion time is measured.

III. An accurately divided scale, upon which the indicator moves and by which its motion is measured.

IV. Mechanism, by which the motion proceeding from the moving power is imparted to the indicator.

V. A regulator, which renders the motion imparted to the indicator uniform, and which fixes its velocity at the required rate.

Thus, for example, in a common clock, the moving power is the weight suspended by cords over a pulley fixed upon the axle of a wheel, to which the weight in descending imparts a motion of rotation. The indicator is the hand. The scale is the dial-plate upon which the hours, minutes, and sometimes the seconds, are marked by equal divisions, over which the point of the hand moves. The mechanism is a train of wheel-work, so constructed that the rate of rotation of the last wheel, upon the axle of which the hand is fixed, shall have a certain proportion to the rate of rotation of the first wheel, upon the axle of which the weight is suspended. And if, as is generally the case, there be two or three hands, then the wheelwork is so constructed that while one of the hands makes one revolution, another shall make twelve revolutions, and the third shall make sixty revolutions during a single revolution of the latter, and, therefore, seven hundred and twenty during a single revolution of the former.

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